"Knowing the path is good but not enough, walking the path with determination leads to destiny"

AS Edexcel
Paper 1 (WMA11)
MARK SCHEME FOR CLASSIFIED QUESTIONS

Contents

Topic 1: ALGEBRA

Indices and surds:

1.

1	$a = 162$	B1
	$b = 5$	B1
	$c = 12$	B1
		(3 marks)

2.

2.	Attempts both sides as powers of 3 $\quad \frac{3^x}{3^{4y}} = 3^3 \times 3^{0.5} \Rightarrow 3^{x-4y} = 3^{"3.5"}$	M1
	Sets powers equal and attempts to makes y the subject : $x - 4y = "3.5" \Rightarrow y = ...$	dM1
	$y = \frac{1}{4}x - \frac{7}{8}$	A1
		(3) **(3 marks)**
Alt1	Multiplies by 3^{4y} first:	
	Attempts both sides as powers of 3 (Addition law on RHS) $\quad 3^x = 27\sqrt{3} \times 3^{4y} \Rightarrow 3^x = 3^{"3.5"+4y}$	M1
	Sets powers equal and makes y the subject $x = "3.5" + 4y \Rightarrow y = ...$	dM1
	$y = \frac{1}{4}x - \frac{7}{8}$	A1
Alt2	Divides by $27\sqrt{3}$ first:	
	Attempts both sides as powers of 3 $\quad \frac{3^x}{3^{4y} \times 27\sqrt{3}} = 1 \Rightarrow 3^{x-4y-"3.5"} = 3^0$ (Subtraction law on LHS)	M1
	Sets powers equal and makes y the subject $x - "3.5" - 4y = 0 \Rightarrow y = ...$	dM1
	$y = \frac{1}{4}x - \frac{7}{8}$	A1

3.

2 (a)	$3^{3x} = \left(3^x\right)^3 = y^3$	B1 **(1)**
(b)	$\frac{1}{3^{x-2}} = \frac{1}{3^x \times 3^{-2}} = \frac{9}{y}$	M1 A1 **(2)**
(c)	$\frac{81}{9^{2-3x}} = \frac{9^2}{9^{2-3x}} = 9^{2-(2-3x)} = 9^{3x} = 3^{6x} = y^6$	M1 A1 **(2)**
		(5 marks)

4.

3(i)	$\sqrt{8} = 2\sqrt{2}$ seen anywhere in the solution (see notes)	B1
	$\left(x+\sqrt{2}\right)^2 + \left(3x-5\sqrt{8}\right)^2$ $= x^2 + 2x\sqrt{2} + 2 + 9x^2 - 30x\sqrt{8} + 25\times 8$	M1
	$= 10x^2 - 58x\sqrt{2} + 202$	A1
		(3)
(ii)	$\sqrt{3}\left(4y-3\sqrt{3}\right) = 5y+\sqrt{3}$ $\Rightarrow 4\sqrt{3}y - 9 = 5y + \sqrt{3}$ $\Rightarrow 4\sqrt{3}y - 5y = 9 + \sqrt{3}$	M1
	$\Rightarrow y = \ldots$ or $\Rightarrow ky = \ldots$ eg $y = "\frac{9+\sqrt{3}}{4\sqrt{3}-5}"$ or $"23"y = "9+\sqrt{3}"$	dM1
	$y = \frac{9+\sqrt{3}}{4\sqrt{3}-5} \times \frac{4\sqrt{3}+5}{4\sqrt{3}+5} \Rightarrow y = \frac{\ldots}{"23"}$	ddM1
	$y = \frac{57}{23} + \frac{41}{23}\sqrt{3}$ (or $y = 2\frac{11}{23} + 1\frac{18}{23}\sqrt{3}$)	A1
		(4)
	(ii) Alternative 1:	
	$\sqrt{3}\left(4p+4q\sqrt{3}-3\sqrt{3}\right) = 5\left(p+q\sqrt{3}\right)+\sqrt{3}$ $\Rightarrow 4p\sqrt{3} + 12q - 9 = 5p + 5q\sqrt{3} + \sqrt{3}$ $\Rightarrow 4p = 5q+1,\ 12q-9 = 5p$	M1 dM1
	$\Rightarrow 4p = 5q+1,\ 12q-9 = 5p$ $\Rightarrow p = \ldots,\ q = \ldots$	ddM1
	$y = \frac{57}{23} + \frac{41}{23}\sqrt{3}$ (or $y = 2\frac{11}{23} + 1\frac{18}{23}\sqrt{3}$)	A1
		Total 7

5.

2.(a)	$\frac{1}{4-2\sqrt{2}} = \frac{1}{4-2\sqrt{2}} \times \frac{4+2\sqrt{2}}{4+2\sqrt{2}}$	M1
	$= \frac{4+2\sqrt{2}}{16-8} = \frac{1}{2} + \frac{1}{4}\sqrt{2}$ oe	A1
		(2)
(b)	$4x = 2\sqrt{2}x + 20\sqrt{2} \Rightarrow \left(4-2\sqrt{2}\right)x = 20\sqrt{2}$	M1
	$\Rightarrow x = \frac{20\sqrt{2}}{\left(4-2\sqrt{2}\right)} = 20\sqrt{2} \times (a)$	dM1
	$\Rightarrow x = 20\sqrt{2} \times \left(\frac{1}{2} + \frac{1}{4}\sqrt{2}\right) = 10 + 10\sqrt{2}$	A1
		(3) **(5 marks)**

6.

3(i)	Writes $\sqrt{180}$ as $6\sqrt{5}$ or $\sqrt{80}$ as $4\sqrt{5}$	**M1**
	Concludes working $\frac{6\sqrt{5}-4\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{\sqrt{5}} = 2$	**A1**
		(2)
(ii)	$\frac{4\sqrt{5}-5}{7-3\sqrt{5}} = \frac{4\sqrt{5}-5}{7-3\sqrt{5}} \times \frac{7+3\sqrt{5}}{7+3\sqrt{5}} = \ldots$	**M1**
	$= \frac{28\sqrt{5} - 35 + 12\left(\sqrt{5}\right)^2 - 15\sqrt{5}}{49 - 9 \times 5}$	**dM1**
	$= \frac{25}{4} + \frac{13}{4}\sqrt{5}$	**A1**
		(3)
		(5 marks)

7.

Question Number	Scheme			Marks
(i)	**Way 1:** $\frac{49}{\sqrt{7}}=\frac{7^2}{7^{\frac{1}{2}}}=7^{2-\frac{1}{2}}$	**Way 2:** $7\sqrt{7}=7^{1+\frac{1}{2}}$	**Way 3:** $7^a=\frac{49}{\sqrt{7}}\Rightarrow a=\frac{\log\frac{49}{\sqrt{7}}}{\log 7}$ or $7^a=\frac{49}{\sqrt{7}}\Rightarrow a=\log_7\frac{49}{\sqrt{7}}$	M1
	$(a=)1\frac{1}{2}$ (oe) or see answer $=7^{1\frac{1}{2}}$			A1
				[2]
(ii)	**Way 1:** $\frac{10(\sqrt{18}+4)}{(\sqrt{18}-4)(\sqrt{18}+4)}$	**Way 2:** $(15\sqrt{2}+20)(\sqrt{18}-4)$		M1
	$=\frac{\cdots}{2}$	$=15\sqrt{36}-60\sqrt{2}+20\sqrt{18}-80$		B1
	$\frac{10}{\sqrt{18}-4}=5\left(3\sqrt{2}+4\right)=15\sqrt{2}+20*$	$=90-60\sqrt{2}+60\sqrt{2}-80$ $=10$ so $\frac{10}{\sqrt{18}-4}=15\sqrt{2}+20*$		A1cso
				[3]
				5 marks

8.

Question Number	Scheme	Marks
(a)	$\frac{1}{3}x$ as the final answer.	B1
(b)	$81x^{-3}$ as the final answer	B1
(c)	$x^{\frac{3}{2}}$ as the final answer	B1
		[3] (3 marks)

9.

Question Number	Scheme	Marks
(a)	$y^{-\frac{1}{2}}=\left(\frac{64x^6}{25}\right)^{-\frac{1}{2}}=\frac{5}{8}x^{-3}$	M1 A1 A1 [3]
(b)	$(25y)^{\frac{2}{3}}=16x^4$	B1, B1 [2]
		5 marks
	Notes	

10.

Question Number	Scheme	Marks
(a)	y^2	B1 (1)
(b)	$8y$	B1 (1)
(c)	$\frac{64}{y^4}$	M1 A1 (2)
		(4 marks)

11.

Question Number	Scheme		Marks
(a)	$2^{2(2x+1)}$ or 2^{4x+2} Accept either $2^{2(2x+1)}$ or 2^{4x+2} **but not** $(2^2)^{(2x+1)}$ unless followed by $2^{2(2x+1)}$ or 2^{4x+2} Also accept $a=4x+2$ or equivalent e.g. $a=2(2x+1)$ Apply isw once a correct answer is seen.		B1
			[1]
(b)	**Examples:** $2^x\times4^{2x+1}=2^x\times2^{'4x+2'}=2^{x+'4x+2'}$ or $4^{\frac{1}{2}x}\times4^{2x+1}=4^{\frac{1}{2}x+2x+1}$ or $16^{\frac{1}{4}x}\times16^{\frac{1}{2}(2x+1)}=16^{\frac{1}{4}x+\frac{1}{2}(2x+1)}$ or $16^{3x}=2^{4\times3x}$ or 2^{12x} or $16^{3x}=4^{2\times3x}$ or 4^{6x}	**Either** A correct application of the addition law on the lhs. Follow through on their $4x+2$ but if they use bases other than 2 then the powers must be correct. **Or** A correct application of the multiplication law on the rhs. As in (a) must be e.g. $2^{4\times3x}$ not $(2^4)^{3x}$ Condone invisible brackets for this mark e.g. $4^{2x+1}=16^{\frac{1}{2}2x+1}$	M1
	Examples: $2^{x+4x+2}=2^{4\times3x}$, $4^{\frac{1}{2}x+2x+1}=2^{12x}$, $2^{5x+2}=16^{3x}$, $16^{\frac{5}{4}x+\frac{1}{2}}=16^{3x}$, $2^{4x+2}=2^{11x}$ Any correct equation or correct follow through from their answer to part (a) in the form $m^{f(x)}=n^{g(x)}$ **which may be implied by their equation below** Note that it is **not** necessary that $m=n$ If 'isw' has been applied in (a), mark positively and allow this mark if possible e.g. if $2^{2(2x+1)}=2^{4x+1}$ is seen in (a), score B1 and then allow M1A1ft in (b) if 2^{4x+1} is used in (b)		A1ft
	Examples: $5x+2=12x, \frac{1}{2}x+2x+1=6x, \frac{1}{4}x+x+\frac{1}{2}=3x, 4x+2=11x$ This is for any **fully correct** linear equation (no inexact decimals from logs) **(not follow through here)** **<u>Note that this is an M mark on ePEN</u>**		A1 **(M1 on ePEN)**
	$\Rightarrow x=\dfrac{2}{7}$	Correct answer and no other values. Allow equivalent exact fractions e.g. $\dfrac{4}{14}$ but not $\dfrac{-2}{-7}$. Note that this mark is cso so cannot be 'recovered' once inexact decimals have been used.	A1cso
			[4]

12.

Question Number	Scheme	Marks
	(a) x^2	B1 [1]
	(b) $\frac{1}{4}x^4$ or $\frac{1}{2^2}x^4$ or $0.25x^4$	B1, B1 [2]
		3 marks
	Notes	

13.

Question Number	Scheme	Marks
(i)	Either $4^{2x+1} = 2^{2(2x+1)}$ and $8^{4x} = 2^{3\times4x}$ or $8^{4x} = 4^{\frac{3}{2}\times4x}$	M1
	$2(2x+1) = 12x \Rightarrow x = \frac{1}{4}$	dM1A1 (3)
(ii)(a)	$3\sqrt{18} - \sqrt{32} = 9\sqrt{2} - 4\sqrt{2} = 5\sqrt{2}$	M1A1 (2)
(b)	$\sqrt{n} = 5\sqrt{2} \Rightarrow n = (5\sqrt{2})^2 = 25\times2 = 50$	M1A1 (2)
		(7 marks)
Alt (i)	Taking logs of both sides and proceeding to $(2x+1)\log4 = 4x\log8$	M1
	$\Rightarrow x = \frac{\log4}{4\log8 - 2\log4}$	
	$\Rightarrow x = \frac{\log4}{\log256} = \frac{1}{4}$	dM1A1 (3)

14.

4.	(i) $\frac{8^y}{4^{2x}} = \frac{\sqrt{2}}{32}$; (ii) $x\sqrt{3} = 4\sqrt{2} + x$		
(i) **Way 1**	$\frac{2^{3y}}{2^{4x}} = \frac{2^{\frac{1}{2}}}{2^5} \Rightarrow 2^{3y-4x} = 2^{\frac{1}{2}-5}$		M1 A1
	$3y - 4x = -\frac{9}{2} \Rightarrow y = \frac{4}{3}x - \frac{3}{2}$ or $y = \frac{1}{6}(8x-9)$ **cso**		dM1 A1 **cso**
(i) **Way 2**	$\log\left(\frac{8^y}{4^{2x}}\right) = \log\left(\frac{\sqrt{2}}{32}\right) \Rightarrow y\log 8 - 2x\log 4 = \log\left(\frac{\sqrt{2}}{32}\right)$		M1
	$y\log 8 - 2x\log 4 = \log(\sqrt{2}) - \log(32)$		A1
	$y = \frac{2x\log 4 + \log(\sqrt{2}) - \log(32)}{\log 8} \Rightarrow y = \frac{2x(2\log 2) + \frac{1}{2}\log 2 - 5\log 2}{3\log 2}$		dM1
	$\Rightarrow y = \frac{4}{3}x - \frac{3}{2}$ or $y = \frac{1}{6}(8x-9)$ **cso**		A1 **cso**
			(4)
(ii)	$x\sqrt{3} - x = 4\sqrt{2} \Rightarrow x(\sqrt{3} - 1) = 4\sqrt{2}$	For sight of **an equation** containing $(\pm\sqrt{3} \pm 1)x$	M1
	$x = \frac{4\sqrt{2}}{\sqrt{3}-1}$	$x = \frac{4\sqrt{2}}{\sqrt{3}-1}$ or $x = \frac{-4\sqrt{2}}{1-\sqrt{3}}$ o.e.	A1
	$x = \frac{4\sqrt{2}}{(\sqrt{3}-1)} \cdot \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$	**dependent on the previous M mark** Attempt to rationalise the denominator	dM1
	$x = \frac{4\sqrt{6} + 4\sqrt{2}}{2} \Rightarrow x = 2\sqrt{6} + 2\sqrt{2}$ **cso**	Uses a non-calculator process to obtain $x = 2\sqrt{6} + 2\sqrt{2}$ or equivalent	A1 **cso**
			(4)

15.

Question Number	Scheme	Marks
(i)	$\dfrac{4(2\sqrt{2}+\sqrt{6})}{(2\sqrt{2}-\sqrt{6})(2\sqrt{2}+\sqrt{6})}$	M1
	$(2\sqrt{2}-\sqrt{6})(2\sqrt{2}+\sqrt{6})=8-6=2$	B1
	$\sqrt{6}=\sqrt{2}\sqrt{3}$ used in numerator - may be implied by a correct factorisation of numerator	B1
	Concludes $\dfrac{4(2\sqrt{2}+\sqrt{6})}{2}=2\sqrt{2}(2+\sqrt{3})$ *	A1 *
		[4]
(ii)	1st two terms $\sqrt{27}=3\sqrt{3}$ and $\sqrt{21}\times\sqrt{7}=7\sqrt{3}$	B1
	3rd term See $2\sqrt{3}$ or $\dfrac{6\sqrt{3}}{3}$	B1
	$3\sqrt{3}+7\sqrt{3}-2\sqrt{3}=8\sqrt{3}$ or $3\sqrt{3}+7\sqrt{3}-\dfrac{6\sqrt{3}}{3}=8\sqrt{3}$ *	B1 *
		[3]
Alternative for (i)	Assume result and multiply both sides by $(2\sqrt{2}-\sqrt{6})$	M1
	$(2\sqrt{2}-\sqrt{6})(4\sqrt{2}+2\sqrt{6})=16-12=4$	B1 B1
	So LHS = RHS and result is true	A1
		[4]
Alternative for (ii)	$\dfrac{\sqrt{81}+\sqrt{21\times7\times3}-6}{\sqrt{3}}$ *Or* $\sqrt{81}+\sqrt{21\times7\times3}-6=8\sqrt{3}\sqrt{3}$	B1
	$\dfrac{9+21-6}{\sqrt{3}}$ $9+21-6=$	B1
	$\dfrac{24}{\sqrt{3}}\times\dfrac{\sqrt{3}}{\sqrt{3}}=8\sqrt{3}$ $9+21-6=24$ **so equation is true**	B1
		[3]
		(7 marks)

16.

Question Number	Scheme	Marks
1.(a)	$\dfrac{6}{\sqrt{5}-\sqrt{2}}=\dfrac{6}{\sqrt{5}-\sqrt{2}}\times\dfrac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}$	M1
	$=\dfrac{6(\sqrt{5}+\sqrt{2})}{5-2}=2\sqrt{5}+2\sqrt{2}$	A1
		(2)
(b)	$\sqrt{5}x=\sqrt{2}x+18\sqrt{5}\Rightarrow(\sqrt{5}-\sqrt{2})x=18\sqrt{5}$	M1
	$\Rightarrow x=\dfrac{18\sqrt{5}}{(\sqrt{5}-\sqrt{2})}=3\sqrt{5}\times(\text{a})$	dM1
	$\Rightarrow x=3\sqrt{5}\times2(\sqrt{5}+\sqrt{2})=30+6\sqrt{10}$	A1
		(3)
		(5 marks)

17.

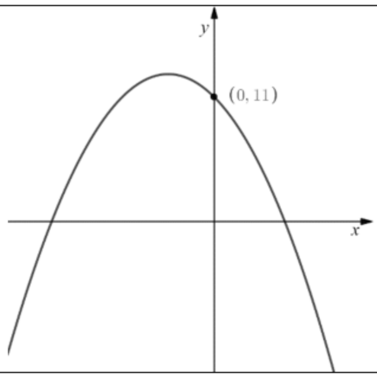
Question Number	Scheme	Marks
2	$\sqrt{27}=3\sqrt{3},\ \frac{6}{\sqrt{3}}=2\sqrt{3}$	
	$x\sqrt{27}+21=\frac{6x}{\sqrt{3}}\Rightarrow 3\sqrt{3}x+21=2\sqrt{3}x$	M1 A1
	$\Rightarrow\sqrt{3}x=-21$	
	$\Rightarrow x=-\frac{21}{\sqrt{3}}\Rightarrow x=-7\sqrt{3}$	M1 A1
		(4 marks)

Quadratic:

1.

9.	$\frac{3}{x}+5=-2x+c$ Multiplies through by x $\quad 3+5x=-2x^2+cx \Rightarrow \pm 2x^2 ....(=0)$	M1
	and writes in quadratic form $\quad \Rightarrow 2x^2+(5-c)x+3(=0)$ oe	A1
	Attempts $"b^2-4ac"=(5-c)^2-24$	M1
	Attempts $"b^2-4ac"=0 \Rightarrow (5-c)^2-24=0 \Rightarrow c=...$	dM1
	$(c=)\ 5\pm 2\sqrt{6}$ oe	A1
	Attempt at inside region	M1
	$5-2\sqrt{6}<c<5+2\sqrt{6}$ oe	A1
		(7) (7 marks)

2.

2(a)	$f(x)=11-4x-2x^2$ $\Rightarrow \ldots \underline{-2}(2x+x^2)\ldots$ or $\Rightarrow \ldots \underline{-2}(2x+x^2\ldots)$	B1
	$\ldots(2x+x^2)\Rightarrow \ldots.((x+1)^2 \pm \ldots)$	M1
	$(f(x)=)\ 13-2(x+1)^2$	A1
		(3)
(b)		M1 A1
		(2)
(c)	$x=-1$	B1ft
		(1)
		Total 6
Alt(a)	$a+b(x^2+2cx+c^2)=11-4x-2x^2$ $b=-2$	B1
	$2bc=-4\Rightarrow c=\ldots\ (=1)$	M1
	$a+bc^2=-4\Rightarrow a=\ldots\ (=13)$ $(f(x)=)\ 13-2(x+1)^2$	A1

3.

8	$x-6x^{\frac{1}{2}}+4=0$ $x^{\frac{1}{2}}=3\pm\sqrt{5}$ oe $x=(3\pm\sqrt{5})^2 \Rightarrow x=14\pm6\sqrt{5}$	M1 A1 M1 A1 A1 **(5 marks)**
8 Alt	$x+4=6x^{\frac{1}{2}}$ $(x+4)^2=36x$ $x^2-28x+16=0\Rightarrow(x-14)^2=180\Rightarrow x=14\pm\sqrt{180}\Rightarrow x=14\pm6\sqrt{5}$	M1 A1 M1 A1 A1 **(5)**

4.

2(a)	$b=2$	B1
	$\ldots\pm\ldots(x\pm3)^2$	M1
	$(\mathrm{f}(x)=)\ 21-2(x-3)^2$	A1
		(3)
(b)	R is $(0,-4)$ or "h" $=4$	B1
	$\mathrm{f}(x)-7=14-2(x-3)^2 \Rightarrow x=\ldots$ or $\mathrm{f}(x)-7=-4+12x-2x^2 \Rightarrow x=\ldots$ $\left(\text{NB } x=3\pm\sqrt{7}\right)$	M1
	$\text{Area}=\frac{1}{2}\times\left(\text{"}3+\sqrt{7}\text{"}-\left(\text{"}3-\sqrt{7}\text{"}\right)\right)\times\text{"}4\text{"}$	dM1
	$=4\sqrt{7}$	A1
		(4)
		(7 marks)

5.

Number		
8.(a)(i)	$4+12x-3x^2=a\pm3(x+c)^2$ or $a+b(x\pm2)^2$	M1
	Two of $16-3(x-2)^2$ or two of $a=16, b=-3, c=-2$	A1
	$16-3(x-2)^2$	A1
(ii)	Coordinates $M=(2,16)$	B1ft B1ft
		(5)
(b)	States or implies that l_2 has equation $y=\text{"}8\text{"}x+k$	M1
	Sets $4+12x-3x^2=\text{"}8x\text{"}+k$ and proceeds to 3TQ	dM1
	Correct 3TQ $3x^2-4x+k-4=0$	A1
	Attempts to use $b^2-4ac=0$ to find k	ddM1
	$k=\frac{16}{3}\Rightarrow y=8x+\frac{16}{3}$	A1
		(5)
		(10 marks)

6.

9(a)	$\frac{1}{2}x^2-10x+22=\frac{1}{2}(x\pm\ldots)^2\pm\ldots$ or states $a=\frac{1}{2}$	B1
	$\frac{1}{2}x^2-10x+22=\frac{1}{2}(x\pm10)^2\pm\ldots$ or states $a=\frac{1}{2}$ and $b=\pm10$	M1
	$\frac{1}{2}x^2-10x+22=\frac{1}{2}(x-10)^2-28$	A1
		(3)
(b)	$("10", "-28")$	B1ftB1ft
		(2)
(c)(i)	Gradient of tangent $=8$	B1
	$\frac{dy}{dx}=x-10=8\Rightarrow x=\ldots$	M1
	$x=18,\ y=4$	A1A1
(c)(ii)	$k-\frac{1}{8}\times"18"="4"\Rightarrow k=\frac{25}{4}$	dM1A1
		(6)
(d)	One of $x \ldots "10"$ or $y „ "\frac{25}{4}"-\frac{1}{8}x$ or $y \ldots \frac{1}{2}x^2-10x+22$	B1ft
	Two of $x \ldots "10"$ or $y „ "\frac{25}{4}"-\frac{1}{8}x$ or $y \ldots \frac{1}{2}x^2-10x+22$	B1ft
	All three of $x \ldots 10$, $y „ \frac{25}{4}-\frac{1}{8}x$ and $y \ldots \frac{1}{2}x^2-10x+22$	B1
		(3)
		(14 marks)

natural science solution
Accelerating Academic Skills

Simultaneous equations:

1.

Question	Scheme	Marks
4	$y = 3x+4 \Rightarrow x^2 + (3x+4)^2 + 6x - 4(3x+4) = 4$ **or** $x = \frac{y-4}{3} \Rightarrow \left(\frac{y-4}{3}\right)^2 + y^2 + 6\left(\frac{y-4}{3}\right) - 4y = 4$	M1
	$5x^2 + 9x - 2\ (=0)$ or $5y^2 - 13y - 46\ (=0)$	M1A1
	$(5x-1)(x+2) = 0 \Rightarrow x = ...$ or $(5y-23)(y+2) = 0 \Rightarrow y = ...$	dM1
	$x = 0.2,\ x = -2$ or $y = 4.6,\ y = -2$	B1
	Substitutes their x into their $y = 3x + 4$ / Substitutes their y into their $x = \frac{y-4}{3}$	M1
	$x = 0.2\left(\text{or } \frac{1}{5}\right),\ y = 4.6\left(\text{or } 4\frac{3}{5} \text{ or } \frac{23}{5}\right)$ **and** $x = -2,\ y = -2$	A1
		(7 marks)

2.

6(a)	$2xy - 3x^2 = 50;\ y - x^3 + 6x = 0$	
	$\Rightarrow 2x(x^3 - 6x) - 3x^2 = 50$	**M1**
	$\Rightarrow 2x^4 - 12x^2 - 3x^2 - 50 = 0 \Rightarrow 2x^4 - 15x^2 - 50 = 0$ * CSO	**A1***
		(2)
(b)	$\Rightarrow (2x^2+5)(x^2-10) = 0 \Rightarrow x^2 = ...$	**M1**
	So $x^2 = 10$	**A1**
	$\Rightarrow y = (\sqrt{10})^3 - 6\sqrt{10} = ...$	**M1**
	one solution pair is $x = \sqrt{10}, y = 4\sqrt{10}$	**A1**
	Solutions are $x = \sqrt{10}, y = 4\sqrt{10}$ and $x = -\sqrt{10}, y = -4\sqrt{10}$ CSO	**A1**
		(5)
		(7 marks)

3.

8.	Equates $y=k(2x-1)$ and $y=x^2+2x+11 \Rightarrow k(2x-1)=x^2+2x+11$ $\Rightarrow x^2+(2-2k)x+11+k \quad (=0)$ Attempts $"b^2-4ac"...0 \Rightarrow (2-2k)^2-4(11+k)...0$ and proceeds to critical values Critical values of $(k=)\ 5,-2$ No roots so $b^2-4ac<0$ so choose inside region $-2<k<5$	M1 A1 M1 A1 M1 A1 **(6)** **(6 marks)**

4.

2 .(a)	Attempts to use the given model at least once. Eg $2^3 = p \times 3^2 + q$ Two correct simplified equations $9p+q=8 \qquad 25p+q=13.8(24)$ Solves simultaneously to get at least one of p or q $p=0.364, q=4.72(4)$	M1 A1 dM1 A1 **(4)**
(b)	Attempts to find T when $H=5$ Eg. Calculates $\sqrt{\frac{125-"q"}{"p"}}$ $(T=)\ 18.2$	M1 A1 **(2)** **(6 marks)**

5.

6.(a)	Sets $4x+c=x(x-3)$ and attempts to write as a 3TQ Uses $b^2=4ac$ for their $x^2-7x-c=0$ Correct equation $49=-4c$ or $49+4c=0$ $c=-12.25$ oe	M1 dM1 A1 A1 **(4)**
(b)	Attempt to solve $x^2-7x-c=0$ with their c Attempt to find the y coordinate for their x coordinate $\left(\frac{7}{2},\frac{7}{4}\right)$ oe	M1 dM1 A1 **(3)** **(7 marks)**

6.

2. (a)	$1.85 = 2a + b$ and $3.45 = 7a + b$ Solves simultaneously to get $a = 0.32, b = 1.21$ (oe)	M1 A1 dM1 A1 **(4)**
(b)	States 1.21 m or 121 cm (oe)	B1ft **(1)**
		(5 marks)

7.

3(a)	$1.05 = 3p + q$ $1.65 = 5p + q$ e.g. $\Rightarrow 2p = 0.6$ or $\Rightarrow 1.05 = 3p + (1.65 - 5p)$ or $\Rightarrow 1.65 = 5p + (1.05 - 3p)$ $p = 0.3 \quad q = 0.15$	M1 A1A1
		(3)
(b)	$2.5 = "0.3"T + "0.15" \Rightarrow T =$ $T = 7.8$	M1 A1
		(2)
		(5 marks)

Cubic Equations:

1.

5. (a)	$20x^3 - 50x^2 - 30x = 0 \Rightarrow 10x\left(2x^2 - 5x - 3\right) = 0$	
	$\Rightarrow 10x(2x+1)(x-3) = 0$	M1
	$\Rightarrow x = 0, -\frac{1}{2}, 3$	A1, A1
		(3)
(b)	Sets or implies $(y+3)^{\frac{1}{2}} = 0$ or $-\frac{1}{2}$ or 3	B1ft
	Full method to find y	M1
	$y = 6$	A1ft,
	$y = -3,\ 6$	A1
		(4)
		(7 marks)

2.

5.(a)	$2x^3 + 3x^2 - 35x = 0 \Rightarrow x\left(2x^2 + 3x - 35\right) = 0$	M1
	$(2x-7)(x+5) = 0 \Rightarrow x = ...$	dM1
	$x = -5, 0, \frac{7}{2}$	A1
		(3)
(b)	$2(y-5)^6 + 3(y-5)^4 - 35(y-5)^2 = 0$	
	States that $y = 5$ is a solution	B1
	$(y-5)^2 = "\frac{7}{2}" \Rightarrow y = ...$	M1
	$y = 5 + \sqrt{\frac{7}{2}}$ or $y = 5 - \sqrt{\frac{7}{2}}$ or exact equiavlent	A1ft
	Both $y = 5 + \sqrt{\frac{7}{2}}$ and $y = 5 - \sqrt{\frac{7}{2}}$ or exact equivalent.	A1
		(4)
		(7 marks)

3.

8 (a)	$x > 4$	B1 **(1)**
(b)	$(3x-2)^2(x-4) = (9x^2-12x+4)(x-4)$ $= 9x^3 - 48x^2 + 52x - 16$	M1 A1 A1 **(3)**
(c)	Sets $9x^3 - 48x^2 + 52x - 16 = -16 \Rightarrow 9x^3 - 48x^2 + 52x \ (=0)$	B1ft
	$\Rightarrow x(9x^2 - 48x + 52) = 0 \Rightarrow x = \dfrac{48 \pm \sqrt{48^2 - 4\times 9\times 52}}{18} = \dfrac{16 \pm 4\sqrt{3}}{6}$	M1 B1
	Distance $PQ = \dfrac{16+4\sqrt{3}}{6} - \dfrac{16-4\sqrt{3}}{6} = \dfrac{4}{3}\sqrt{3}$	M1 A1
		(5) **(9 marks)**

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4.

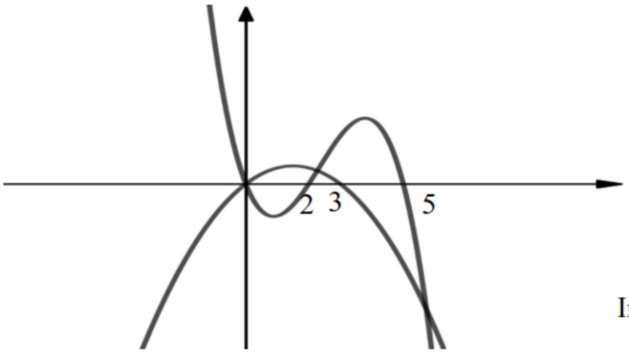
2(a)(i)		
	$-a+6a+8+a^2=32 \Rightarrow a^2+5a-24=0$	M1
	$(a+8)(a-3)=0$	dM1
	$a=3$ or $a=-8$ and chooses $a=3$ with reason *	A1* cso
		(3)
(ii)	$3x^3+26x^2-9x=0 \Rightarrow x(3x^2+26x-9)=0$ $x(3x-1)(x+9)$	M1
	$(x=)\ 0, \frac{1}{3}, -9$	A1
		(2)
(b)(i)	$(y=)\ 0$	B1
	$y^{\frac{1}{3}}="\frac{1}{3}"$ or $y^{\frac{1}{3}}="-9" \Rightarrow y=...$ (or $(-9)^3=...$ or $\left(\frac{1}{3}\right)^3=...$)	M1
	$(y=)\ \frac{1}{27},\ -729$	A1
		(3)
(b)(ii)	$9^z="\frac{1}{3}" \rightarrow z=...$	M1
	$(z=)\ -\frac{1}{2}$ only	A1
		(2)
		(10 marks)

5.

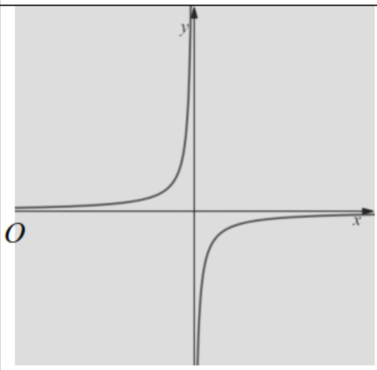
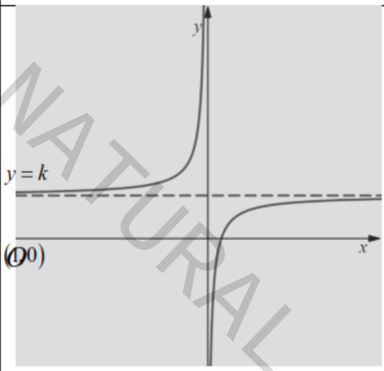
4(a)	$x^2(2x+1)-15x \Rightarrow 2x^3+x^2-15x = x(2x^2+x-15)$	M1
	$x(2x-5)(x+3)=0 \Rightarrow x=...$	dM1
	Two of $x=0, \frac{5}{2}, -3$	B1
	$x=0, \frac{5}{2}, -3$	A1
		(4)
(b)	$y^{\frac{2}{3}}=\frac{5}{2} \Rightarrow y=\left(\frac{5}{2}\right)^{\frac{3}{2}}$	M1
	$\frac{5}{4}\sqrt{10}$	A1cso
		(2)
		(6 marks)

Sketching curves:

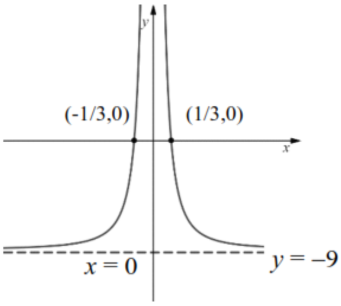
1.

11. (a)		$\cap$ shaped quadratic	B1
		Intercepts at O and 3	B1
		-ve cubic	B1
		Intercepts at O, 2 and 5	B1
			(4)
(b)	Sets $x(x-2)(5-x)=x(3-x)$		M1
	$3x-x^2=-x^3+7x^2-10x \Rightarrow \pm(x^3-8x^2+13x)\ (=0)$ OR $\pm x\{(x-2)(5-x)-(3-x)\}\ (=0)$		dM1
	Proceeds to $x(x^2-8x+13)=0$ *		A1*
			(3)
(c)	Solves $x^2-8x+13=0 \Rightarrow x=4\pm\sqrt{3}$		M1 A1
	Substitutes $x=\text{"}4-\sqrt{3}\text{"}$ into $y=x(3-x)$ oe		M1
	$y=(4-\sqrt{3})(-1+\sqrt{3})=-4+4\sqrt{3}+\sqrt{3}-3=\ldots$		M1
	$y=-7+5\sqrt{3}$		A1
			(5)
			(12 marks)

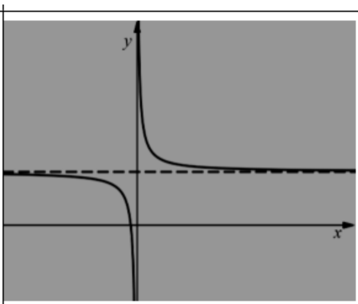
2.

6.(a)	 Negative reciprocal shape Fully correct	M1 A1 **(2)**
(b)	 Graph is part (a) translated ↑ Correct asymptote or intercept Correct asymptote and intercept	B1ft B1 B1 **(3)**
(c)	Sets $3x+4=-\frac{k}{x}+k \Rightarrow 3x^2+(4-k)x+k=0$ Attempts use $b^2-4ac=0$ to find the critical values Uses $b^2-4ac<0$ and selects inside region for critical values $10-2\sqrt{21}<k<10+2\sqrt{21}$	M1, A1 M1 dM1 A1 **(5)** **(10 marks)**

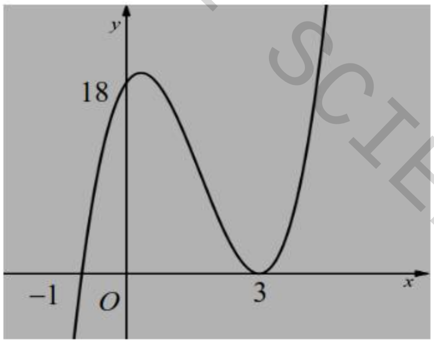
3.

10(a)		B1B1B1B1
		(4)
(b)	$k < 0$	B1
	$\frac{1}{x^2} - 9 = kx^2 \Rightarrow 1 - 9x^2 = kx^4$	M1
	$1 - 9x^2 = kx^4 \Rightarrow kx^4 + 9x^2 - 1 = 0$ Require $b^2 - 4ac \Rightarrow 81 - 4 \times k \times -1$	M1
	Critical value $(k =) -\frac{81}{4}$	A1
	$-\frac{81}{4} < k < 0$	A1cso
		(5)
		Total 9

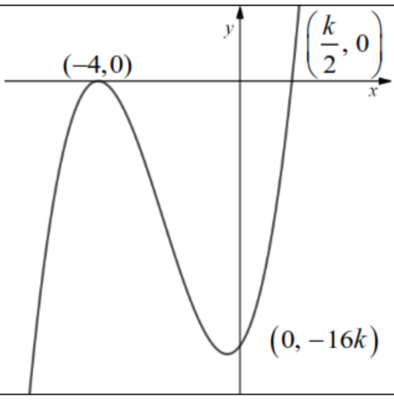
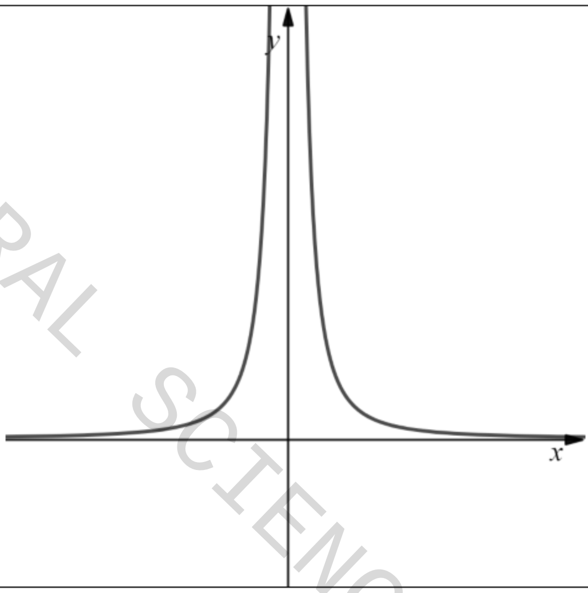
4.

6. (a)	 Shape	B1
	States asymptote as $y = k$	B1
	States intercept as $-\frac{4}{k}$	B1
		(3)
(b)	$10 - 2x = \frac{4}{x} + k \Rightarrow 10x - 2x^2 = 4 + kx$	M1
	$\Rightarrow 2x^2 + (k - 10)x + 4 = 0$	A1
	Attempts "$b^2 - 4ac$" $= 0 \Rightarrow (k-10)^2 - 4 \times 2 \times 4 = 0$	M1
	$k = 10 \pm 4\sqrt{2}$ oe	M1 A1
		(5)
		(8 marks)

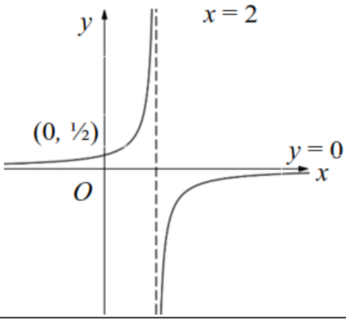
5.

6.(a)		Positive cubic shape anywhere with 1 maximum and 1 minimum	M1
		Positive cubic shape that at least reaches the x-axis at $x = -1$ and with a minimum on the x-axis at $x = 3$	A1
		y intercept at 18. Must correspond with their sketch	B1
	For the intercepts allow as numbers as above or allow as coordinates e.g. (18, 0), (0, –1), (0, 3) as long as they are marked in the correct place.		
			(3)
(b)	E.g. $(2x+2)(x^2 - 6x + 9) = \ldots$		M1
	$= 2x^3 - 10x^2 + 6x + 18$		A1 A1
			(3)
(c)	$(\mathrm{f}'(x) =) 6x^2 - 20x + 6$		B1ft
	$\mathrm{f}'\left(\frac{1}{3}\right) = 6\left(\frac{1}{3}\right)^2 - 20\left(\frac{1}{3}\right) + 6$		M1
	$\mathrm{f}'\left(\frac{1}{3}\right) = 0$		A1
	$y = \frac{512}{27}$		A1
			(4)
			(10 marks)

6.

6(a)(i)		B1B1B1
		(3)
(a)(ii)		B1B1
		(2)
(b)	One root because the two graphs intersect each other once	B1
		(1)
		(6 marks)

7.

7(a)	 B1 Negative reciprocal shape B1 Intercept at $\left(0, \frac{1}{2}\right)$ B1 $x = 2,\ y = 0$	B1B1B1
		(3)
(b)	$4x + k = \frac{1}{2-x} \Rightarrow (4x+k)(2-x) = 1 \Rightarrow 8x + 2k - 4x^2 - kx - 1 (= 0)$ oe	M1
	$4x^2 + (k-8)x + 1 - 2k = 0$ $a = 4,\ b = k - 8,\ c = 1 - 2k$ or $a = -4,\ b = 8 - k,\ c = 2k - 1$	A1
	$(k-8)^2 - 4 \times 4(1-2k)(> 0)$ oe	M1
	$k^2 - 16k + 64 - 16 + 32k > 0 \Rightarrow k^2 + 16k + 48 > 0$*	A1*
		(4)
(c)	$k^2 + 16k + 48 = 0 \Rightarrow (k+12)(k+4) = 0 \Rightarrow k = ...$	M1
	$k = -12,\ -4$	A1
	$k < -12$ or $k > -4$	M1A1
		(4)
		(11 marks)

Inequality and Region:

1.

5(a)	E.g. $12-a(x+2)^2$ or $a(x-1)(x+5)$ or $y=ax^2+bx+c \Rightarrow 4a-2b+c=12$ and $25a-5b+c=0$	**M1A1**
	E.g. $0=12-a(-5+2)^2 \Rightarrow a=...$ or $12=a(-2-1)(-2+5) \Rightarrow a=...$ or $2a(-2)+b=0, 25a-5b+c=0, 4a-2b+c=12 \Rightarrow a=..., b=..., c=...$	**dM1**
	$12-\frac{4}{3}(x+2)^2$ or $-\frac{4}{3}(x-1)(x+5)$ or $-\frac{4}{3}x^2-\frac{16}{3}x+\frac{20}{3}$ oe	**A1**
		(4)
(b)	Gradient of l_2 is $\frac{-5}{4}$ o.e.	**M1**
	Equation of l_2 is $y="-\frac{5}{4}"(x+5)$	**M1**
	$y=-\frac{5}{4}x-\frac{25}{4}$	**A1**
		(3)
(c)	For two of $y \geqslant "-\frac{5}{4}x-\frac{25}{4}"$; $y \geqslant \frac{4}{5}x$; $y \leqslant "-\frac{4}{3}x^2-\frac{16}{3}x+\frac{20}{3}"$ Or with strict inequalities	**M1**
	For all three of $y \geqslant "-\frac{5}{4}x-\frac{25}{4}"$; $y \geqslant \frac{4}{5}x$; $y \leqslant "-\frac{4}{3}x^2-\frac{16}{3}x+\frac{20}{3}"$ Or with strict inequalities	**A1ft**
		(2)
		(9 marks)

2.

3. (a)	$x^2-5x+13=(x-2.5)^2-2.5^2+13=(x-2.5)^2+6.75$ Coordinates $M=(2.5, 6.75)$	M1 A1 A1 **(3)**
(b)	Attempts the equation of l using their M $y=\frac{6.75}{2.5}x$ $(y=2.7x)$ Attempts to solve their $y=2.7x$ with $y=x^2-5x+13$ $\Rightarrow 2.7x=x^2-5x+13 \Rightarrow x^2-7.7x+13=0 \Rightarrow (x-2.5)(x-5.2)=0$ $x=5.2$ oe Coordinates $N=(5.2, 14.04)$	M1 M1 A1 dM1 A1 **(5)**
(c)	States two of $y<x^2-5x+13, y>2.7x, 0 \leqslant x<2.5$ States all three of $y<x^2-5x+13, y>2.7x, 0 \leqslant x<2.5$	M1 A1ft **(2)** **(10 marks)**

3.

4.	When ---- represents < or > and —— represents $\le$ or $\ge$ Either $2y \le x$ or $y \ge 2x - \frac{1}{8}x^2$	B1
	$2x - \frac{1}{8}x^2 = 0 \Rightarrow x = 16 \Rightarrow x < \ldots$ or $x \le \ldots$	M1
	$x < 16$, $2y \le x$ and $y \ge 2x - \frac{1}{8}x^2$	A1
		(3) **(3 marks)**
Alt1	When ---- represents $\le$ or $\ge$ and —— represents < or > Either $2y < x$ or $y > 2x - \frac{1}{8}x^2$	B1
	$2x - \frac{1}{8}x^2 = 0 \Rightarrow x = 16 \Rightarrow x < \ldots$ or $x \le \ldots$	M1
	$x \le 16$, $2y < x$ and $y > 2x - \frac{1}{8}x^2$	A1
		(3)

4.

4.(a)	Gradient $PQ = -3$	B1
	Attempts to find equation of l Eg. $y - 13 = -3(x+2)$	M1
	$y = -3x + 7$	A1
		(3)
(b)	Attempts to use minimum is $(4,-5)$ Eg $y = \ldots(x-4)^2 - 5$	M1
	Attempts to use $(-2,13)$ with $y = a(x-4)^2 - 5 \Rightarrow a = \ldots$	dM1
	$\Rightarrow y = \frac{1}{2}(x-4)^2 - 5$ or $y = \frac{1}{2}x^2 - 4x + 3$ oe	A1
		(3)
(c)	Two of $y > -3x + 7$, $y < \frac{1}{2}(x-4)^2 - 5$ $x < -2$	M1
	All three of $y > -3x + 7$, $y < \frac{1}{2}(x-4)^2 - 5$ $x < -2$	A1
		(2) **(8 marks)**

5.

4(a)	$x+y=6,\ y=6x-2x^2+1$ $\Rightarrow 6-x=6x-2x^2+1$ $\Rightarrow 2x^2-7x+5=0$ oe	M1
	$2x^2-7x+5=0 \Rightarrow (2x-5)(x-1)=0$ $\Rightarrow x=\frac{5}{2},\ 1$	M1
	$x="\frac{5}{2}" \Rightarrow y=\frac{7}{2}$ or $x="1" \Rightarrow y=5$	dM1
	$(1,\ 5)$ and $(2.5,\ 3.5)$	A1
		(4)
(b)	$y \geqslant 6x-2x^2+1$ oe	M1
	$x+y \leqslant 6$ oe $x \geqslant a$ where $1 \leqslant a \leqslant 2.5$	A1
	$($or $a \leqslant x \leqslant b$ where $1 \leqslant a \leqslant 2.5,\ b \geqslant 6)$ $y \geqslant 0$ (or $0 \leqslant y \leqslant c$ where $c \geqslant 3.5$)	A1
	Allow strict or non-strict inequalities	
		(3)
		Total 7

6.

3.(i)	$\frac{3}{x}>4 \Rightarrow 3x>4x^2 \Rightarrow x(4x-3)<0 \Rightarrow 0,\ \frac{3}{4}$	B1
	$0<x<\frac{3}{4}$	M1 A1
		(3)
(ii)	$y-0=3(x+5)$	B1
	E.g. $y<2x^2-50,\ y>3x+"15"$	M1
	E.g. $y<2x^2-50,\ y>3x+15,\ x<-5$	A1
		(3)
		(6 marks)

7.

5(a)	$P_B - P_A = 44.2 - (53 - 0.4 \times 8^2) = \ldots$	M1
	awrt (£) 16.8 million	A1
		(2)
(b)	(£) 53 (million)	B1
		(1)
(c)	$-1.6t + 44.2 = 53 - 0.4(t-8)^2$	M1
	$\Rightarrow 0.4t^2 - 8t + \frac{84}{5} = 0 \Rightarrow t = \ldots$	M1
	$t = 10 - \sqrt{58} =$ awrt 2.38 (years)	A1
	$"2.38" < t \;(\leqslant 15)$	A1ft
		(4)
(d)	"The **share value** would be negative" / "the **model** is known to hold for 15 years only (and 20 years is more than 15)."	B1
		(1)
		(8 marks)

8.

3.(a)	Attempts perimeter of garden = $2 \times 5x + 2 \times (6x - 2)$	M1
	Sets $2 \times 5x + 2 \times (6x-2) > 29 \Rightarrow 22x > 33$	dM1
	$\Rightarrow x > \frac{33}{22} \Rightarrow x > 1.5$ *	A1*
		(3)
(b)	Attempts area of garden = $2x(2x-1) + 3x(6x-2)$	M1
	Sets $A < 72 \Rightarrow 22x^2 - 8x - 72 < 0$	A1
	Finds critical values $11x^2 - 4x - 36 \Rightarrow x = -\frac{18}{11}, 2$	M1
	Chooses inside region	ddM1
	$-\frac{18}{11} < x < 2$	A1
		(5)
(c)	$1.5 < x < 2$	B1
		(1)
		(9 marks)

Equations of straight lines:

1.

3.(a)	Attempts to make y the subject States $-\frac{3}{5}$ or exact equivalent	M1 A1 (2)
(b)	Uses perpendicular gradients rule $\Rightarrow$ gradient $l_2 = \frac{5}{3}$ Forms equation of l_2 using (6,-2) $y+2 = "\frac{5}{3}"(x-6)$ $y = \frac{5}{3}x - 12$	M1 M1 A1 (3) **(5 marks)**
Alt1(a)	Eg Coordinates of two points on the line $(0,1.4)$ and $(1,0.8)$ Gradient $= \frac{0.8-1.4}{1-0}$ Gradient $= -0.6$	M1 A1

2.

6.(a)	Attempts to find the gradient of $3x-4y+20=0 \Rightarrow y=\underline{\underline{\frac{3}{4}}}x+5$	M1
	Equation l_2 is $y-0=\text{"}\frac{3}{4}\text{"}(x-8) \Rightarrow 3x-4y-24=0$ oe	M1, A1
		(3)
(b)	$P=\left(-\frac{20}{3},0\right), Q=(0,5)$	B1
	Area $PQRS=$ $PR\times OQ=\left(8+\text{"}\frac{20}{3}\text{"}\right)\times\text{"}5\text{"}=\frac{220}{3}$	M1, A1
		(3)
(c)	$\overrightarrow{QR}=\begin{pmatrix}8\\-5\end{pmatrix} \Rightarrow S=\left(-\frac{20}{3}+8, 0-5\right)=\left(\frac{4}{3},-5\right)$	M1, A1
		(2) **(8 marks)**
Alt(c)	Solve their $y=\text{"}\frac{3}{4}x-6\text{"}$ with their $\text{"}y=-\frac{5}{8}\left(x+\frac{20}{3}\right)\text{"}$	
	$\frac{3}{4}x-6=-\frac{5}{8}\left(x+\frac{20}{3}\right) \Rightarrow \frac{11}{8}x=\frac{11}{6} \Rightarrow x=...$	M1
	$S=\left(\frac{4}{3},-5\right)$	A1
		(2)

3.

8(a)	$\frac{2}{5}$ or decimal equivalent	B1
		(1)
(b)	$m_N=-1\div\text{"}\frac{2}{5}\text{"}$	M1
	$y+2=\text{"}-\frac{5}{2}\text{"}(x-6)$	M1
	$y=-\frac{5}{2}x+13$	A1
		(3)
(c)	$\text{"}-\frac{5}{2}x+13\text{"}=\text{"}\frac{2}{5}x+\frac{7}{5}\text{"} \Rightarrow \text{"}\frac{29}{10}x\text{"}=\text{"}\frac{58}{5}\text{"} \Rightarrow x=...(=4)$ or $\text{"}\frac{5}{2}y-\frac{7}{2}\text{"}=\text{"}-\frac{2}{5}y+\frac{26}{5}\text{"} \Rightarrow \text{"}\frac{29}{10}y\text{"}=\text{"}\frac{87}{10}\text{"} \Rightarrow y=...(=3)$	M1
	$x=\text{"}4\text{"} \Rightarrow y=...$ or $y=\text{"}3\text{"} \Rightarrow x=...$	dM1
	$(4,3)$	A1
		(3)
(d)	$(2,8)$	B1B1
		(2)
		Total 9

4.

6(a)	E.g. $m=\dfrac{2-11}{8+4}$ or $m=\dfrac{11-2}{-4-8}$	M1
	$m=-\dfrac{3}{4}$	A1
		(2)
(b)	M is $\left(2,\ \dfrac{13}{2}\right)$	B1
	$m_N=-1\div"-\dfrac{3}{4}"$	M1
	$y-"\dfrac{13}{2}"="\dfrac{4}{3}"(x-"2")$	M1
	$8x-6y+23=0$	A1
		(4)
(c)	$AB=\sqrt{(-4-8)^2+(11-2)^2}(=15)$ or $AB^2=(-4-8)^2+(11-2)^2(=225)$	M1
	$\dfrac{1}{2}\times MC\times AB=37.5\Rightarrow MC=\dfrac{75}{15}\ (=5)$ or $MC^2=25$	M1
	$m_N=\dfrac{4}{3}, MC=5\Rightarrow\ C$ is $\left("2"-3,\ \dfrac{"13"}{2}-4\right)$ or $\left("2"+3,\ \dfrac{"13"}{2}+4\right)$	dM1
	$(-1,\ 2.5)$ **or** $(5,\ 10.5)$ **or** $x=-1, x=5$ **or** $y=2.5,\ y=10.5$	A1
	$(-1,\ 2.5)$ **and** $(5,\ 10.5)$	A1
		(5)
		(11 marks)

5.

5(a)	$3y-2x=30\Rightarrow m=\dfrac{2}{3}$	B1
	$y=-\dfrac{3}{2}(x-24),\ y=-\dfrac{3}{2}x+36,\ 2y+3x=72,\ \dfrac{y-0}{x-24}=-\dfrac{3}{2}$	M1 A1ft
	Full method to find one co-ordinate of P E.g. Solves $\dfrac{2}{3}x+10=-\dfrac{3}{2}(x-24)$	M1
	Coordinates of P $(12,18)$	A1
		(5)
(b)	$y=0,\ 3y-2x=30\Rightarrow x=\ldots$	M1 (B1 on EPEN)
	Area $BPA=\dfrac{1}{2}\times(24+"15")\times"18"=351$	dM1 A1 cso
		(3)
		(8 marks)

TOPIC 2: GEOMETRY

Sine and cosine laws/rules:

1.

7.(a)	Attempts $\frac{\sin\angle ACB}{6.5}=\frac{\sin 35}{4.7}$	M1	
	$\angle ACB=\text{awrt}(52 \text{ or } 53)°$ or $\text{awrt}(127\text{or}128)°$	A1	
	$\angle ACB = 127.5°$	A1	
			(3)
(b)	Eg $\frac{(AC)}{\sin"17.5°"}=\frac{6.5}{\sin"127.5°"}$ or $=\frac{4.7}{\sin 35°}$	M1	
	$\left[\frac{(CD)}{\sin"75°"}=\frac{4.7}{\sin"127.5°"}\Rightarrow(CD)=...\Rightarrow(AC)+(CD)\right]$=awrt 8.2	A1	
	Total length of wood $=8.1+6.5+4.7+4.7=\text{awrt}24.1$	A1	
		(6 marks)	(3)
Alt1(a)	$\cos 35=\frac{AC^2+6.5^2-4.7^2}{2\times 6.5\times AC}\Rightarrow AC^2-13\cos(35)AC+20.16=0\Rightarrow AC=...$		
	$\cos\angle ACB=\frac{"AC"^2+4.7^2-6.5^2}{2\times"AC"\times 4.7}$ oe	M1	

2.

3(a)	$\left(2\sqrt{2}\right)^2 = p^2 + q^2 - 2pq\cos 60°$ oe $p^2 + q^2 - pq = 8$ *	M1 A1*
		(2)
(b)	$q = p + 2 \Rightarrow 8 = p^2 + ("p+2")^2 - p("p+2")$ $p^2 + 2p - 4 = 0$ or $q^2 - 2q - 4 = 0$ $p = \dfrac{-2 \pm \sqrt{2^2 - 4\times1\times(-4)}}{2}$ or $q = \dfrac{2 \pm \sqrt{(-2)^2 - 4\times1\times(-4)}}{2}$ $p = -1 + \sqrt{5}$ or $q = 1 + \sqrt{5}$ $p = -1 + \sqrt{5}$ and $q = 1 + \sqrt{5}$ only	M1 A1 M1 B1 (A1 on EPEN) A1cso
		(5)
(c)	$\text{Area} = \dfrac{1}{2}\times("-1+\sqrt{5}")("1+\sqrt{5}")\times\sin 60°$ $\text{Area} = \sqrt{3}$ (m^2)	M1 A1
		(2)
Alt(a)	Forming a line *BX* which is perpendicular to *AC* where *X* is on the line *AC*. $AX = p\cos 60 = \dfrac{p}{2}$ $BX = \sqrt{p^2 - \left(\dfrac{p}{2}\right)^2} = \dfrac{\sqrt{3}}{2}p$ or $BX = p\sin 60$ $\left(\dfrac{\sqrt{3}}{2}p\right)^2 + \left(q - \dfrac{p}{2}\right)^2 = \left(2\sqrt{2}\right)^2$ or $(p\sin 60)^2 + \left(q - \dfrac{p}{2}\right)^2 = \left(2\sqrt{2}\right)^2$ $\dfrac{3p^2}{4} + q^2 - pq + \dfrac{p^2}{4} = 8$ $p^2 + q^2 - pq = 8$ *	M1 A1*
		(9 marks)

3.

2(a)	$AB = 21\text{ cm}, BC = 13\text{ cm}, \angle BAC = 25°, \angle ACB = x°$	
	$\frac{\sin x°}{21} = \frac{\sin 25°}{13}$ o.e	**M1**
	$\sin x° = 0.6827$ (awrt)	**A1**
		(2)
(b)	$\sin^{-1}(0.6827) = ...(43.05°)$	**M1**
	($AC < AB$ so $\angle ABC < \angle ACB$ so) required angle is $180° - \sin^{-1}(0.6827) = ...$	**M1**
	So $x =$ awrt 136.95	**A1**
		(3)
		(5 marks)

4.

4. (a)	Area $ABCD$ is 40 cm^2 $\Rightarrow 40 = 6 \times 10 \times \sin\theta$ oe	M1
	$\sin\theta = \frac{2}{3} \Rightarrow \theta = 180° - 41.8°$	M1
	$\angle DAB =$ awrt $138.19°$	A1
		(3)
(b)	Attempts $DB^2 = 10^2 + 6^2 - 2\times10\times6\cos"138.19°"$	M1
	$DB =$ awrt 15.0 (cm)	A1
		(2)
		(5 marks)

Arc length and sector:

1.

10. (a)	Correct equations $\frac{1}{2}r^2\theta=6, \quad 2r+r\theta=10$	B1 B1
	Eliminates $r=\frac{10}{2+\theta}\Rightarrow\frac{1}{2}\left(\frac{10}{2+\theta}\right)^2\theta=6$	M1
	$\Rightarrow 50\theta=6\left(4+4\theta+\theta^2\right)\Rightarrow\ 3\theta^2-13\theta+12=0$ *	A1*
		(4)
(b)	$(3\theta-4)(\theta-3)=0\Rightarrow\theta=\frac{4}{3},3$	B1
	$\theta=\frac{4}{3},r=3 \qquad \theta=3,r=2$	M1 A1
		(3) (7 marks)

2.

4 (a)	States or uses $\cos AOD=\frac{4}{12}\Rightarrow$ angle $AOD=1.231$ *	M1 A1*
		(2)
(b)	Attempts $\frac{1}{2}r^2\theta$ with $r=12$ and $\theta=\pi\pm1.231$ or 1.231	M1
	Attempts area $AOD=\frac{1}{2}\times"4"\times\sqrt{12^2-"4"^2}$ oe $(22.627...)$	M1
	Attempts sector – triangle $=\frac{1}{2}\times12^2\times(\pi+1.231)-\frac{1}{2}\times"4"\times\sqrt{12^2-"4"^2}$ $(314.8....)\ -\ (22.627...)$ **or** Attempts circle-sector-triangle $\pi\times12^2-\frac{1}{2}\times12^2\times(\pi-1.231)-\frac{1}{2}\times"4"\times\sqrt{12^2-"4"^2}$ $452.38...\ -\ 137.562....\ -\ 22.627......$	ddM1
	$=$ awrt 292.2 (m^2)	A1
		(4)
(c)	Attempts $s=r\theta$ with $r=12$ and $\theta=\pi\pm1.231$ or 1.231	M1
	Attempts $P=16+\sqrt{12^2-4^2}+12(\pi+1.231)$ oe	dM1
	$=$ awrt 79.8 (m)	A1
		(3) **(9 marks)**
Alt(a)	$AD=\sqrt{12^2-4^2}=8\sqrt{2}$ $\cos AOD=\frac{12^2+4^2-(8\sqrt{2})^2}{2\times12\times4}\Rightarrow$ angle $AOD=\cos^{-1}\left(\frac{1}{3}\right)=1.231$	M1A1*

3.

5 (a)	Attempts the sine rule $\frac{\sin\alpha}{14} = \frac{\sin 0.43}{6}$	M1
	$\Rightarrow \alpha = 1.337$ (radians) Accept awrt 1.33/1.34 or awrt 76.6/76.7 (°)	A1
	angle $AOD = \pi - 1.337 =$ awrt 1.805 (radians)	A1
		(3)
(b)	Attempts $s = r\theta$ with $r = 6$ and an allowable θ	M1
	Arc length ABC = awrt 26.9 m	A1
		(2)
(c)	Attempts $\frac{1}{2}r^2\theta$ with $r = 6$ and an allowable θ in radians (= 80.6)	M1
	Attempts area $AOD = \frac{1}{2}\times 6\times 14\times\sin(\text{"}0.91\text{"})$ oe (= 33.1)	M1
	Attempts sector + triangle with correct attempt at angles	dM1
	= 113.7 (m^2)	A1
		(4)
		(9 marks)

4.

5(a)	$\angle BOD = \pi - 2\times 0.7 = 1.742$ *	B1*
		(1)
(b)	Area of $BOD = \frac{1}{2}\times 3^2 \sin 1.742 \quad (= \text{awrt } 4.43)$	M1
	Area of R is: $\frac{1}{2}\times 3^2 \times 1.742 - \frac{1}{2}\times 3^2 \sin 1.742$ or $\frac{1}{2}\times\pi\times 3^2 - \frac{1}{2}\times 3^2 \sin 1.742 - 2\times\frac{1}{2}\times 3^2\times 0.7$	dM1
	= awrt 3.4 (m^2)	A1
		(3)
(c)	$BD = \sqrt{3^2 + 3^2 - 2\times 3\times 3\cos 1.742} \quad (= \text{awrt } 4.59)$ or $BD = 2\times 3\sin\left(\frac{1.742}{2}\right)$ or $BD = 2\times 3\cos 0.7$ or $BD = \dfrac{3\sin 1.742}{\sin\left(\frac{\pi - 1.742}{2}\right)}$ or arc $BCD = 3\times 1.742 \quad (= 5.226)$	M1
	Perimeter of R is: $3\times 1.742 + "BD"$	dM1
	= awrt 9.8 (m)	A1
		(3)
		Total 7

5.

7.(a)	Attempts to use $\frac{1}{2}r^2\theta$ with $r=6$ and any allowable angle θ	M1
	Full method to find area $\frac{1}{2}\times6^2\times(2\pi-0.7)$ or $\pi\times6^2-\frac{1}{2}\times6^2\times0.7$	M1
	$=100.5\text{ cm}^2$ (awrt)	A1
		(3)
(b)	Attempts $\frac{\sin\angle ADO}{6}=\frac{\sin0.7}{5}\Rightarrow\sin\angle ADO=0.77...$	M1 A1
	$\angle ADO=2.258$ (awrt)	A1
		(3)
(c)	Attempts arc length $ABC=6\times(2\pi-0.7)$ 33.50	M1
	Attempts length OD $\frac{\sin(\pi-0.7-"2.258")}{OD}=\frac{\sin0.7}{5}\Rightarrow OD=...$ 1.42	M1
	Full method to find perimeter = "33.50"+5+ 6−"1.42"	ddM1
	= 43.1 cm	A1
		(4)
		(10 marks)
Alt (c)	Alternative for arc length $ABC=12\pi-6\times0.7$	M1
	Alternative for finding OD using the cosine rule $OD^2=6^2+5^2-2\times6\times5\cos(\pi-0.7-"2.258")\Rightarrow OD$	M1
	Solutions where candidate changes to degrees Look for angle AOD = awrt 40° to score M marks	
7.(a)	Attempts to use $\frac{\theta}{360}\pi r^2$ with $r=6$ and angle $\theta=$ awrt 40 or 320	M1
	Full method to find area $\frac{(360-\text{awrt }40)}{360}\times\pi6^2$ or $\pi\times6^2-\frac{\text{awrt }40}{360}\times\pi6^2$	M1
	$=100.5\text{ cm}^2$ (awrt)	A1
		(3)
(b)	Attempts $\frac{\sin\angle ADO}{6}=\frac{\sin40°}{5}\Rightarrow\sin\angle ADO=0.77...$	M1 A1
	$\angle ADO=129.4°$ (awrt)	A1
		(3)
(c)	Attempts arc length $ABC=\frac{(360-40)}{360}\times2\pi6$ 33.50	M1
	Attempts length OD $\frac{\sin(180-40-"129.4")}{OD}=\frac{\sin40}{5}\Rightarrow OD=...$ 1.42	M1
	Full method to find perimeter = "33.50"+5+ 6−"1.42"	ddM1
	= 43.1 cm	A1
		(4)
		(10 marks)

6.

8(a)	With θ being the angle subtended by arc AB and ϕ being the angle subtended by arc CD	
	$15 = 9\times\theta \Rightarrow \theta = \frac{5}{3} = (1.67)$	**M1**
	Therefore $\phi = \frac{2\pi}{3} - \frac{5}{3} = (0.4277...)$	**dM1**
	So length of arc $CD = 84\times\left(\frac{2\pi}{3}-\frac{5}{3}\right) = 35.929... = 35.9\,\text{cm}$ (1 d.p.)* CSO	**A1***
		(3)
(b)	Perimeter $= 3\times(15+35.9...)+6\times(84-9)$	**M1**
	$=$ awrt 603 cm (602.787…)	**A1**
		(2)
(c)	FOR EXAMPLE Area of a "blade" is $\frac{1}{2}\times 84^2\times "\left(\frac{2\pi-5}{3}\right)"$ = awrt (1510)	**M1**
	Area of sector of inner circle between "blades" is $\frac{1}{2}\times 9^2\times "\frac{5}{3}"$ = (67.5)	**dM1** **A1**
	Total area is $3\left(\frac{1}{2}\times 84^2\times "\left(\frac{2\pi}{3}-\frac{5}{3}\right)" + \frac{1}{2}\times 9^2\times "\frac{5}{3}"\right) = ...(4729.577764\,\text{cm}^2)$	**ddM1**
	So area is awrt 0.473 m^2 or awrt 4730 cm^2	**A1**
		(5)
		(10 marks)

7.

1.(a)	Sets $\frac{1}{2}r^2\times 1.25 = 15 \Rightarrow r^2 = 24$	M1
	$\Rightarrow r = \sqrt{24}$ or $2\sqrt{6}$ (only)	A1
		(2)
(b)	Attempts $s = r\theta = 2\sqrt{6}\times 1.25$	M1
	Attempts $P = 2r + r\theta = 2\times 2\sqrt{6} + 2\sqrt{6}\times 1.25$	dM1
	$= \frac{13\sqrt{6}}{2}$ oe	A1
		(3)
		(5 marks)

8.

4	$y = 3x+4 \Rightarrow x^2 + (3x+4)^2 + 6x - 4(3x+4) = 4$ **or** $x = \frac{y-4}{3} \Rightarrow \left(\frac{y-4}{3}\right)^2 + y^2 + 6\left(\frac{y-4}{3}\right) - 4y = 4$	M1
	$5x^2 + 9x - 2\ (=0)$ or $5y^2 - 13y - 46\ (=0)$	M1A1
	$(5x-1)(x+2) = 0 \Rightarrow x = ...$ or $(5y-23)(y+2) = 0 \Rightarrow y = ...$	dM1
	$x = 0.2,\ x = -2$ or $y = 4.6,\ y = -2$	B1
	Substitutes their x into their $y = 3x + 4$ / Substitutes their y into their $x = \frac{y-4}{3}$	M1
	$x = 0.2\left(\text{or } \frac{1}{5}\right),\ y = 4.6\left(\text{or } 4\frac{3}{5} \text{ or } \frac{23}{5}\right)$ **and** $x = -2,\ y = -2$	A1
		(7 marks)

9.

7.(a)	$OB^2 = 0.6^2 + 1.4^2 - 2\times 0.6\times 1.4\cos 2 \Rightarrow OB^2 = \ldots$ or $OB = \ldots$	M1
	$OB = 1.738$	A1
	$\frac{\sin AOB}{1.4} = \frac{\sin 2}{"1.738"} \Rightarrow AOB = 0.822$ or e.g. $\frac{\sin ABO}{0.6} = \frac{\sin 2}{"1.738"} \Rightarrow ABO = 0.319\ldots \Rightarrow AOB = \pi - 2 - 0.319\ldots$	dM1
	$\theta = 2\times AOB = 2\times 0.822 = 1.64$ *	A1*
		(4)
(b)	Attempts $0.6\times\alpha$ with $\alpha = 2\pi - 1.64$ or $\alpha = \pi - 1.64$ or Attempts $2\times\pi\times 0.6 - 0.6\times 1.64$	M1
	$0.6\times(2\pi - 1.64) + 2.8 = 5.6\,\text{m}$	A1
		(2)
(c)	Attempts $\frac{1}{2}\times 0.6^2\times\alpha$ with $\alpha = 2\pi - 1.64$ or $\alpha = \pi - 1.64$ or Attempts $\pi\times 0.6^2 - \frac{1}{2}\times 0.6^2\times 1.64$	M1
	Attempts $0.6\times 1.4\sin 2$	M1
	Full method $\frac{1}{2}\times 0.6^2\times(2\pi - 1.64) + 0.6\times 1.4\sin 2 = 1.6\,\text{m}^2$	ddM1 A1
		(4)
		(10 marks)

10.

8(a)(i) **(ii)**	$2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}$ Area of sector $= \frac{1}{2}\times 3^2\times "\frac{4}{3}\pi" = 6\pi\ \ (\text{m}^2)$ Length of arc $= 3\times "\frac{4}{3}\pi" \Rightarrow$ Perimeter $= 4\pi + 6$ (m)	B1 M1A1 M1A1
		(5)
(b)	$\frac{1}{2}\times 3^2\times\sin\left(\frac{2}{3}\pi\right) = \frac{9\sqrt{3}}{4}\ \ (\text{m}^2)$	M1A1
		(2)
(c)	Eg $AB^2 = 3^2 + 3^2 - 2\times 3\times 3\times\cos\left(\frac{2}{3}\pi\right) \Rightarrow AB^2 = 27 \Rightarrow AB = 3\sqrt{3}$ (m) *	M1A1*
		(2)
(d)	$\frac{\sin BAC}{8} = \frac{\sin\left(\frac{\pi}{6}\right)}{3\sqrt{3}} \Rightarrow \sin BAC = \ldots.$ or $BAC = \ldots$ $\sin BAC = \text{awrt}\ \frac{4\sqrt{3}}{9}$ or $BAC = \text{awrt } 0.88\ \ (0.8785\ldots)$ Area $ABC = \frac{1}{2}\times 3\sqrt{3}\times 8\times\sin(\pi - \frac{\pi}{6} - "0.88")\ \ (= 20.4896\ldots.)$ Total area $=$ "18.8"+"3.90"+"20.5" $=$ awrt 43 (m^2)	M1 A1 M1 dM1 A1
		(5)
		(14 marks)

TOPIC 3: Calculus (differentiation/integration)

Differentiation, gradient, and equations of lines:

1.

1. (a)	$y=2x^3-5x^2-\frac{3}{2x}+7\Rightarrow\frac{dy}{dx}=6x^2-10x+\frac{3}{2x^2}$	M1 A1 A1
		(3)
(b)	$x=\frac{1}{2}\Rightarrow y=3$	B1
	Substitutes $x=\frac{1}{2}$ into their $\frac{dy}{dx}=6x^2-10x+\frac{3}{2x^2}=\ldots\left(=\frac{5}{2}\right)$	M1
	Uses the perpendicular gradient rule Eg. $\frac{5}{2}\to-\frac{2}{5}$	dM1
	Attempts the equation of the normal at P $y-3=-\frac{2}{5}\left(x-\frac{1}{2}\right)$	M1
	$2x+5y-16=0$ oe	A1
		(5)
		(8 marks)

2.

1.(a)	$\frac{dy}{dx}=\frac{1}{8}\times3x^2-24\times-\frac{1}{2}x^{-\frac{3}{2}}$	M1
	$\frac{dy}{dx}=\frac{3}{8}x^2+12x^{-\frac{3}{2}}$	A1 A1
		(3)
(b)	$\left.\frac{dy}{dx}\right\|_{x=4}=\frac{3}{8}\times4^2+12\times4^{-\frac{3}{2}}=(7.5)$	M1
	$y+3=7.5(x-4)\Rightarrow y=7.5x-33$	M1 A1
		(3)
		(6 marks)

3.

1(a)	$\left(\frac{dy}{dx}=\right)\ ...x^{1}+...x^{-\frac{3}{2}}+...x^{-2}$	M1
	$\left(\frac{dy}{dx}=\right)\ \frac{2}{3}x\ -2x^{-\frac{3}{2}}\ -\frac{8}{3}x^{-2}$	A1A1A1
		(4)
(b)	$\frac{dy}{dx}="\frac{2}{3}"\times4-"2"\times4^{"-\frac{3}{2}"}-"\frac{8}{3}"\times4^{"-2"}=\frac{9}{4}$	M1
	$\frac{9}{4}\rightarrow-\frac{4}{9}$	M1
	$y-3="-\frac{4}{9}"(x-4)$	dM1
	$4x+9y-43=0$	A1
		(4)
		(8 marks)

4.

5.(a)	$\frac{dy}{dx}=\frac{1}{2}x^{2}+2x^{-\frac{1}{2}}$	M1A1 A1
		(3)
(b)	$\left.\frac{dy}{dx}\right\|_{x=4}=\frac{1}{2}\times4^{2}+2\times\frac{1}{\sqrt{4}}=(9)$	M1
	Gradient of normal is $-\frac{1}{9}$	dM1
	$y-\frac{11}{3}=-\frac{1}{9}(x-4)\Rightarrow x+9y-37=0$	M1 A1
		(4)
		(7 marks)

5.

8(a)	$y=(x-2)(x^2-8x+16) \Rightarrow y=x^3-8x^2+16x-2x^2+16x-32 \Rightarrow$ $y=x^3 \pm ...x^2 \pm ...x \pm 32$	M1
	$=x^3-10x^2+32x-32$	A1
	$\frac{dy}{dx}=3x^2-20x+32$ *	M1A1*
		(4)
(b)	$x=6 \Rightarrow y=(6-2)(6-4)^2=16$	B1
	$\frac{dy}{dx}=3(6)^2-20(6)+32=20$	B1
	$y-"16"="20"(x-6)$	M1
	$y=20x-104$	A1
		(4)
(c)	$3x^2-20x+32="20" \Rightarrow 3x^2-20x+12=0$	M1
	$3x^2-20x+12=0 \Rightarrow (3x-2)(x-6)=0 \Rightarrow x=...$	dM1
	$\alpha=\frac{2}{3}$	A1
		(3)
		(11 marks)

6.

1(a)	$\left(\frac{dy}{dx}=\right)\ \ldots x^2+\ldots x+\ldots x^{-2}$	M1
	$\left(\frac{dy}{dx}=\right)\ \frac{3}{4}x^2-2x-17x^{-2}$	A1A1
		(3)
(b)	$\left(\frac{dy}{dx}=\right)\frac{3}{4}(2)^2-2(2)-17(2)^{-2}=-\frac{21}{4}$	M1
	$y-\frac{13}{2}=\text{"}-\frac{21}{4}\text{"}(x-2)$	dM1
	$21x+4y-68=0$	A1
		(3)
		(6 marks)

7.

9.	$\frac{4x^2+9}{2\sqrt{x}} = \frac{4x^2}{2\sqrt{x}} + \frac{9}{2\sqrt{x}} = 2x^{\frac{3}{2}} + \frac{9}{2}x^{-\frac{1}{2}}$	M1 A1
	$\left(\frac{dy}{dx}=\right)\ 3x^{\frac{1}{2}} - \frac{9}{4}x^{-\frac{3}{2}}$	M1 A1
	$\left(\frac{dy}{dx}=\right)\ 3x^{\frac{1}{2}} - \frac{9}{4}x^{-\frac{3}{2}} = 0 \Rightarrow x^2 = \frac{3}{4} \Rightarrow x = \frac{\sqrt{3}}{2}$	M1 A1
		(6) **(6 marks)**
Alt(I)	Quotient rule	
	$u = 4x^2+9, u' = 8x, v = 2\sqrt{x}, v' = x^{-\frac{1}{2}}$	M1A1
	$\left(\frac{dy}{dx}=\right)\ \frac{2\sqrt{x}\times 8x - (4x^2+9)\times x^{-\frac{1}{2}}}{4x}$	M1A1
	$\left(\frac{dy}{dx}=\right)\ \frac{2\sqrt{x}\times 8x - (4x^2+9)\times x^{-\frac{1}{2}}}{4x} = 0 \Rightarrow x^2 = \frac{3}{4} \Rightarrow x = \frac{\sqrt{3}}{2}$	M1A1
Alt(II)	Product rule	
	$u = 4x^2+9, u' = 8x, v = \frac{1}{2}x^{-\frac{1}{2}}, v' = \frac{1}{4}x^{-\frac{3}{2}}$	M1A1
	$\left(\frac{dy}{dx}=\right)\ (4x^2+9)\times\frac{1}{4}x^{-\frac{3}{2}} + 8x\times\frac{1}{2}x^{-\frac{1}{2}}$	M1A1
	$\left(\frac{dy}{dx}=\right)\ \frac{2\sqrt{x}\times 8x - (4x^2+9)\times x^{-\frac{1}{2}}}{4x} = 0 \Rightarrow x^2 = \frac{3}{4} \Rightarrow x = \frac{\sqrt{3}}{2}$	M1A1

8.

6.(a)	$f'(x) = 5x^{\frac{3}{2}} - 40$	M1A1
	Attempts $5x^{\frac{"3"}{2}} - 40 = 0 \Rightarrow x^{\frac{"3"}{2}} = ..$ $x = 4$	M1 A1 cao
		(4)
(b)	$f''(x) = \frac{15}{2}x^{\frac{1}{2}} = 5$	M1
	$\Rightarrow x^{\frac{1}{2}} = .. \Rightarrow x = ..^2$ $\qquad x = \frac{4}{9}$	M1 A1
		(3) **(7 marks)**

9.

7. (a)	$2x - 3\sqrt{x} - 5 = 9 \Rightarrow 2x - 3\sqrt{x} - 14 = 0$ and treats as quadratic equation	M1
	$\Rightarrow (2\sqrt{x} - 7)(\sqrt{x} + 2) = 0 \Rightarrow (\sqrt{x} =) \frac{7}{2}, (-2)$	A1
	$\Rightarrow x = \left(\frac{7}{2}\right)^2 = \frac{49}{4}$	dM1 A1
		(4)
(b)	$(f'(x) =)\ 2 - \frac{3}{2}x^{-\frac{1}{2}}$	B1
	$(f''(x) =)\ \frac{3}{4}x^{-\frac{3}{2}}$	M1 A1
	Attempts $\frac{3}{4}x^{-\frac{3}{2}} = 6 \Rightarrow x^{-\frac{3}{2}} = 8 \Rightarrow x = \frac{1}{4}$	dM1 A1
		(5) **(9 marks)**

10.

2.	$y = 3x^5 + 4x^3 - x + 5 \Rightarrow \left(\frac{dy}{dx} =\right) 15x^4 + 12x^2 - 1$	M1 A1
	$15x^4 + 12x^2 - 1 = 2 \Rightarrow 15x^4 + 12x^2 - 3 = 0$	dM1
	$\Rightarrow 3(5x^2 - 1)(x^2 + 1) = 0$ o.e	ddM1
	$\Rightarrow x = \pm\frac{1}{\sqrt{5}}$ o.e.	A1
		(5)
		(5 marks)

11.

10 (a)	$P=\left(-\frac{1}{2},0\right)$	B1
		(1)
(b)	$\text{f}(x)=(x-4)(2x+1)^2 \Rightarrow \text{f}(x)=ax^3+bx^2+cx+d$	M1
	$=4x^3-12x^2-15x-4$ oe	A1
	$\text{f}'(x)=12x^2-24x-15$	dM1 A1
		(4)
(c)	Attempts $\text{f}'(2.5)=12\times2.5^2-24\times2.5-15=0$	M1A1
	Finds y coordinate for $x=2.5$ $y=-54$	A1
		(3)
(d)	$a=-\frac{1}{2},(+)4$	B1, B1
		(2)
		(10 marks)

12.

7.(a)	Attempts $\frac{\text{d}y}{\text{d}x}=4x$ at $x=2$	M1
	At $x=2$ gradient of tangent $=8$	A1
		(2)
(b)	$(y_Q=)\ 2(2+h)^2+5$	B1
	Gradient $PQ=\frac{\text{their } y_Q-13}{2+h-2}$	M1
	$\left(=\frac{8h+2h^2}{h}\right)=8+2h$	A1
		(3)
(c)	States as $h\to0$ Gradient $PQ\to8$ = Gradient of tangent	B1
		(1)
		(6 marks)

13.

3.(a)	Attempts $\left(\frac{\text{d}y}{\text{d}x}=\right)$ $2x+3$ at $x=3$	M1
	At $x=3$ gradient of tangent $=9$	A1
		(2)
(b)	$(y_Q=)\ (3+h)^2+3(3+h)-2$	B1
	Gradient $PQ=\frac{(3+h)^2+3(3+h)-2-16}{3+h-3}=\frac{9h+h^2}{h}=9+h$	M1 A1
		(3)
(c)	States as $h\to0$ Gradient $PQ\to9$ = Gradient of tangent	B1
		(1)
		(6 marks)

14.

10(a)	$\frac{dy}{dx}=\frac{6}{7}x^2+\frac{2}{7}x-\frac{5}{2}$	**M1** **A1**
		(2)
(b)	At $x=-\frac{7}{2}$, $\frac{dy}{dx}=\frac{6}{7}\left(-\frac{7}{2}\right)^2+\frac{2}{7}\left(-\frac{7}{2}\right)-\frac{5}{2}=...(=7)$	**M1**
	So at B we know $\frac{dy}{dx}="-\frac{1}{7}"$	**M1**
	hence $\frac{6}{7}x^2+\frac{2}{7}x-\frac{5}{2}=-\frac{1}{7}$	**dM1**
	$\Rightarrow 12x^2+4x-35=-2\Rightarrow 12x^2+4x-33=0*$	**A1***
		(4)
(c)	E.g. $12x^2+4x-33=0\Rightarrow(2x-3)(6x+11)=0\Rightarrow x=...$	**M1**
	From graph we can see the x coordinate is positive, so $x=\frac{3}{2}$ at B	**A1**
		(2)
(d)	Equation of l is $y="-\frac{1}{7}"x-1$	**M1**
	Finds coordinates of A $x=-\frac{7}{2}\Rightarrow y="-\frac{1}{7}"\times-\frac{7}{2}-1=\left(-\frac{1}{2}\right)$	**dM1**
	Substitutes $x=-\frac{7}{2}, y="-\frac{1}{2}"$ into $y=\frac{2}{7}x^3+\frac{1}{7}x^2-\frac{5}{2}x+k\Rightarrow k=...$	**ddM1**
	$k=\frac{5}{4}$ CSO	**A1**
		(4)
		(12 marks)

Integration, gradient, and equations of lines:

1.

1.	$\int\left(\frac{8}{3}x^3-\frac{1}{2}x^{-\frac{1}{2}}-5\right)dx=\frac{8}{3}\times\frac{x^4}{4}-\frac{1}{2}\times 2x^{\frac{1}{2}}-5x+c$	M1 A1
	$=\frac{2}{3}x^4-x^{\frac{1}{2}}-5x+c$	A1 A1
		(4 marks)

2.

1.	$\int\left(\frac{8x^3}{5}-\frac{2}{3x^4}-1\right)dx=\frac{1}{4}\times\frac{8x^4}{5}-\frac{2}{3}\times\frac{1}{-3}x^{-3}-x$	M1 A1
	$\frac{2}{5}x^4+\frac{2}{9}x^{-3}-x+c$	A1 A1
		(4)
		Total 4

3.

4.	$\frac{4x^2+1}{2\sqrt{x}}=\frac{4x^2}{2\sqrt{x}}+\frac{1}{2\sqrt{x}}=2x^{\frac{3}{2}}+\frac{1}{2}x^{-\frac{1}{2}}$	M1 A1
	$\int\frac{4x^2+1}{2\sqrt{x}}dx=\frac{4}{5}x^{\frac{5}{2}}+x^{\frac{1}{2}}+c$	M1 A1 A1
		(5 marks)

4.

4	$\int\frac{3x^{\frac{3}{2}}-15x^{\frac{1}{2}}+2x-10}{4\sqrt{x}}dx=\int\frac{3}{4}x-\frac{15}{4}+\frac{1}{2}x^{\frac{1}{2}}-\frac{5}{2}x^{-\frac{1}{2}}dx$	M1A1A1
	$x^n\to x^{n+1}$	dM1
	$\frac{3}{8}x^2-\frac{15}{4}x+\frac{1}{3}x^{\frac{3}{2}}-5x^{\frac{1}{2}}+C$	A1A1
		(6 marks)

5.

1	$\int 10x^5+6x^3-\frac{3}{x^2}dx=10\times\frac{x^6}{6}+6\times\frac{x^4}{4}-3\times\frac{x^{-1}}{-1}(+c).$	**M1 A1**
	$=\frac{5x^6}{3}+\frac{3x^4}{2}+\frac{3}{x}+c$	**A1A1**
		(4)
		(4 marks)

6.

1.	$\int 12x^3 + \frac{1}{6\sqrt{x}} - \frac{3}{2x^4}\,dx = 12\times\frac{x^4}{4} + \frac{1}{6}\times 2x^{\frac{1}{2}} - \frac{3}{2}\times\frac{x^{-3}}{-3}$	M1
	$= 3x^4 + \frac{1}{3}x^{\frac{1}{2}} + \frac{1}{2}x^{-3} + c$	A1A1A1A1
		(5)
		(5 marks)

7.

2(a)	$a = 2$	B1
	$b = -3$	B1
		(2)
(b)	Any two term of $\int \frac{2x^3 - 3x^2 - 32x - 15}{5\sqrt{x}}\,dx = \int \frac{2}{5}x^{\frac{5}{2}} - \frac{3}{5}x^{\frac{3}{2}} - \frac{32}{5}x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}\,dx$	M1A1
	$x^n \to x^{n+1}$	M1
	$\frac{4}{35}x^{\frac{7}{2}} - \frac{6}{25}x^{\frac{5}{2}} - \frac{64}{15}x^{\frac{3}{2}} - 6x^{\frac{1}{2}} + c$	A1A1
		(5)
		(7 marks)

8.

12.(a)	Substitutes $x = 4$ in $\frac{dy}{dx} = 3x\sqrt{x} - 10x^{-\frac{1}{2}} = 3\times 4\times 2 - \frac{10}{2} = 19$	M1A1
	Attempts $(y - (-2)) = "19"\times(x - 4) \Rightarrow y = 19x - 78$	M1A1 cao
		(4)
(b)	$f'(x) = 3x^{\frac{3}{2}} - 10x^{-\frac{1}{2}} \Rightarrow f(x) = \frac{6}{5}x^{\frac{5}{2}} - 20x^{\frac{1}{2}} + c$	M1 A1 A1
	$x = 4, f(x) = -2 \Rightarrow$	
	$-2 = 38.4 - 40 + c \Rightarrow c = ...(-0.4)$	M1
	$[f(x) =]\ \frac{6}{5}x^{\frac{5}{2}} - 20x^{\frac{1}{2}} - 0.4$	A1 cso
		(5)
		(9 marks)

9.

1.	$\int \frac{2}{3}x^3 - \frac{1}{2x^3} + 5\,dx = \frac{2}{3}\times\frac{x^4}{4} - \frac{1}{2}\times\frac{x^{-2}}{-2} + 5x + c$	M1 A1
	$= \frac{1}{6}x^4 + \frac{1}{4}x^{-2} + 5x + c$	A1 A1
		(4 marks)

10.

11 (a)	Gradient of normal $= \frac{1}{4}$	B1
	Equation of normal $(y+50) = "\frac{1}{4}"(x-4) \Rightarrow y = \frac{1}{4}x - 51$	M1 A1
		(3)
(b)	$(f''(x) =)\ \frac{6}{\sqrt{x^3}} + x = 6x^{-\frac{3}{2}} + x \Rightarrow f'(x) = -12x^{-\frac{1}{2}} + \frac{1}{2}x^2 + k$	M1 A1
	Substitutes $x = 4,\ f'(x) = -4 \Rightarrow k = -6$	dM1 A1
	$(f'(x) =)\ -12x^{-\frac{1}{2}} + \frac{1}{2}x^2 - 6 \Rightarrow (f(x) =)\ -24x^{\frac{1}{2}} + \frac{1}{6}x^3 - 6x + d$	dM1 A1ft
	Substitutes $x = 4,\ f(x) = -50 \Rightarrow d = \frac{34}{3}$	dddM1
	$(f(x) =) -24x^{\frac{1}{2}} + \frac{1}{6}x^3 - 6x + \frac{34}{3}$	A1
		(8) **(11 marks)**

11.

9. (i)	$\frac{(3x+2)^2}{4\sqrt{x}} = \frac{9x^2 + 12x + 4}{4\sqrt{x}} = \frac{9}{4}x^{\frac{3}{2}} + 3x^{\frac{1}{2}} + x^{-\frac{1}{2}}$	M1
	$\int \frac{(3x+2)^2}{4\sqrt{x}}\,dx = \frac{2}{5}\times\frac{9}{4}x^{\frac{5}{2}} + \frac{2}{3}\times 3x^{\frac{3}{2}} + 2\times x^{\frac{1}{2}}(+c) = \frac{9}{10}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$	dM1 A1 A1 A1
		(5)
(ii)	$f'(x) = x^2 + ax + b$	
	Attempts to use $f'(3) = 2 \Rightarrow$ $2 = 9 + 3a + b$	M1
	Attempts to integrate $(f(x) =)\ \frac{1}{3}x^3 + \frac{1}{2}ax^2 + bx + c$	M1
	Attempts to use y intercept $= -8$ and $(3,-2)$ in $f(x) = \frac{1}{3}x^3 + \frac{1}{2}ax^2 + bx + c$	dM1
	Correct equation in a and b $-2 = 9 + \frac{9}{2}a + 3b - 8$	A1
	Solves simultaneously to get values for a and b	ddM1
	$a = -4, b = 5 \Rightarrow (f(x) =)\ \frac{1}{3}x^3 - 2x^2 + 5x - 8$	A1
		(6) **(11 marks)**

12.

6(a)(i)	$x=4 \Rightarrow f'(4)=\frac{(4+3)^2}{4\sqrt{4}}=\frac{49}{8}$ (6.125)	B1
(ii)	$y-20="\frac{49}{8}"(x-4)$	M1A1ft
	$49x-8y-36=0$	A1
		(4)
(b)	$f'(x)=\frac{(x+3)^2}{x\sqrt{x}}=\frac{x^2+6x+9}{x\sqrt{x}}$ $=\frac{x^2}{x\sqrt{x}}+\frac{6x}{x\sqrt{x}}+\frac{9}{x\sqrt{x}}=...$	M1
	$=x^{\frac{1}{2}}+6x^{-\frac{1}{2}}+9x^{-\frac{3}{2}}$	A1
	$(f(x)=)\frac{2}{3}x^{\frac{3}{2}}+12x^{\frac{1}{2}}-18x^{-\frac{1}{2}}+c$	dM1 A1A1
	$20=\frac{2}{3}(4)^{\frac{3}{2}}+12(4)^{\frac{1}{2}}-18(4)^{-\frac{1}{2}}+c \Rightarrow c=...$	M1
	$(f(x)=)\frac{2}{3}x^{\frac{3}{2}}+12x^{\frac{1}{2}}-18x^{-\frac{1}{2}}-\frac{1}{3}$	A1
		(7)
		Total 11

13.

8.(a)	Substitutes $x=4$ in $f'(4)=4\times2-2-\frac{8}{3\times4^2}=\left(\frac{35}{6}\right)$	M1
	Attempts to find the gradient of the perpendicular $=-\frac{6}{35}$	dM1
	Attempts the normal $y-1=-\frac{6}{35}\times(x-4)\Rightarrow 6x+35y-59=0$	M1A1
		(4)
(b)	$f'(x)=4x^{\frac{1}{2}}-2-\frac{8}{3x^2}\Rightarrow f(x)=\frac{8}{3}x^{\frac{3}{2}}-2x+\frac{8}{3x}(+c)$	M1 A1 A1
	$x=4, f(x)=1\Rightarrow 1=\frac{8}{3}\times8-8+\frac{2}{3}+c\Rightarrow c=...(-13)$	dM1
	$f(x)=\frac{8}{3}x^{\frac{3}{2}}-2x+\frac{8}{3x}-13$	A1
		(5) **(9 marks)**

14.

7(a)	$f'(x)=2x^{-\frac{1}{2}}+Ax^{-2}+3 \Rightarrow f''(x)=\ldots x^{-\frac{1}{2}-1}+-\ldots x^{-2-1}$	M1
	$\Rightarrow f''(x)=2\times-\frac{1}{2}x^{-\frac{3}{2}}+-2Ax^{-3}=-x^{-\frac{3}{2}}-2Ax^{-3}$	A1
	$f''(4)=0\Rightarrow -4^{-\frac{3}{2}}-2A\times4^{-3}=0\Rightarrow A=\ldots$	dM1
	$-\frac{1}{8}-\frac{2A}{64}=0\Rightarrow A=-4$	A1
		(4)
(b)	$f(x)=\int 2x^{-\frac{1}{2}}+Ax^{-2}+3\,dx=\frac{2x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+\frac{Ax^{-2+1}}{-2+1}+3x(+c)$	M1
	$=4x^{\frac{1}{2}}-\frac{A}{x}+3x(+c)$	A1ft
	$f(12)=8\sqrt{3}\Rightarrow 4\sqrt{12}-\frac{A}{12}+36+c=8\sqrt{3}\Rightarrow c=\ldots$	dM1
	$c=8\sqrt{3}-4\sqrt{12}-36-\frac{4}{12}=-\frac{109}{3}$ or follow through $c=\frac{A}{12}-36$	A1ft
	So $(f(x))=4x^{\frac{1}{2}}+\frac{4}{x}+3x-\frac{109}{3}$ oe	A1
		(5)
		(9 marks)

15.

11. (a)	Attempts $y-\frac{32}{3}=5(x-4)\Rightarrow y=5x-\frac{28}{3}$	M1 A1 (2)
(b)	$f''(x)=\frac{4}{\sqrt{x}}-3\Rightarrow f'(x)=8x^{\frac{1}{2}}-3x+k$	M1 A1
	Substitutes $x=4,\ f'(x)=5\Rightarrow k=1$	dM1 A1
	$f'(x)=8x^{\frac{1}{2}}-3x+1\Rightarrow f(x)=\frac{16}{3}x^{\frac{3}{2}}-\frac{3}{2}x^2+x+d$	dM1 A1
	Substitutes $x=4,\ f(x)=\frac{32}{3}\Rightarrow d=-12$	ddM1
	$f(x)=\frac{16}{3}x^{\frac{3}{2}}-\frac{3}{2}x^2+x-12$	A1
		(8) (10 marks)

16.

9	$$\frac{21x^3-5x}{2\sqrt{x}}=\alpha x^{\frac{5}{2}}+\ldots \text{ or } \frac{21x^3-5x}{2\sqrt{x}}=\ldots+\beta x^{\frac{1}{2}}$$	M1
	$$\mathrm{f}(x)=\frac{27}{3}x^3-\frac{21}{2}\times\frac{2}{7}x^{\frac{7}{2}}+\frac{5}{2}\times\frac{2}{3}x^{\frac{3}{2}}(+c) \quad \left(=9x^3-3x^{\frac{7}{2}}+\frac{5}{3}x^{\frac{3}{2}}(+c)\right)$$	M1A1A1
	$$\mathrm{f}(9)=10\Rightarrow 9(9)^3-3(9)^{\frac{7}{2}}+\frac{5}{3}(9)^{\frac{3}{2}}+c=10\Rightarrow c=\ldots$$	dM1
	$$(\mathrm{f}(x)=)\ 9x^3-3x^{\frac{7}{2}}+\frac{5}{3}x^{\frac{3}{2}}-35$$	A1
		(6 marks)

17.

10(a)	$$\mathrm{f}'(x)=ax-12x^{\frac{1}{3}}\Rightarrow \mathrm{f}''(x)=a-4x^{-\frac{2}{3}}$$	B1
	Sets $\mathrm{f}''(27)=0\Rightarrow 0=a-4\times\frac{1}{9} \qquad \Rightarrow a=\frac{4}{9}$	M1 A1
		(3)
(b)	$$\mathrm{f}'(x)=ax-12x^{\frac{1}{3}}\Rightarrow(\mathrm{f}(x)=)\frac{1}{2}ax^2-9x^{\frac{4}{3}}+c$$	M1 A1ft
	Substitutes $x=1, \mathrm{f}(x)=-8\Rightarrow c=\ldots$	dM1
	$$(\mathrm{f}(x)=)\frac{2}{9}x^2-9x^{\frac{4}{3}}+\frac{7}{9}$$	A1
		(4)
		(7 marks)

18.

5(a)	$f'(x) = 12x^{-\frac{1}{2}} + \frac{x}{3} - 4$	
	One of $x^{-\frac{1}{2}} \to x^{\frac{1}{2}}$, $-4 \to -4x$, $x \to x^2$	M1
	$f(x) = \int 12x^{-\frac{1}{2}} + \frac{x}{3} - 4\,dx = 24x^{\frac{1}{2}} + \frac{x^2}{6} - 4x + c$	A1A1
	$8 = 24(9)^{\frac{1}{2}} + \frac{(9)^2}{6} - 4(9) + c \Rightarrow c = ...$	dM1
	$(f(x) =)\ 24x^{\frac{1}{2}} + \frac{x^2}{6} - 4x - \frac{83}{2}$	A1
		(5)
(b)	$f'(9) = \frac{12}{\sqrt{9}} + \frac{9}{3} - 4 \quad (= 3)$	M1
	$3 \to -\frac{1}{3}$	dM1
	$y - 8 = "-\frac{1}{3}"(0 - 9)$	M1
	$(0, 11)$	A1
		(4)
		(9 marks)

19.

6(a)	$$f'(8)=\frac{32}{3\times 8^2}+3-2\sqrt[3]{8}\quad\left(=-\frac{5}{6}\right)$$	M1
	$$y-2="-\frac{5}{6}"(x-8)$$	dM1
	$$y=-\frac{5}{6}x+\frac{26}{3}$$	A1
		(3)
(b)	$$f'(x)=\frac{32}{3x^2}+3-2\sqrt[3]{x}=...x^{-2}+3+...x^{\frac{1}{3}}$$	
	$$x^{-2}\to x^{-1},\ 3\to 3x,\ x^{\frac{1}{3}}\to x^{\frac{4}{3}}$$	M1
	$$f(x)=\int\frac{32}{3}x^{-2}+3-2x^{\frac{1}{3}}dx=-\frac{32}{3}x^{-1}+3x-\frac{3}{2}x^{\frac{4}{3}}+c$$	A1A1
	$$2=-\frac{32}{3}\times 8^{-1}+3\times 8-\frac{3}{2}\times 8^{\frac{4}{3}}+c\Rightarrow c=...$$	dM1
	$$(f(x)=)\ -\frac{32}{3}x^{-1}+3x-\frac{3}{2}x^{\frac{4}{3}}+\frac{10}{3}$$	A1
		(5)
		(8 marks)

TOPIC 4: Transformation and Trigonometry

Transformations:

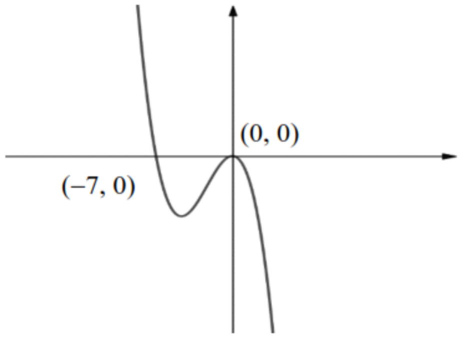
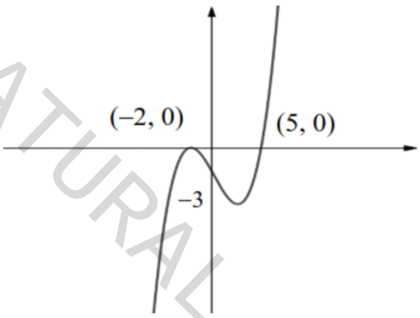
1.

7(a)	$4\times -2\times -9=72$ $p="72"-50$	M1
	$(p=)\ 22$	A1
		(2)
(b)	$(q=)-4,\ 2,\ 4.5$	B1B1
		(2)
(c)	$\mathrm{f}(x)=(x+4)(x-2)(2x-9)$ $\mathrm{f}(x)=(x^2+2x-8)(2x-9)=...x^3\pm...x^2\pm...x\ (\pm...)$	M1
	$=2x^3-5x^2-34x\ \ (+72)$	A1
	$(\mathrm{f}'(x)=)\ 6x^2-10x-34$	M1A1
		(4)
(d)	$"6x^2-10x-34"=-18$ $"6x^2-10x-16"=0\Rightarrow x=...\left(-1,\ \frac{8}{3}\right)$	M1
	$-1<x<\frac{8}{3}$	dM1A1
		(3)
		Total 11

2.

10 (a)	$\mathrm{f}(x)\leqslant 0\Rightarrow\quad x\leqslant -\frac{5}{2},\quad x=3$	M1 A1
		(2)
(b)	$\mathrm{f}(x)=(2x+5)(x-3)^2=(2x+5)(x^2-6x+9)$	M1
	$=2x^3-12x^2+18x+5x^2-30x+45$	M1
	$=2x^3-7x^2-12x+45$	A1
		(3)
(c)	(i) $P(0,45)$	B1ft
	(ii) Gradient $=-12$	B1ft
		(2)
(d)	(i) $\mathrm{g}(x)=(2(x-2)+5)(x-2-3)^2=(2x+1)(x-5)^2$	M1 A1
	(ii) 25	B1
		(3)
		(10 marks)

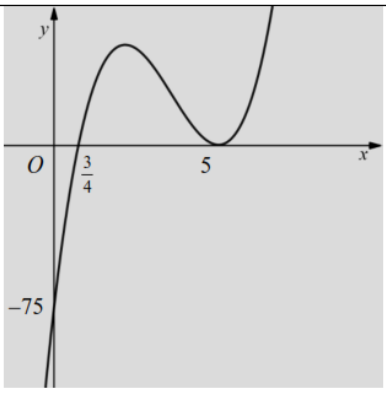
3.

5(i)(a)		B1 Horizontal translation ← B1 Maximum at origin B1 $(-7, 0)$	B1B1B1
			(3)
(b)		B1 Reflection in y-axis B1 Touches at $(-2, 0)$ and passes through $(5, 0)$ B1 Passes through $(0, -3)$	B1B1B1
			(3)
(ii)a	$x = 0 \Rightarrow y = k\cos\left(\frac{\pi}{6}\right) = \sqrt{3}$ $k\frac{\sqrt{3}}{2} = \sqrt{3} \Rightarrow k = 2$		B1
(b)	$(p =)\frac{\pi}{3}$ **or** $(q =)\frac{4\pi}{3}$ $(p =)\frac{\pi}{3}$ **and** $(q =)\frac{4\pi}{3}$		B1 B1
			(3)
			(9 marks)

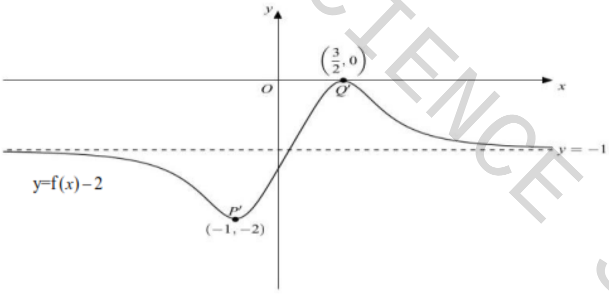
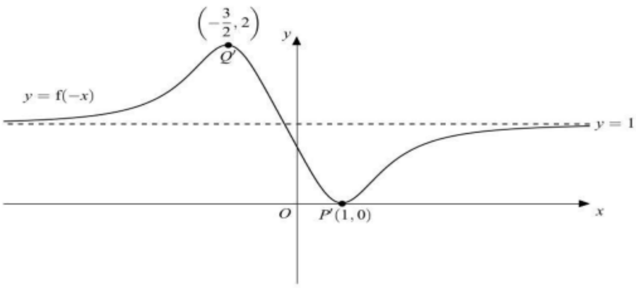
4.

8 (a)	States $y = 4$	B1	(1)
(b)	States $(16, 9)$ only	B1	(1)
(c)	$k \leqslant 4, k = 9$	B1, B1	(2)
(d) (i)	$a = 6$	B1	
(ii)	$y = \mathrm{f}(x-3)$	B1	(2)
			(6 marks)

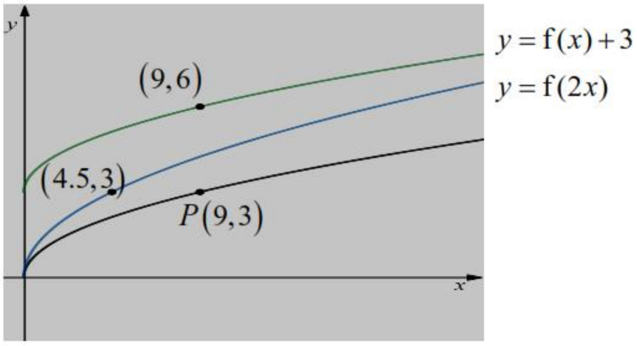
5.

10.(a)	 Shape for +ve x^3	B1
	Cuts x-axis at $\left(\frac{3}{4},0\right)$ and meets at $(5,0)$	B1
	Crosses y-axis at $(0,-75)$	B1
		(3)
(b)(i)	$(x=)\ 3, 20$	B1ft
(ii)	$(p=)75$	B1ft
		(2)
(c) (i)	$(\mathrm{g}(x)=)\ \left(4(x+1)-3\right)(x+1-5)^2=(4x+1)(x-4)^2$	M1 A1
(ii)	16	B1
		(3)
		(8 marks)

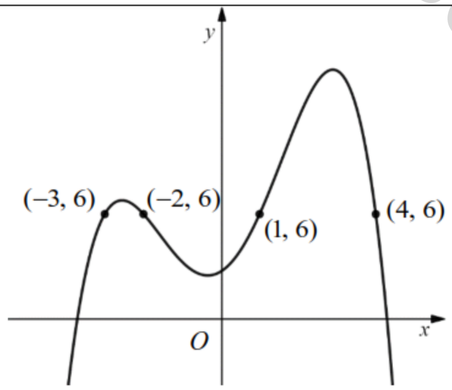
6.

4(i)		Correct shape, translated down.	**B1**
		Correct horizontal asymptote labelled	**B1**
		Correct **maximum** and **minimum** points labelled	**B1**
			(3)
(ii)		Correct shape, reflected in y axis	**B1**
		Correct horizontal asymptote labelled	**B1**
		Correct **maximum** and **minimum** points labelled.	**B1**
			(3)
			(6 marks)

7.

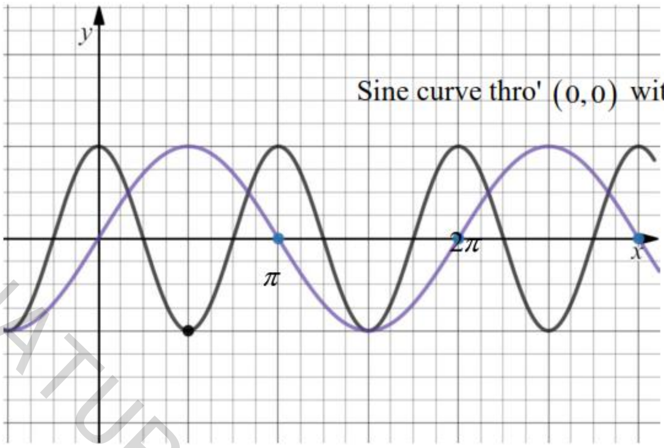
9(a)		
	One correct sketch drawn and labelled correctly	M1
	One correct sketch drawn and labelled and with correct point	A1
	Completely correct sketches with both points	A1
		(3)
(b)	Sets $\sqrt{x} + 3 = \sqrt{2x}$	B1
	$3 = \left(\sqrt{2} - 1\right)\sqrt{x}$	M1
	$\sqrt{x} = \frac{3}{\left(\sqrt{2}-1\right)} \times \frac{\left(\sqrt{2}+1\right)}{\left(\sqrt{2}+1\right)} = 3\left(\sqrt{2}+1\right)$ *	A1*
		(3)
(c)	$\sqrt{x} = 3\left(\sqrt{2}+1\right) \Rightarrow x = 9\left(\sqrt{2}+1\right)^2 = \ldots$	M1
	$\Rightarrow x = 9\left(3 + 2\sqrt{2}\right),\ y = 3\sqrt{2} + 6$	A1, B1
		(3)
		(9 marks)

8.

7(a)	$-1 < x < 2$ $x < -4,\ x > 3$	M1A1 B1
		(3)
(b)	$(x =)\ 1.5$	B1
		(1)
(c)(i)		B1B1B1
(ii)	$-3 \leqslant x \leqslant -2$	B1
		(4)
		(8 marks)

Trigonometry and graphs:

1.

5. (a)	$\left(\frac{\pi}{2}, -1\right)$	B1 B1
		(2)
(b)	 Sine curve thro' $(0,0)$ with max/min of ± 1	M1
	Fully correct	A1
		(2)
(c)	(i) 30 but follow through on $10\times$ the number of their solutions $0 \to 2\pi$	B1ft
	(ii) 32	B1
		(2)
		(6 marks)

2.

7. (a)(i)	$P = (90°, 3)$	M1, A1
(ii)	$Q = (540°, 0)$	B1
		(3)
(b)	$(270°, 4)$	M1, A1
		(2)
		(5 marks)

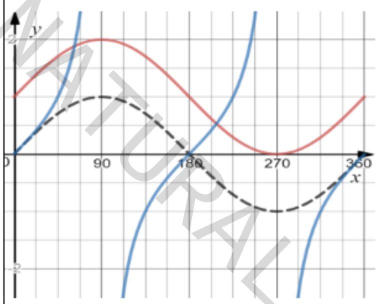
3.

3. (a)(i)	$P(-180, -4)$	B1, B1
(ii)	$Q(450, 0)$	B1
		(3)
(b)	$R(360, 7)$	B1, B1
		(2)
		(5 marks)

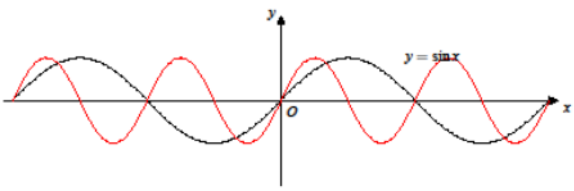
4.

Number		
9(a)	$(A=)-3$	B1
		(1)
(b)	$y=3$	B1
	eg $x=30+5\times180$ or $x=210+720$ or $x=180+2\times360+30$	M1
	$x=930$	A1
		(3)
		Total 4

5.

9. (a)	$(270°,-4)$	B1 B1 **(2)**
(b)	 For $y=1+\sin\theta$ $y=\tan\theta$	B1 B1 **(2)**
(c)	(i) $6\times2=12$ (ii) 11	M1 A1 B1 ft **(3)**
		(7 marks)

6.

9(a)	(i) $2p$		**B1**
	(ii) $-p$		**B1**
	(iii) $3-p$		**B1**
			(3)
(b)		Correct shape, same height starting at *O*, scaling may be incorrect.	**M1**
		Two repeats of the sinx graph each side	**A1**
			(2)
(c)	For $x=\frac{\alpha}{2}$		**B1**
	Attempt at second root E.g. $x=\frac{180°-\alpha}{2}$		**M1**
	$x=90°-\frac{\alpha}{2}$		**A1**
			(3)
			(8 marks)

7.

9. (a)	24π	B1 **(1)**
(b)	$(18\pi, -1)$	B1ft **(1)**
(c)(i)	$-12\pi - \alpha$	B1 ft
(ii)	$6\pi - \alpha$	B1 ft **(2)**
		(4 marks)

8.

4.(a)(i)	$(90, -1)$	B1 B1
(ii)	225	B1
		(3)
(b)	One of $-1 < p < 0$, $p = 1$	M1
	Both $-1 < p < 0$, $p = 1$	A1
		(2)
		(5 marks)