“Knowing the path is good but not enough, walking the path with determination leads to destiny”



# AS CAMBRIDGE Paper 1/9709 CLASSIFIED QUESTIONS

# Cambridge AS Mathematics

## Mark scheme for topic-wise practice workbook

Contents

# Algebra

## Topic 1: Quadratics

1.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 9(a) | $\left[2(x+3)^2\right]\ [-7]$ | B1B1 | Stating $a=3, b=-7$ gets B1B1 |
| | | 2 | |

2.

| Question | Answer | Marks |
|---|---|---|
| 9(a) | $\left[(x-2)^2\right]\ [-1]$ | B1 B1 |
| | | 2 |

3.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(a) | $\left[(x+3)^2\right]\ [-4]$ | B1 B1 | |
| | | 2 | |

4.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | $u=2x-3$ leading to $u^4-3u^2-4\ [=0]$ | M1 | Or $u=(2x-3)^2$ leading to $u^2-3u-4\ [=0]$ |
| | $(u^2-4)(u^2+1)\ [=0]$ | M1 | Or $(u-4)(u+1)\ [=0]$ |
| | $2x-3=[\pm]2$ | A1 | |
| | $x=\frac{1}{2}, \frac{5}{2}$ **only** | A1 | |
| | | 4 | |

5.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(a) | $(4x-3)^2$ or $(4x+(-3))^2$ or $a=-3$ | B1 | $k(4x-3)^2$ where $k\neq 1$ scores B0 but mark final answer, allow recovery. |
| | $+1$ or $b=1$ | B1 | |
| | | 2 | |
| 1(b) | [For one root] $k=1$ or '*their b*' | B1 FT | Either by inspection or solving or from $24^2-4\times 16\times(10-k)=0$ WWW |
| | [Root or $x=$]$\frac{3}{4}$ or 0.75 | B1 | **SC B2** for correct final answer WWW. |
| | | 2 | |

6.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | $kx^2+2x-k=kx-2$ leading to $kx^2+(-k+2)x-k+2\ [=0]$ | *M1 | Eliminate *y* and form 3-term quadratic. Allow 1 error. |
| | $(-k+2)^2-4k(-k+2)$ | DM1 | Apply $b^2-4ac$ ; allow 1 error but *a*, *b* and *c* must be correct for *their* quadratic. |
| | $5k^2-12k+4$ or $(-k+2)(-k+2-4k)$ | A1 | May be shown in quadratic formula. |
| | $(-k+2)(-5k+2)$ | DM1 | Solving a 3-term quadratic in *k* (all terms on one side) by factorising, use of formula or completing the square. Factors must expand to give *their* coefficient of $k^2$. |
| | $\frac{2}{5}<k<2$ | A1 | WWW, accept two separate correct inequalities. If M0 for solving quadratic, **SC B1** can be awarded for correct final answer. |
| | | **5** | |

7.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(a) | $\left\{-3(x-2)^2\right\}$ $\{+14\}$ | B1 B1 | B1 for each correct term; condone $a=2$, $b=14$. |
| | | **2** | |

1.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 9(a) | $\left[2(x+3)^2\right][-7]$ | B1B1 | Stating $a=3, b=-7$ gets B1B1 |
| | | 2 | |
| 9(b) | $y=2(x+3)^2-7 \rightarrow 2(x+3)^2=y+7 \rightarrow (x+3)^2=\frac{y+7}{2}$ | M1 | First 2 operations correct. Condone sign error or with *x*/*y* interchange |
| | $x+3=(\pm)\sqrt{\frac{y+7}{2}} \rightarrow x=(\pm)\sqrt{\frac{y+7}{2}}-3 \rightarrow f^{-1}(x)=-\sqrt{\frac{x+7}{2}}-3$ | A1FT | FT on *their a* and *b*. Allow *y* = ... |
| | Domain: $x \geqslant -5$ **or** $\geqslant -5$ **or** $[-5, \infty)$ | B1 | Do not accept $y=\ldots,\ f(x)=\ldots, f^{-1}(x)=\ldots$ |
| | | 3 | |
| 9(c) | $fg(x) = 8x^2 - 7$ | B1FT | SOI. FT on *their* –7 from part **(a)** |
| | $8x^2-7=193 \rightarrow x^2=25 \rightarrow x=-5$ only | B1 | |
| | **Alternative method for question 9(c)** | | |
| | $g(x) = f^{-1}(193) \rightarrow 2x-3=-\sqrt{100}-3$ | M1 | FT on *their* $f^{-1}(x)$ |
| | $x=-5$ only | A1 | |
| | | 2 | |
| 9(d) | (Largest *k* is) $-\frac{1}{2}$ | B1 | Accept $-\frac{1}{2}$ or $k \leqslant -\frac{1}{2}$ |
| | | 1 | |

2.

| Question | Answer | Marks |
|---|---|---|
| 4(a) | $-1 \leqslant f(x) \leqslant 2$ | B1 B1 |
| | | 2 |

3.

| Question | Answer | Marks |
|---|---|---|
| 6(a) | $3(3x+b)+b=9x+4b \rightarrow 10 = 18 + 4b$ | M1 |
| | $b=-2$ | A1 |
| | **Either** $f(14) = 2$ **or** $f^{-1}(x) = 2(x + a)$ etc. | M1 |
| | $a = 5$ | A1 |
| | | 4 |
| 6(b) | $gf(x) = 3\left(\frac{1}{2}x-5\right)-2$ | M1 |
| | $gf(x) = \frac{3}{2}x-17$ | A1 |
| | | 2 |

4.

| Question | Answer | Marks |
|---|---|---|
| 5(a) | $ff(x) = a - 2(a - 2x)$ | M1 |
| | $ff(x) = 4x - a$ | A1 |
| | $f^{-1}(x) = \frac{a-x}{2}$ | M1 A1 |
| | | 4 |
| 5(b) | $4x - a = \frac{a-x}{2} \rightarrow 9x = 3a$ | M1 |
| | $x = \frac{a}{3}$ | A1 |
| | | 2 |

5.

| Question | Answer | Marks |
|---|---|---|
| 9(a) | f($x$) from −1 to 5 | B1B1 |
| | g($x$) from −10 to 2<br>(**FT** from part (**a**)) | B1FT |
| | | 3 |

6.

| Question | Answer | Marks |
|---|---|---|
| 9(a) | $\left[(x-2)^2\right][-1]$ | B1 B1 |
| | | 2 |
| 9(b) | Smallest $c = 2$<br>(**FT** on *their* part (**a**)) | B1FT |
| | | 1 |
| 9(c) | $y = (x-2)^2 - 1 \rightarrow (x-2)^2 = y + 1$ | *M1 |
| | $x = 2(\pm)\sqrt{y+1}$ | DM1 |
| | $(f^{-1}(x)) = 2 + \sqrt{x+1}$ for $x > 8$ | A1 |
| | | 3 |
| 9(d) | $gf(x) = \frac{1}{(x-2)^2 - 1 + 1} = \frac{1}{(x-2)^2}$ OE | B1 |
| | Range of gf is $0 < gf(x) < \frac{1}{9}$ | B1 B1 |
| | | 3 |

7.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 11(a) | fg(x) = $(2x+1)^2+3$ | B1 | OE |
| | | 1 | |
| 11(b) | $y=(2x+1)^2+3 \rightarrow 2x+1=(\pm)\sqrt{y-3}$ | M1 | 1st two operations. Allow one sign error or *x*/*y* interchanged |
| | $x=(\pm)\frac{1}{2}\left(\sqrt{y-3}\ -1\right)$ | M1 | OE 2nd two operations. Allow one sign error or *x*/*y* interchanged |
| | $\left(\text{fg}^{-1}(x)=\right)\ \frac{1}{2}\left(\sqrt{x-3}\ -1\right)$ for $(x)>3$ | A1 B1 | Allow $(3, \infty)$ |
| | | 4 | |
| 11(c) | gf(x) = $2(x^2+3)+1$ | B1 | SOI |
| | $(2x+1)^2+3-3=2(x^2+3)+1 \rightarrow 2x^2+4x-6\ (=0)$ | *M1 | Express as 3-term quadratic |
| | $(2)(x+3)(x-1)\ (=0)$ | DM1 | Or quadratic formula or completing the square |
| | $x=1$ | A1 | |
| | | 4 | |

8.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | 0 | B1 | |
| | | 1 | |
| 5(b) | $(\text{f}^{-1}(x)) = \frac{x+2}{4}$, $(\text{g}^{-1}(x)) = \frac{4-x}{x}$ or $\frac{4}{x}-1$ | B1 B1 | OE. Sight of correct inverses. |
| | $x^2 + 6x - 16\ (= 0)$ | B1 | Equating inverses and simplifying. |
| | $(x+8)$and$(x-2)$ | M1 | Correct attempt at solution of *their* 3-term quadratic-factorising, completing the square or use of formula. |
| | $(x =)$ 2 or –8 | A1 | Do not accept answers obtained with no method shown. |
| | | 5 | |

9.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | $y=\frac{2x}{3x-1} \rightarrow 3xy-y=2x \rightarrow 3xy-2x=y$ (or $-y=2x-3xy$) | *M1 | For 1st two operations. Condone a sign error |
| | $x(3y-2)=y \rightarrow x=\frac{y}{3y-2}$ (or $x=\frac{-y}{2-3y}$) | DM1 | For 2nd two operations. Condone a sign error |
| | $\left(\text{f}^{-1}(x)\right)=\frac{x}{3x-2}$ | A1 | Allow $\left(\text{f}^{-1}(x)\right)=\frac{-x}{2-3x}$ |
| | | 3 | |
| 6(b) | $\left[\frac{2(3x-1)+2}{3(3x-1)}\right] = \left[\frac{6x}{3(3x-1)} = \frac{2x}{3x-1}\right]$ | B1 B1 | AG, WWW<br>First B1 is for a correct single unsimplified fraction.<br>An intermediate step needs to be shown. Equivalent methods accepted. |
| | | 2 | |
| 6(c) | $(\text{f}(x)) > \frac{2}{3}$ | B1 | Allow $(y) > \frac{2}{3}$. Do not allow $x > \frac{2}{3}$ |
| | | 1 | |

10.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | $[f(x)=](x+1)^2+2$ | B1 B1 | Accept $a=1, b=2$. |
| | Range [of f is $(y)$] $\geqslant 2$ | B1FT | OE. Do not allow $x \geqslant 2$, FT on *their* *b*. |
| | | 3 | |
| 7(b) | $y=(x+1)^2+2$ leading to $x=[\pm]\sqrt{y-2}-1$ | M1 | Or by using the formula. Allow one sign error. |
| | $f^{-1}(x)=-\sqrt{x-2}-1$ | A1 | |
| | | 2 | |
| 7(c) | $2(x^2+2x+3)+1=13$ | B1 | Or using a correct completed square form of f(*x*). |
| | $2x^2+4x-6[=0]$ leading to $(2)(x-1)(x+3)[=0]$ | B1 | Or $x=1, x=-3$ using formula or completing square. Must reach 2 solutions. |
| | $x=-3$ **only** | B1 | |
| | | 3 | |

11.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 9(a) | Range of f is f(*x*) $\geqslant -4$ | B1 | Allow *y*, f or 'range' or $[-4, \infty)$ |
| | | 1 | |
| 9(b) | $y=(x-2)^2-4 \Rightarrow (x-2)^2=y+4 \Rightarrow x-2=+\sqrt{(y+4)}$ or $\pm\sqrt{(y+4)}$ | M1 | May swap variables here |
| | $[f^{-1}(x)]=\sqrt{(x+4)}+2$ | A1 | |
| | | 2 | |
| 9(c) | $(x-2)^2-4=-\frac{5}{3}x+2 \Rightarrow x^2-4x+4-4=-\frac{5}{3}x+2\ [\Rightarrow x^2-\frac{7}{3}x-2=0]$ | M1 | Equating and simplifying to a 3-term quadratic |
| | $(3x+2)(x-3)[=0]$ or $\frac{7\pm\sqrt{7^2-4(3)(-6)}}{6}$ OE | M1 | Solving quadratic |
| | $x=3$ only | A1 | |
| | | 3 | |
| 9(d) | $f^{-1}(12)=6$ | M1 | Substitute 12 into *their* $f^{-1}(x)$ and evaluate |
| | $g(f^{-1}(12))=6a+2$ | M1 | Substitute *their* '6' into g(*x*) |
| | $g(g(f^{-1}(12)))=a(6a+2)+2=62$ | M1 | Substitute the result into g(*x*) and = 62 |
| | $6a^2+2a-60\ [=0]$ | M1 | Forming and solving a 3-term quadratic |
| | $a=-\frac{10}{3}$ or 3 | A1 | |
| | **Alternative method for Question 9(d)** | | |
| | $g(f^{-1}(x))=a\left(\sqrt{x+4}+2\right)+2$ or $gg(x)=a(ax+2)+2$ | M1 | Substitute *their* $f^{-1}(x)$ or g(*x*) into g(*x*) |
| | $g(g(f^{-1}(x)))=a\left(a\left(\sqrt{x+4}+2\right)+2\right)+2$ | M1 | Substitute the result into g(*x*) |
| | $g(g(f^{-1}(12)))=a(6a+2)+2=62$ | M1 | Substitute 12 and = 62 |
| | $6a^2+2a-60\ [=0]$ | M1 | Forming and solving a 3-term quadratic |
| | $a=-\frac{10}{3}$ or 3 | A1 | |
| | | 5 | |

12.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | $ff(x) = 2(2x^2 + 3)^2 + 3$ | M1 | Condone = 0. |
| | $8x^4 + 24x^2 + 21$ | A1 | ISW if correct answer seen. Condone = 0. |
| | | **2** | |
| 5(b) | $8x^4 + 24x^2 + 21 = 34x^2 + 19 \Rightarrow 8x^4 + 24x^2 - 34x^2 + 21 - 19$ [= 0] | M1 | Equating $34x^3 + 19$ to *their* 3-term ff(x) and collect all terms on one side condone ± sign errors. |
| | $8x^4 - 10x^2 + 2$[= 0] | A1 | |
| | [2]$(x^2 - 1)(4x^2 - 1)$ | M1 | Attempt to solve 3-term quartic or 3-term quadratic by factorisation, formula or completing the square or factor theorem. |
| | $\left[x^2 = 1 \text{ or } \frac{1}{4} \text{ leading to}\right] x = 1 \text{ or } x = \frac{1}{2}$ | A1 | If factorising, factors must expand to give $8x^4$ or $4x^4$ 4 or *their* $ax^4$ otherwise M0A0 due to calculator use. Condone $\pm1$, $\pm\frac{1}{2}$ but not $\sqrt{\frac{1}{4}}$ or $\sqrt{1}$. |
| | | **4** | |

13.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(a) | $[fg(x)=]1/(2x+1)^2 - 1$ | **B1** | SOI |
| | $1/(2x+1)^2 - 1 = 3$ leading to $4(2x+1)^2 = 1$ or $\frac{1}{(2x+1)} = [\pm]2$ or $16x^2 + 16x + 3 = 0$ | **M1** | Setting $fg(x) = 3$ and reaching a stage before $2x+1 = \pm½$ or reaching a 3 term quadratic in $x$ |
| | $2x+1 = \pm½$ or $2x+1 = -½$ or $(4x+1)(4x+3)[=0]$ | **A1** | Or formula or completing square on quadratic |
| | $x = -\frac{3}{4}$ only | **A1** | |
| | **Alternative method for Question 8(a)** | | |
| | $x^2 - 1 = 3$ | **M1** | |
| | $g(x) = -2$ | **A1** | |
| | $\frac{1}{(2x+1)} = -2$ | **M1** | |
| | $x = -\frac{3}{4}$ only | **A1** | |
| | | **4** | |
| 8(b) | $y = \frac{1}{(2x+1)^2} - 1$ leading to $(2x+1)^2 = \frac{1}{y+1}$ leading to $2x+1 = [\pm]\frac{1}{\sqrt{y+1}}$ | ***M1** | Obtain $2x+1$ or $2y+1$ as the subject |
| | $x = [\pm]\frac{1}{2\sqrt{y+1}} - \frac{1}{2}$ | **DM1** | Make $x$ (or $y$) the subject |
| | $-\frac{1}{2\sqrt{x+1}} - \frac{1}{2}$ | **A1** | OE e.g. $-\frac{\sqrt{x+1}}{2x+2} - \frac{1}{2}$, $-\left(\sqrt{\frac{-x}{4x+4} + \frac{1}{4}} + \frac{1}{2}\right)$ |
| | | **3** | |

14.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(a) | $\{-3(x-2)^2\}$ $\{+14\}$ | **B1 B1** | B1 for each correct term; condone $a = 2$, $b = 14$. |
| | | **2** | |

| Question | Answer | Marks | Guidance |
| --- | --- | --- | --- |
| 8(b) | [$k$ =] 2 | B1 | Allow [$x$] ⩽ 2. |
| | | 1 | |

| Question | Answer | Marks | Guidance |
| --- | --- | --- | --- |
| 8(c) | [Range is] [$y$] ⩽ –13 | B1 | Allow [f($x$)] ⩽ –13, [f ] ⩽ –13 but NOT $x$ ⩽ –13. |
| | | 1 | |
| 8(d) | $y=-3(x-2)^2+14$ leading to $(x-2)^2=\frac{14-y}{3}$ | M1 | Allow $\frac{y-14}{-3}$. Allow 1 error in rearrangement if $x$, $y$ on opposite sides. |
| | $x=2(\pm)\sqrt{\frac{14-y}{3}}$ | A1 | Allow $\frac{y-14}{-3}$. |
| | $[f^{-1}(x)]=2-\sqrt{\frac{14-x}{3}}$ | A1 | OE. Allow $\frac{x-14}{-3}$. Must be $x$ on RHS; must be negative square root only. |
| | **Alternative method for question 8(d)** | | |
| | $x=-3(y-2)^2+14$ leading to $(y-2)^2=\frac{14-x}{3}$ | M1 | Allow $\frac{x-14}{-3}$. Allow 1 error in rearrangement if $x$, $y$ on opposite sides. |
| | $=2(\pm)\sqrt{\frac{14-x}{3}}$ | A1 | Allow $\frac{x-14}{-3}$. |
| | $[f^{-1}(x)]=2-\sqrt{\frac{14-x}{3}}$ | A1 | OE. Allow $\frac{x-14}{-3}$. Must be $x$ on RHS; must be negative square root only. |
| | | 3 | |
| 8(e) | [g($x$) =] $\{-3(x+3-2)^2\}+\{14+1\}$ | B2, 1, 0 | OR $\{-3(x+3)^2\}+\{12(x+3)\}+\{3\}$ |
| | $g(x)=-3x^2-6x+12$ | B1 | |
| | | 3 | |

1.

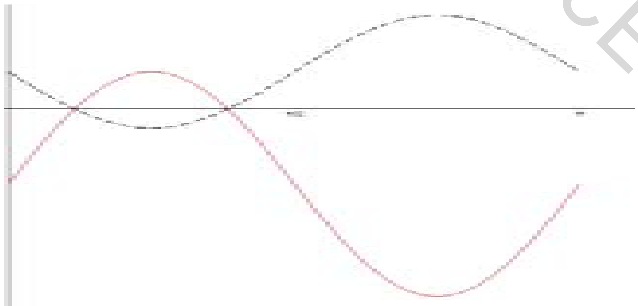
| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | [Stretch] [factor 2, $x$ direction (or $y$-axis invariant)] | *B1 DB1 | |
| | [Translation or Shift] [1 unit in $y$ direction] **or** [Translation/Shift] $\left[\begin{pmatrix}0\\1\end{pmatrix}\right]$ | B1B1 | Accept transformations in either order. Allow (0, 1) for the vector |
| | | 4 | |

2.

| Question | Answer | Marks |
|---|---|---|
| 4(a) | $-1 \leqslant f(x) \leqslant 2$ | B1 B1 |
| | | 2 |
| 4(b) | $k = 1$ | B1 |
| | Translation by 1 unit upwards parallel to the y-axis | B1 |
| | | 2 |
| 4(c) | $y = -\frac{3}{2}\cos 2x - \frac{1}{2}$ | B1 |
| | | 1 |



3.

| Question | Answer | Marks |
|---|---|---|
| 9(a) | f($x$) from −1 to 5 | B1B1 |
| | g($x$) from −10 to 2<br>(**FT** from part (**a**)) | B1FT |
| | | 3 |
| 9(b) |  | B2, 1 |
| | | 2 |
| 9(c) | Reflect in $x$-axis | B1 |
| | Stretch by factor 2 in the $y$ direction | B1 |
| | Translation by $-\pi$ in the $x$ direction OR translation by $\begin{pmatrix}0\\-\pi\end{pmatrix}$. | B1 |
| | | 3 |

4.

| Question | Answer | Marks |
|---|---|---|
| 3(a) | $(y)=\mathrm{f}(-x)$ | B1 |
| | | 1 |
| 3(b) | $(y)=2\mathrm{f}(x)$ | B1 |
| | | 1 |
| 3(c) | $(y)=\mathrm{f}(x+4)-3$ | B1 B1 |
| | | 2 |

5.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4 | $(y=)[3]+[2]\left[\cos\frac{1}{2}\theta\right]$ | B1 B1 B1 | |
| | | 3 | |

6.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 11(d) | Stretch by (scale factor) $\frac{1}{2}$, parallel to $x$-axis or in $x$ direction (or horizontally) | B1 | |
| | Translation of $\begin{pmatrix}0\\4\end{pmatrix}$ | B1 | Accept translation/shift<br>Accept translation 4 units in positive $y$-direction. |
| | | 2 | |
| 11(e) | Translation of $\begin{pmatrix}-\frac{\pi}{2}\\0\end{pmatrix}$ | B1 | Accept translation/shift<br>Accept translation $-\frac{\pi}{2}$ units in $x$-direction. |
| | Stretch by (scale factor) 2 parallel to $y$-axis (or vertically). | B1 | |
| | | 2 | |

7.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(a) | $\left[(x+3)^2\right]$ $[-4]$ | B1 B1 | |
| | | 2 | |
| 1(b) | [Translation or shift] $\begin{pmatrix}-3\\-4\end{pmatrix}$ | B1<br>B1 FT | Accept [translation/shift] $\begin{pmatrix}-\textit{their } a\\ \textit{their } b\end{pmatrix}$<br>OR translation –3 units in $x$-direction and (translation) –4 units in $y$-direction. |
| | | 2 | |

8.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | (Stretch) (factor 3 in $y$ direction **or** parallel to the y-axis) | B1 B1 | |
| | (Translation) $\begin{pmatrix}4\\0\end{pmatrix}$ | B1 B1 | Allow Translation 4 (units) in $x$ direction.<br>N.B. Transformations can be given in either order. |
| | | 4 | |
| 5(b) | $[y =]\ 3\mathrm{f}(x-4)$ | B1 B1 | B1 for 3 , B1 for $(x-4)$ with no extra terms. |
| | | 2 | |

9.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(a) | Translation $\begin{pmatrix}1\\0\end{pmatrix}$ | B1 | Allow shift and allow by 1 in $x$-direction or [parallel to/on/in/ along/against] the $x$-axis or horizontally.<br>'Translation by 1 to the right' only, scores B0 |
| | Stretch | B1 | Stretch. **SC B2** for amplitude doubled. |
| | Factor 2 in $y$-direction | B1 | With/by **factor** 2 in $y$-direction or [parallel to/on/in/along/against] the $y$-axis or vertically or with $x$ axis invariant<br>'With/by factor 2 upwards' only, scores B0.<br>Accept SF as an abbreviation for scale factor. |
| | | **3** | **Note:** Transformations can be in either order |
| 2(b) | [$-\sin 6x$][ + $15x$] or [$\sin(-6x)$][ + $15x$] OE | B1 B1 | Accept an unsimplified version. ISW.<br>B1 for each correct component – square brackets indicate each required component. |
| | | | If B0, **SC B1** for either $\sin(-2x) + 5x$ or $-\sin(2x) + 5x$ or $\sin 6x - 15x$ or $\sin\left(-\frac{2}{3}x\right)+\frac{5}{3}x$ |
| | | **2** | |

10.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | $f(x)=(x-1)^2+4$ | **B1** | |
| | $g(x)=(x+2)^2+9$ | **B1** | |
| | $g(x)=f(x+3)+5$ | **B1 B1** | B1 for each correct element.<br>Accept $p=3, q=5$ |
| | | **4** | |
| 6(b) | Translation or Shift | **B1** | |
| | $\begin{pmatrix}-3\\5\end{pmatrix}$ or acceptable explanation | **B1 FT** | If given as 2 single translations both must be described correctly e.g. $\begin{pmatrix}-3\\0\end{pmatrix}$ & $\begin{pmatrix}0\\5\end{pmatrix}$<br>FT from *their* $f(x+p)+q$ *or their* $f(x)\to g(x)$<br>Do not accept $\begin{pmatrix}1\\4\end{pmatrix}$ or $\begin{pmatrix}-2\\9\end{pmatrix}$ |
| | | **2** | |

# TOPIC 4: Binomial

**1.**

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | $5C2\,[2(x)]^3\left[\frac{a}{(x^2)}\right]^2$ | B1 | SOI<br>Can include correct $x$'s |
| | $10\times8\times a^2\left(\frac{x^3}{x^4}\right)=720\left(\frac{1}{x}\right)$ | B1 | SOI<br>Can include correct $x$'s |
| | $a=\pm3$ | B1 | |
| | | 3 | |
| 6(b) | $5C4\,[2(x)]\left[\frac{their\ a}{(x^2)}\right]^4$ | B1 | SOI<br>*Their a* can be just one of their values (e.g. just 3).<br>Can gain mark from within an expansion but must use *their* value of $a$ |
| | 810 identified | B1 | Allow with $x^{-7}$ |
| | | 2 | |

**2.**

| Question | Answer | Marks |
|---|---|---|
| 1(a) | $(2+3x)\left(x-\frac{2}{x}\right)^6$<br>Term in $x^2$ in $\left(x-\frac{2}{x}\right)^6 = 15x^4\times\left(\frac{-2}{x}\right)^2$ | B1 |
| | Coefficient = 60 | B1 |
| | | 2 |
| 1(b) | Constant term in $\left(x-\frac{2}{x}\right)^6 = 20x^3\times\left(\frac{-2}{x}\right)^3$ (−160) | B2, 1 |
| | Coefficient of $x^2$ in $(2+3x)\left(x-\frac{2}{x}\right)^6$ = 120 – 480 = −360 | B1FT |
| | | 3 |

**3.**

| Question | Answer | Marks |
|---|---|---|
| 4(a) | $1+5a+10a^2+10a^3+\ldots$ | B1 |
| | | 1 |
| 4(b) | $1+5(x+x^2)+10(x+x^2)^2+10(x+x^2)^3+\ldots$ SOI | M1 |
| | $1+5(x+x^2)+10(x^2+2x^3+\ldots)+10(x^3+\ldots)+\ldots$ SOI | A1 |
| | $1+5x+15x^2+30x^3+\ldots$ | A1 |
| | | 3 |

**4.**

| Question | Answer | Marks |
|---|---|---|
| 2 | $\left(kx+\frac{1}{x}\right)^5+\left(1-\frac{2}{x}\right)^8$<br>Coefficient in $\left(kx+\frac{1}{x}\right)^5$ = 10 × $k^2$<br>(**B1** for 10. **B1** for $k^2$) | B1B1 |
| | Coefficient in $\left(1-\frac{2}{x}\right)^8$ = 8 × −2 | B2,1,0 |
| | $10k^2 - 16 = 74 \rightarrow k = 3$ | B1 |
| | | 5 |

**5.**

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | $6C2\times\left[2(x^2)\right]^4\times\left[\frac{a}{(x)}\right]^2$ , $6C3\times\left[2(x^2)\right]^3\times\left[\frac{a}{(x)}\right]^3$ | B1 B1 | SOI Can be seen in an expansion |
| | $15\times2^4\times a^2=20\times2^3\times a^3$ | M1 | SOI Terms must be from a correct series |
| | $a=\frac{15\times2^4}{20\times2^3}=\frac{3}{2}$ | A1 | OE |
| | | 4 | |
| 5(b) | 0 | B1 | |
| | | 1 | |

**6.**

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | Coefficient of $x^3$ in $(1-2x)^5$ is –80 | B1 | Can be seen in an expansion but must be simplified correctly. |
| | Coefficient of $x^2$ in $(1-2x)^5$ is 40 | B1 | |
| | Coefficient of $x^3$ in $(1+kx)(1-2x)^5$ is $40k-80=20$ | M1 | Uses the relevant two terms to form an equation = 20 and solves to find $k$. Condone $x^3$ appearing in some terms if recovered. |
| | $(k=)\frac{5}{2}$ | A1 | |
| | | 4 | |

**7.**

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5 | $\left[7C1a^6b(x)\right]$, $\left[7C2a^5b^2(x^2)\right]$, $\left[7C4a^3b^4(x^4)\right]$ | B2, 1, 0 | SOI, can be seen in an expansion. |
| | $\frac{7C2a^5b^2(x^2)}{7C1a^6b(x)}=\frac{7C4a^3b^4(x^4)}{7C2a^5b^2(x^2)}\rightarrow\frac{21a^5b^2}{7a^6b}=\frac{35a^3b^4}{21a^5b^2}$ | M1 A1 | M1 for a correct relationship OE (Ft from *their* 3 terms). For A1 binomial coefficients must be correct & evaluated. |
| | $\frac{a}{b}=\frac{5}{9}$ | A1 | OE |
| | | 5 | |

**8.**

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(a) | $1+5x+10x^2$ | B1 | |
| | | 1 | |
| 1(b) | $1-12x+60x^2$ | B2, 1, 0 | B2 all correct, B1 for two correct components. |
| | | 2 | |
| 1(c) | $(1+5x+10x^2)(1-12x+60x^2)$ leading to $60-60+10$ | M1 | 3 products required |
| | 10 | A1 | Allow $10x^2$ |
| | | 2 | |

**9.**

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | 243 | **B1** | |
| | $-810x$ | **B1** | |
| | $+1080x^2$ | **B1** | |
| | | **3** | |
| 3(b) | $(4+x)^2 = 16 + 8x + x^2$ | **B1** | |
| | Coefficient of $x^2$ is $16 \times 1080 + 8 \times (-810) + 243$ | **M1** | Allow if at least 2 pairs used correctly |
| | 11043 | **A1** | Allow $11043x^2$ |
| | | **3** | |

**10.**

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4 | [Coefficient of $x$ or $p$ =] 480 | B1 | SOI. Allow $480x$ even in an expansion. |
| | $\left[\text{Term in } \frac{1}{x} \text{ or } q =\right][10\times](2x)^3\left(\frac{k}{x^2}\right)^2$ | M1 | Appropriate term identified and selected. |
| | $[10 \times 2^3k^2 =]\ 80k^2$ | A1 | Allow $\frac{80k^2}{x}$ |
| | $p = 6q$ used $(480 = 6 \times 80k^2$ or $80 = 80k^2)$ | M1 | Correct link used for *their* coefficient of $x$ and $\frac{1}{x}$ ($p$ and $q$) with no $x$'s. |
| | $[k^2 = 1 \Rightarrow]\ k = \pm 1$ | A1 | A0 if a range of values given. Do not allow $\pm\sqrt{1}$ . |
| | | 5 | |

**11.**

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | $(a-x)^6 = a^6 - 6a^5x + 15a^4x^2 - 20a^3x^3 + \ldots$ | **B2, 1, 0** | Allow extra terms.<br>Terms may be listed. Allow $a^6x^0$. |
| | | **2** | |
| 7(b) | $\left(1+\frac{2}{ax}\right)(\ldots 15a^4x^2 - 20a^3x^3 + \ldots)$ leading to $[x^2](15a^4 - 40a^2)$ | **M1** | Attempting to find 2 terms in $x^2$ |
| | $15a^4 - 40a^2 = -20$ leading to $15a^4 - 40a^2 + 20[=0]$ | **A1** | Terms on one side of the equation |
| | $(5a^2 - 10)(3a^2 - 2)\ [=0]$ | **M1** | OE.<br>M1 for attempted factorisation or solving for $a^2$ or $u$ (=$a^2$) using e.g. formula or completing the square |
| | $a = \pm\sqrt{2},\ \pm\sqrt{\frac{2}{3}}$ | **B1 B1** | OE exact form only<br>If B0B0 scored then **SC B1** for $\sqrt{2}, \sqrt{\frac{2}{3}}$ WWW<br>or $\pm 1.41, \pm 0{,}816$ WWW |
| | | **5** | |

**12.**

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(a) | $1-\frac{1}{x}+\frac{1}{4x^2}$ | B1 | OE. Multiply or use binomial expansion. Allow unsimplified. |
| | | 1 | |
| 1(b) | $1+12x+60x^2+160x^3$ | B2, 1, 0 | Withhold 1 mark for each error; B2, 1, 0. ISW if more than 4 terms in the expansion. |
| | | 2 | |
| 1(c) | $their(1\times12)+their(-1\times60)+their(\frac{1}{4}\times160)$ | M1 | Attempts at least 2 products where each product contains one term from each expansion. |
| | $[12-60+40=]\ -8$ | A1 | Allow $-8x$. |
| | | 2 | |

**13.**

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(a) | Terms required for $x^2$: $-5\times2^4\times ax+10\times2^3\times a^2x^2\left[=-80ax+80a^2x^2\right]$ | B1 | Can be seen as part of an expansion or in correct products. |
| | $2\times$ ($\pm$*their* coefficient of $x$) + $4\times$ ($\pm$*their* coefficient of $x^2$) | *M1 | |
| | $x^2$ coefficient is $320a^2-160a=-15$ $\Rightarrow 64a^2-32a+3\Rightarrow(8a-3)(8a-1)$ | DM1 | Forming a 3-term quadratic in $a$, with all terms on the same side or correctly setting up prior to completing the square and solving using factorisation, formula or completing the square. If factorising, factors must expand to give *their* coefficient of $a^2$. |
| | $a=\frac{1}{8}$ or $a=\frac{3}{8}$ | A1 | OE. **Special case:** If DM0 for solving quadratic, SC B1 can be awarded for correct final answers. |
| | | 4 | |
| 8(b) | $320a^2-160a=k\Rightarrow 320a^2-160a-k[=0]$ | M1 | Forming a 3-term quadratic in $a$ with all terms on the same side. Allow $\pm$ sign errors. |
| | *Their* $b^2-4ac$ [= 0], $[160^2-4\times320\times(-k)=0]$ | M1 | Any use of discriminant on a 3-term quadratic. |
| | $k=-20$ | A1 | |
| | $a=\frac{1}{4}$ | B1 | Condone $a=\frac{1}{4}$ from $k=20$. |
| | **Alternative method for question 8(b)** | | |
| | $320a^2-160a=k$ and divide by 320 $\left[a^2-\frac{a}{2}=\frac{k}{320}\right]$ | M1 | Allow $\pm$ sign errors. |
| | Attempt to complete the square $\left[\left(a-\frac{1}{4}\right)^2-\frac{1}{16}=\frac{k}{320}\right]$ | M1 | Must have $\left(a-\frac{1}{4}\right)^2$ |
| | $a=\frac{1}{4}$ | A1 | |
| | $k=-20$ | B1 | |

**14.**

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(a) | $1+6ax+15a^2x^2$ | B1 | Terms must be evaluated. |
| | | 1 | |
| 2(b) | *their* $15a^2 \pm (3\times$ *their* $6a)$ | *M1 | Expect $15a^2-18a$. |
| | $15a^2-18a=-3$ | A1 | |
| | $(3)(a-1)(5a-1)\ [=0]$ | DM1 | Dependent on 3-term quadratic. Or solve using formula or completing the square. |
| | $a=1,\ \frac{1}{5}$ | A1 | WWW. If DM0 awarded **SC B1** if both answers correct. |
| | | 4 | |

**15.**

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | ${}^6C_2\times(3x)^4\left(\frac{2}{x^2}\right)^2$ | B1 | Can be seen within an expansion. |
| | $15\times3^4\times2^2$ | B1 | Identified. Powers must be correct. |
| | 4860 | B1 | Without any power of $x$ |
| | | 3 | |
| 3(b) | *Their* 4860 and one other relevant term | M1 | Using *their* 4860 and an attempt to find a term in $x^{-3}$ |
| | Other term = $6C3(3x)^3\left(\frac{2}{x^2}\right)^3$ or $6C3\times3^3\times2^3$ or 4320 | A1 | Must be identified. If M0 scored then **SC B1** for 4320 as the only answer. |
| | [4860 – 4320 =] 540 | A1 | |
| | | 3 | |

**16.**

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | $x^4$ term is $[10\times](2x^2)^3\left(\frac{k^2}{x}\right)^2$ | M1 | For selecting the term in $x^4$. |
| | $80k^4x^4 \Rightarrow a=80k^4$ | A1 | For correct value of $a$. Allow $80k^4x^4$. |
| | [$x^2$ term is [6 ×]$(2kx)^2\times1=24k^2x^2\Rightarrow$] $b=24k^2$ | B1 | For correct value of $b$. Allow $24k^2x^2$. |
| | | 3 | |
| 3(b) | $80k^4+24k^2-216[=0]$ $\quad[\Rightarrow 10k^4+3k^2-27=0]$ | M1 | Forming a 3-term equation in $k$ (all terms on one side) with *their* $a$ and $b$ and no $x$'s. |
| | $(2k^2-3)(5k^2+9)\ [=0]\ [\Rightarrow k^2=\frac{3}{2}$ or $-\frac{9}{5}]$ | M1 | Attempt to solve 3-term quartic (or quadratic in another variable) by factorisation, formula or completing the square – see guidance. |
| | $[k]=\pm\sqrt{\frac{3}{2}}$ | A1 | OE e.g. $\pm\frac{\sqrt{6}}{2}$, $\pm\sqrt{1.5}$, AWRT $\pm1.22$<br>Omission of ± A0.<br>Additional answers A0.<br>If M1 M0, **SC B1** can be awarded for correct final answer, max 2/3. |
| | | 3 | |

**17.**

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | Coefficient of $x^4 = 15$ | B1 | Condone inclusion of $x^4$. Can be seen as part of an expansion. |
| | Coefficient of $x^2 = 240a^2$ | B1 | Condone inclusion of $x^2$. Can be seen as part of an expansion. |
| | '*Their* $240'a^2$ – '*their* 15' | M1 | Forming an equation of the form $pa^2 = q$, where $p$ and $q$ are constants. Condone inclusion of powers of $x$ as long as they then disappear. |
| | $a = \frac{1}{4}$ or 0.25 | A1 | OE<br>Do not condone extra 'answer' of $-\frac{1}{4}$ ,or allow $\sqrt{\frac{1}{16}}$ or similar. |
| | | 4 | |

**18.**

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | $4C1 \times p \times \frac{1}{p^3} x^3$ | B1 | OE<br>soi Can be seen in an expansion. |
| | $\frac{4}{p^2} = 144$ | B1 | OE<br>Correct with correct power of $p$ and only one $p$ term. |
| | $p = \pm\frac{1}{6}$ | B1 B1 | OE $\pm\frac{2}{12}$ etc. Allow ±0.167 for B1 B1.<br>**SC B1** for $\pm\sqrt{\frac{1}{36}}$ B1 only, |
| | | 4 | |

**19.**

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| .4 | Coefficient of $x^2$ in $\left(1+\frac{2}{p}x\right)^5$ is $10\left(\frac{2}{p}\right)^2 = \frac{10\times 2^2}{p^2}\left[=\frac{40}{p^2}\right]$ | B1 | Accept with $x^2$ present. Must evaluate ${}^5C_2$ |
| | Coefficient of $x^2$ in $(1+px)^6$ is $15(p)^2\ [=15p^2]$ | B1 | Accept with $x^2$ present. Must evaluate ${}^6C_2$ |
| | $\frac{40}{p^2} + 15p^2 = 70$ | *M1 | Forming an equation in $p$ with *their* coefficients, the given 70, no $x$ terms and no extra terms. |
| | $15p^4 - 70p^2 + 40\ [=0]$ or $3p^4 - 14p^2 + 8\ [=0]$ | DM1 | Forming a 3-term equation in $p$ (or another variable) with all terms on one side and *their* coefficients. |
| | $[5](p^2-4)(3p^2-2)\ [=0]$ or $\frac{70\pm\sqrt{70^2-4(15)(40)}}{30}$ or $\frac{14\pm\sqrt{14^2-4(3)(8)}}{6}$ | DM1 | Attempt to solve 3-term quartic (or quadratic in another variable) by factorisation, formula or completing the square. |
| | $p = \pm 2,\ \pm\sqrt{\frac{2}{3}}$ | A1 | OE e.g. $\pm\frac{\sqrt{6}}{3}$ or AWRT ±0.816<br>If *M1 DM1 DM0, allow **SC B1** for 4 correct values. |
| | | 6 | |

**20.**

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | $1+10x+40x^2$ May be part of a complete expansion | **B2, 1, 0** | $1^5$ must be simplified to 1, allow if the '1' is seen in a more complete expansion but not the final answer.<br>Mis-reads not condoned in this question. |
| | | **2** | |
| 3(b) | $1-12x+54x^2$ May be part of a complete expansion | **B2, 1, 0** | $1^4$ must be simplified to 1, allow if the '1' is seen in a more complete expansion but not the final answer.<br>Mis-reads not condoned in this question. |
| | | **2** | |
| 3(c) | $54-120+40$ | **M1** | Forming exactly 3 products correctly using their terms. |
| | $-26$ | **A1** | Allow $-26x^2$ If in a list with other terms it must be clear this is the required term otherwise A0. |
| | | **2** | |

TOPIC 5: Series

1.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(a) | 2% | **B1** | |
| | | **1** | |
| 8(b) | Bonus = 600 + 23 × 100 = 2900 | **B1** | |
| | Salary = $30000 \times 1.03^{23}$ | **M1** | Allow $30000 \times 1.03^{24}$ (60984) |
| | = 59207.60 | **A1** | Allow answers of 3significant figure accuracy or better |
| | $\frac{their\ 2900}{their\ 59200}$ | **M1** | SOI |
| | 4.9(0)% | **A1** | |
| | | **5** | |

2.

| Question | Answer | Marks |
|---|---|---|
| 1 | $117 = \frac{9}{2}(2a + 8d)$ | **B1** |
| | **Either** $91 = S_4$ with '$a$' as $a + 4d$ **or** $117 + 91 = S_{13}$<br>(**M1** for overall approach. **M1** for $S_n$) | **M1M1** |
| | Simultaneous Equations → $a = 7$, $d = 1.5$ | **A1** |
| | | **4** |

3.

| Question | Answer | Marks |
|---|---|---|
| 3(a) | $36 000 × $(1.05)^n$<br>(**B1** for $r$ = 1.05. **M1** method for $r$th term) | **B1M1** |
| | $53 200 after 8 years. | **A1** |
| | | **3** |
| 3(b) | $S_{10} = 36000\frac{(1.05^{10} - 1)}{(1.05 - 1)}$ | **M1** |
| | $453 000 | **A1** |
| | | **2** |

4.

| Question | Answer | Marks |
|---|---|---|
| 4 | 1st term is −6, 2nd term is −4.5<br>(**M1** for using $k$th terms to find both $a$ and $d$) | **M1** |
| | → $a = -6$, $d = 1.5$ | **A1 A1** |
| | $S_n = 84$ → $3n^2 - 27n - 336 = 0$ | **M1** |
| | Solution $n = 16$ | **A1** |
| | | **5** |

5.

| Question | Answer | Marks |
|---|---|---|
| 8(a) | $r = \cos^2\theta$ SOI | M1 |
| | $S_\infty = \frac{\sin^2\theta}{1-\cos^2\theta}$ | M1 |
| | 1 | A1 |
| | | 3 |
| 8(b)(i) | $d = \sin^2\theta\cos^2\theta - \sin^2\theta$ | M1 |
| | $\sin^2\theta(\cos^2\theta - 1)$ | M1 |
| | $-\sin^4\theta$ | A1 |
| | | 3 |
| 8(b)(ii) | Use of $S_{16} = \frac{16}{2}[2a+15d]$ | M1 |
| | With both $a = \frac{3}{4}$ and $d = -\frac{9}{16}$ | A1 |
| | $S_{16} = -55\frac{1}{2}$ | A1 |
| | | 3 |

6.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(a) | $S = \frac{a}{1-r}$ , $2S = \frac{a}{1-R}$ | B1 | SOI at least one correct |
| | $\frac{2a}{1-r} = \frac{a}{1-R}$ | M1 | SOI |
| | $2 - 2R = 1 - r \rightarrow r = 2R - 1$ | A1 | AG |
| | | 3 | |
| 8(b) | $ar^2 = aR \rightarrow (a)(2R-1)^2 = R(a)$ | *M1 | |
| | $4R^2 - 5R + 1\ (=0) \rightarrow (4R-1)(R-1)\ (=0)$ | DM1 | Allow use of formula or completing square. |
| | $R = \frac{1}{4}$ | A1 | Allow $R$ = 1 in addition |
| | $S = \frac{2a}{3}$ | A1 | |
| | **Alternative method for question 8(b)** | | |
| | $ar^2 = aR \rightarrow (a)r^2 = ½(r+1)(a)$ | *M1 | Eliminating 1 variable |
| | $2r^2 - r - 1\ (=0) \rightarrow (2r+1)(r-1)\ (=0)$ | DM1 | Allow use of formula or completing square. Must solve a quadratic. |
| | $r = -\frac{1}{2}$ | A1 | Allow $r$ = 1 in addition |
| | $S = \frac{2a}{3}$ | A1 | |
| | | 4 | |

7.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | $(-2p)^2 = (2p+6)\times(p+2)$ or $\frac{-2p}{2p+6} = \frac{p+2}{-2p}$ | M1 | OE. Using "$a, b, c$ then $b^2 = ac$" or $a = 2p+6$, $ar = -2p$ and $ar^2 = p+2$ to form a correct relationship in terms of $p$ only |
| | $(2p^2 - 10p - 12 = 0)$ $p = 6$ | A1 | |
| | $a = 18$ and $r = -⅓$ | A1 | |
| | $(s_\infty) =$ *their* $a \div (1 -$ *their* $r)$ $\left(=18\div\frac{5}{3}\right)$ | M1 | Correct formula used with their values for $a$ and $r$, $\lvert r\rvert < 1$ Both $a$ & $r$ from the same value of p. |
| | $(s_\infty =)10.8$ | A1 | OE. A0 if an extra solution given |
| | | | **SC B2** for $s_\infty = \frac{2p+6}{1-\frac{-2p}{2p+6}}$ *or* $\frac{2p+6}{1-\frac{p+2}{-2p}}$ ignore any subsequent algebraic simplification. |
| | | 5 | |

8.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4 | $S_x$ *and* $S_{x+1}$ | M1 | Using two values of $n$ in the given formula |
| | $a = 5, d = 2$ | A1 A1 | |
| | $a + (n-1)d > 200 \rightarrow 5 + 2(k-1) > 200$ | M1 | Correct formula used with their $a$ and $d$ to form an equation or inequality with 200, condone use of $n$ |
| | $(k =)$ 99 | A1 | Condone $\geqslant 99$ |
| | **Alternative method for question 4** | | |
| | $\frac{n}{2}(2a + (n-1)d) \equiv n^2 + 4n \rightarrow \left(\frac{d}{2} = 1, a - \frac{1}{2}d = 4\right)$ | M1 | Equating two correct expressions of $S_n$ and equating coefficients of $n$ and $n^2$ |
| | $d = 2, a = 5$ | A1 A1 | |
| | $a + (n-1)d > 200 \rightarrow 5 + 2(k-1) > 200$ | M1 | Correct formula used with their $a$ and $d$ to form an equation or inequality with 200, condone use of $n$ |
| | $(k =)$ 99 | A1 | Condone $\geqslant 99$ |
| | **Alternative method for question 4** | | |
| | $sum_k - sum_{k-1} \rightarrow k^2 + 4k - (k-1)^2 - 4(k-1)$ | M1 A1 | Using given formula with consecutive expressions subtracted. Allow $k$+1 and $k$. |
| | $2k + 3 > 200$ or $= 200$ | M1 A1 | Simplifying to a linear equation or inequality |
| | $(k =)$ 99 | A1 | Condone $\geqslant 99$ |
| | | 5 | |

9.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | $(d =) -\frac{\tan^2\theta}{\cos^2\theta} - \frac{1}{\cos^2\theta}$ | B1 | Allow sign error(s). Award only at form $(d =)$... stage |
| | $-\frac{\sin^2\theta}{\cos^4\theta} - \frac{1}{\cos^2\theta}$ or $\frac{-\sec^2\theta}{\cos^2\theta}$ | M1 | Allow sign error(s). Can imply B1 |
| | $\frac{-\sin^2\theta - \cos^2\theta}{\cos^4\theta}$ or $\frac{-\frac{1}{\cos^2\theta}}{\cos^2\theta}$ | M1 | |
| | $-\frac{1}{\cos^4\theta}$ | A1 | AG, WWW |
| | | 4 | |

| | | | |
|---|---|---|---|
| 7(b) | $a = \frac{4}{3}$, $d = -\frac{16}{9}$ | B1 | SOI, both required. Allow $a = \frac{\frac{1}{3}}{4}$, $d = -\frac{\frac{1}{9}}{16}$ |
| | $u_{13} = \frac{1}{\cos^2\theta} - \frac{12}{\cos^4\theta} = \frac{4}{3} + 12\left(\frac{-16}{9}\right)$ | M1 | Use of correct formula with *their a* and *their d*. The first 2 steps could be reversed |
| | –20 | A1 | WWW |
| | | 3 | |

10.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 9(a)(i) | $\frac{\cos\theta}{1-r} = \frac{1}{\cos\theta}$ | B1 | |
| | $1 - r = \cos^2\theta$ leading to $r = 1 - \cos^2\theta$ | M1 | Eliminate fractions |
| | $r = \sin^2\theta$ leading to 2nd term = $\cos\theta\sin^2\theta$ | A1 | AG |
| | | 3 | |
| 9(a)(ii) | $S_{12} = \frac{\cos\left(\frac{\pi}{3}\right)\left[1-\left(\sin^2\left(\frac{\pi}{3}\right)\right)^{12}\right]}{1-\sin^2\left(\frac{\pi}{3}\right)} = \frac{0.5\left[1-(0.75)^{12}\right]}{1-0.75}$ | M1 | Evidence of correct substitution, use of $S_n$ formula and attempt to evaluate |
| | 1.937 | A1 | |
| | | 2 | |
| 9(b) | $[d =] \cos\theta\sin^2\theta - \cos\theta$ | M1 | Use of $d = u_2 - u_1$ |
| | $-\frac{1}{8}$ | A1 | |
| | [85th term =] $\frac{1}{2} + 84 \times -\frac{1}{8}$ | M1 | Use of $a + 84d$ with a calculated value of $d$ |
| | −10 | A1 | |
| | | 4 | |

11.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | $10(2a + 19d) = 405$ | B1 | |
| | $20(2a + 39d) = 1410$ | B1 | |
| | Solving simultaneously two equations obtained from using the correct sum formulae [$a = 6$, $d = 1.5$] | M1 | Reach $a =$ or $d =$ |
| | Using the correct formula for 60th term with their $a$ and $d$ | M1 | |
| | 60th term = 94.5 | A1 | OE, e.g. $\frac{189}{2}$ |
| | | 5 | |

12.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5 | $(-12)^2 = 8k \times 2k$ | M1 | Forming an equation in $k$ |
| | $k = -3$ | A1 | |
| | Using correct formula for $S_\infty$ [$r = 0.5$, $a = -384$] | M1 | With $-1 < r < 1$ |
| | $S_\infty = -768$ | A1 | |
| | **Alternative method for Question 5** | | |
| | $r^2 = \frac{2k}{8k}$ | M1 | |
| | $r = [\pm]0.5$ | A1 | |
| | Using correct formula for $S_\infty$ [$r = 0.5$, $a = -384$] | M1 | $-1 < r < 1$ |
| | $S_\infty = -768$ | A1 | |
| | | 4 | |

13.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(a) | $\left(a + b = 2\times\frac{3}{2}a\right) \Rightarrow b = 2a$ | B1 | SOI |
| | $18^2 = a(b + 3)$ OE or 2 correct statements about $r$ from the GP, e.g. $r = \frac{18}{a}$ and $b + 3 = 18r$ or $r^2 = \frac{b+3}{a}$ | B1 | SOI |
| | $324 = a(2a + 3) \Rightarrow 2a^2 + 3a - 324[= 0]$<br>or<br>$b^2 + 3b - 648[= 0]$<br>or<br>$6r^2 - r - 12[= 0]$<br>or<br>$4d^2 + 3d - 162[= 0]$ | M1 | Using the correct connection between AP and GP to form a 3-term quadratic with all terms on one side. |
| | $(a - 12)(2a + 27)[= 0]$<br>or<br>$(b-24)(b+27)[=0]$<br>or<br>$(2r-3)(3r+4)[=0]$<br>or<br>$(d-6)(4d+27)[=0]$ | M1 | Solving *their* 3-term quadratic by factorisation, formula or completing the square to obtain answers for $a$, $b$, $r$ or $d$. |
| | $a = 12$, $b = 24$ | A1 | WWW. Condone extra 'solution' $a = -13.5, b = -27$ only. |
| | | 5 | |
| 8(b) | Common difference $d = 6$ | B1 FT | SOI. FT *their* $\frac{a}{2}$ |
| | $S_{20} = \frac{20}{2}(2\times12 + 19\times6)$ | M1 | Using correct sum formula with *their* $a$, *their* calculated $d$ and 20. |
| | 1380 | A1 | |
| | | 3 | |

14.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 9(a) | $ar = \frac{24}{100} \times \frac{a}{1-r}$ | M1 | Form an equation using a numerical form of the percentage and correct formula for $u_2$ and $S_\infty$ |
| | $100r^2 - 100r + 24[=0]$ | A1 | OE. All 3 terms on one side of an equation. |
| | $(20r-8)(5r-3)[=0] \to r = \frac{2}{5}, \frac{3}{5}$ | A1 | Dependent on factors or formula seen from their quadratic. |
| | | 3 | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 9(b) | $3 \times \{(a+4d)\} = \{(2(a+1)+11(d+1))\}$ | *M1 | SOI Attempt to cross multiply with contents of at least one { } correct |
| | Simplifies to $a+d=13$ | A1 | |
| | $\left[\frac{5}{2}\right] \times 3\{(2a+4d)\} = \left[\frac{5}{2}\right] \times 2\{(4(a+1)+4(d+1))\}$ | *M1 | SOI Attempt to cross multiply with contents of at least one { } correct |
| | Simplifies to $-a+2d=8$ | A1 | |
| | Solve 2 linear equations simultaneously | DM1 | Elimination or substitution expected |
| | $d=7, \quad a=6$ | A1 | **SC B1** for $a$=6, $d$=7 without complete working |
| | | 6 | |

15.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | $\frac{5a}{1-(\pm\frac{1}{4})}$ | B1 | Use of correct formula for sum to infinity. |
| | $\frac{8}{2}[2a+7(-4)]$ | *M1 | Use of correct formula for sum of 8 terms and form equation; allow 1 error. |
| | $4a = 8a - 112$ leading to $a = [28]$ | DM1 | Solve equation to reach a value of $a$. |
| | $a = 28$ | A1 | Correct value. |
| | | 4 | |
| 4(b) | *their* $28 + (k-1)(-4) = 0$ | M1 | Use of correct method with *their* $a$. |
| | $[k=]8$ | A1 | |
| | | 2 | |

# Coordinate geometry

1.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 12(a) | Centre = (2, –1) | B1 | |
| | $r^2 = [2-(-3)]^2 + [-1-(-5)]^2$ or $[2-7]^2 + [-1-3]^2$ OE | M1 | OR $\frac{1}{2}\left[(-3-7)^2 + (-5-3)^2\right]$ OE |
| | $(x-2)^2 + (y+1)^2 = 41$ | A1 | Must not involve surd form<br>**SCB3** $(x+3)(x-7)+(y+5)(y-3)=0$ |
| | | 3 | |
| 12(b) | Centre = *their* (2, –1) + $\begin{pmatrix}8\\4\end{pmatrix}$ = (10, 3) | B1FT | SOI<br>FT on *their* (2, –1) |
| | $(x-10)^2 + (y-3)^2 = their\ 41$ | B1FT | FT on *their* 41 even if in surd form<br>**SCB2** $(x-5)(x-15)+(y+1)(y-7)=0$ |
| | | 2 | |
| 12(c) | Gradient $m$ of line joining centres = $\frac{4}{8}$ OE | B1 | |
| | Attempt to find mid-point of line. | M1 | Expect (6, 1) |
| | Equation of $RS$ is $y-1=-2(x-6)$ | M1 | Through *their* (6, 1) with gradient $\frac{-1}{m}$ |
| | $y=-2x+13$ | A1 | AG |
| | **Alternative method for question 12(c)** | | |
| | $(x-2)^2+(y+1)^2-41=(x-10)^2+(y-3)^2-41$ OE | M1 | |
| | $x^2-4x+4+y^2+2y+1=x^2-20x+100+y^2-6y+9$ OE | A1 | Condone 1 error **or** errors caused by 1 error in the first line |
| | $16x+8y=104$ | A1 | |
| | $y=-2x+13$ | A1 | AG |
| | | 4 | |
| 12(d) | $(x-10)^2+(-2x+13-3)^2=41$ | M1 | Or eliminate y between $C_1$ and $C_2$ |
| | $x^2-20x+100+4x^2-40x+100=41 \rightarrow 5x^2-60x+159=0$ | A1 | AG |
| | | 2 | |

2.

| Question | Answer | Marks |
|---|---|---|
| 5(a) | $x(mx+c) = 16 \rightarrow mx^2+cx-16=0$ | B1 |
| | Use of $b^2 - 4ac = c^2 + 64m$ | M1 |
| | Sets to 0 $\rightarrow m = \frac{-c^2}{64}$ | A1 |
| | | 3 |
| 5(b) | $x(-4x+c) = 16$<br>Use of $b^2 - 4ac \rightarrow c^2 - 256$ | M1 |
| | $c > 16$ and $c < -16$ | A1 A1 |
| | | 3 |

3.

| Question | Answer | Marks |
|---|---|---|
| 10(a) | Centre is (3, 1) | **B1** |
| | Radius = 5 (Pythagoras) | **B1** |
| | Equation of C is $(x-3)^2+(y-1)^2=25$<br>(**FT** on *their* centre) | **M1**<br>**A1FT** |
| | | **4** |
| 10(b) | Gradient from (3, 1) to (7, 4) = ¾ (this is the normal) | **B1** |
| | Gradient of tangent $= -\frac{4}{3}$ | **M1** |
| | Equation is $y-4=-\frac{4}{3}(x-7)$ or $3y+4x=40$ | **M1A1** |
| | | **4** |
| 10(c) | *B* is centre of line joining centres → (11, 7) | **B1** |
| | Radius = 5<br>New equation is $(x-11)^2+(y-7)^2=25$<br>(**FT** on coordinates of B) | **M1**<br>**A1FT** |
| | | **3** |

4.

| Question | Answer | Marks |
|---|---|---|
| 11(a) | Simultaneous equations $\frac{8}{x+2}=4-\frac{1}{2}x$ | **M1** |
| | $x=0$ or $x=6$ → *A* (0, 4) and *B* (6, 1) | **B1A1** |
| | At *C* $\frac{-8}{(x+2)^2}=-\frac{1}{2}$ → *C* (2, 2)<br>(**B1** for the differentiation. **M1** for equating and solving) | **B1**<br>**M1A1** |
| | | **6** |

5.

| Question | Answer | Marks |
|---|---|---|
| 6(a) | $2x^2+kx+k-1=2x+3 \rightarrow 2x^2+(k-2)x+k-4=0$ | **M1** |
| | Use of $b^2-4ac=0 \rightarrow (k-2)^2=8(k-4)$ | **M1** |
| | $k=6$ | **A1** |
| | | **3** |
| 6(b) | $2x^2+2x+1=2\left(x+\frac{1}{2}\right)^2+1-\frac{1}{2}$<br>$a=\frac{1}{2}, b=\frac{1}{2}$ | **B1 B1** |
| | vertex $\left(-\frac{1}{2},\frac{1}{2}\right)$<br>(**FT** on *a* and *b* values) | **B1FT** |
| | | **3** |

6.

| Question | Answer | Marks |
|---|---|---|
| 11(a) | Express as $(x-4)^2+(y+2)^2=16+4+5$ | M1 |
| | Centre $C(4, -2)$ | A1 |
| | Radius = $\sqrt{25}$ = 5 | A1 |
| | | 3 |
| 11(b) | $P(1,2)$ to $C(4,-2)$ has gradient $-\frac{4}{3}$<br>(**FT** on coordinates of $C$) | B1FT |
| | Tangent at $P$ has gradient = $\frac{3}{4}$ | M1 |
| | Equation is $y-2=\frac{3}{4}(x-1)$ or $4y = 3x + 5$ | A1 |
| | | 3 |
| 11(c) | $Q$ has the same coordinate as $P$ $y = 2$ | B1 |
| | $Q$ is as far to the right of $C$ as $P$ $x = 3 + 3 + 1 = 7$ $Q$ (7, 2) | B1 |
| | | 2 |
| 11(d) | Gradient of tangent at $Q = -\frac{3}{4}$ by symmetry<br>(**FT** from part (**b**)) | B1FT |
| | Eqn of tangent at $Q$ is $y-2=-\frac{3}{4}(x-7)$ or $4y + 3x = 29$ | M1 |
| | $T\left(4, \frac{17}{4}\right)$ | A1 |
| | | 3 |

7.

| Question | Answer | Marks |
|---|---|---|
| 1 | $3x^2+2x+4=mx+1 \rightarrow 3x^2+x(2-m)+3\ (=0)$ | B1 |
| | $(2-m)^2-36$ SOI | M1 |
| | $(m + 4)(m - 8)$ (>/= 0) **or** $2 - m$ >/= 6 and $2 - m$ </= –6 OE | A1 |
| | $m < -4$, $m > 8$ WWW | A1 |
| | **Alternative method for question 1** | |
| | $\frac{dy}{dx} = 6x + 2 \rightarrow m = 6x + 2 \rightarrow 3x^2+2x+4=(6x+2)x+1$ | M1 |
| | $x=\pm1$ | A1 |
| | $m=\pm6+2 \rightarrow m=8$ or $-4$ | A1 |
| | $m < -4$, $m > 8$ WWW | A1 |
| | | 4 |

8.

| Question | Answer | Marks |
|---|---|---|
| 10(a) | Mid-point is (–1, 7) | B1 |
| | Gradient, $m$, of $AB$ is 8/12 OE | B1 |
| | $y-7=-\frac{12}{8}(x+1)$ | M1 |
| | $3x+2y=11$ **AG** | A1 |
| | | **4** |
| 10(b) | Solve simultaneously $12x-5y=70$ and *their* $3x+2y=11$ | M1 |
| | $x=5,\ y=-2$ | A1 |
| | Attempt to find distance between *their* (5, –2) and either (–7,3) or (5, 11) | M1 |
| | $(r)=\sqrt{12^2+5^2}$ or $\sqrt{13^2+0}=13$ | A1 |
| | Equation of circle is $(x-5)^2+(y+2)^2=169$ | A1 |
| | | **5** |

9.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | $2x^2+5=mx-3 \rightarrow 2x^2-mx+8\ (=0)$ | B1 | Form 3-term quadratic |
| | $m^2-64$ | M1 | Find $b^2-4ac$ . |
| | $-8<m<8$ | A1 | Accept (-8, 8) and equality included |
| | | 3 | |

10.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 9(a) | $m_{AB}=\frac{4-2}{-1-3}=-\frac{1}{2}$ | B1 | |
| | Equation of tangent is $y-2=2(x-3)$ | B1 FT | (3, 2) with *their* gradient $-\frac{1}{m_{AB}}$ |
| | | 2 | |
| 9(b) | $AB^2=4^2+2^2=20$ or $r^2=20$ or $r=\sqrt{20}$ or $AB=\sqrt{20}$ | B1 | |
| | Equation of circle centre $B$ is $(x-3)^2+(y-2)^2=20$ | M1 A1 | FT *their* 20 for M1 |
| | | 3 | |
| 9(c) | $(x-3)^2+(2x-6)^2=$ *their* 20 | M1 | Substitute *their* $y-2=2x-6$ into *their* circle, centre $B$ |
| | $5x^2-30x+25=0$ or $5(x-3)^2=20$ | A1 | |
| | $[(5)(x-5)(x-1)$ or $x-3=\pm2]$ $x=5,\ 1$ | A1 | |
| | | 3 | |

11.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 12(a) | $4x^{\frac{1}{2}}-2x=3-x \rightarrow x-4x^{\frac{1}{2}}+3(=0)$ | *M1 | 3-term quadratic. Can be expressed as e.g. $u^2-4u+3$ (=0) |
| | $\left(x^{\frac{1}{2}}-1\right)\left(x^{\frac{1}{2}}-3\right)(=0)$ or $(u-1)(u-3)(=0)$ | DM1 | Or quadratic formula or completing square |
| | $x^{\frac{1}{2}}=1, 3$ | A1 | SOI |
| | $x=1, 9$ | A1 | |
| | **Alternative method for question 12(a)** | | |
| | $\left(4x^{\frac{1}{2}}\right)^2=(3+x)^2$ | *M1 | Isolate $x^{\frac{1}{2}}$ |
| | $16x=9+6x+x^2 \rightarrow x^2-10x+9\ (=0)$ | A1 | 3-term quadratic |
| | $(x-1)(x-9)\ (=0)$ | DM1 | Or formula or completing square on a quadratic obtained by a correct method |
| | $x=1, 9$ | A1 | |
| | | 4 | |

12.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | $2x^2+m(2x+1)-6x-4(=0)$ | *M1 | y eliminated and all terms on one side with correct algebraic steps. Condone ± errors |
| | Using $b^2-4ac$ on $2x^2+x(2m-6)+m-4\ \ (=0)$ | DM1 | Any use of discriminant with their $a$, $b$ and $c$ identified correctly. |
| | $4m^2-32m+68$ **or** $2m^2-16m+34$ **or** $m^2-8m+17$ | A1 | |
| | $(2m-8)^2+k$ **or** $(m-4)^2+k$ **or** minimum point $(4,k)$ **or** finds $b^2-4ac\ \ (=-4,-16,-64)$ | DM1 | OE. Any valid method attempted on their 3-term quadratic |
| | $(m-4)^2+1$ oe + always $>0 \rightarrow$ 2 solutions for all values of $m$ **or** Minimum point (4,1) + (fn) always $>0 \rightarrow$ 2 solutions for all values of $m$ **or** $b^2-4ac\ <0$ + no solutions $\rightarrow$ 2 solutions for the original equation for all values of $m$ | A1 | Clear and correct reasoning and conclusion without wrong working. |
| | | 5 | |

13.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 9(a) | $r=\sqrt{(6^2+3^2)}$ or $r^2=45$ | B1 | Sight of r = 6.7 implies B1 |
| | $(x-5)^2+(y-1)^2=r^2$ or $x^2-10x+y^2-2y=r^2-26$ | M1 | Using centre given and *their* radius or $r$ in correct formula |
| | $(x-5)^2+(y-1)^2=45$ or $x^2-10x+y^2-2y=19$ | A1 | Do not allow $\left(\sqrt{45}\right)^2$ for $r^2$ |
| | | 3 | |
| 9(b) | $C$ has coordinates (11, 4) | B1 | |
| | 0.5 | B1 | OE, Gradient of $AB$, $BC$ or $AC$. |
| | Grad of CD $=-2$ | M1 | Calculation of gradient needs to be shown for this M1. |
| | $(\frac{1}{2}\times-2=-1\ )$ then states + perpendicular $\rightarrow$ hence shown or tangent | A1 | Clear reasoning needed. |

| | | | |
|---|---|---|---|
| | **Alternative method for question 9(b)** | | |
| | $C$ has coordinates (11, 4) | **B1** | |
| | 0.5 | **B1** | OE, Gradient of $AB$, $BC$ or $AC$. |
| | Gradient of the perpendicular is –2<br>→ Equation of the perpendicular is $y-4=-2(x-11)$ | **M1** | Use of $m_1m_2=-1$ with *their* gradient of $AB$, $BC$ or $AC$ and correct method for the equation of the perpendicular.<br>Could use $D(5, 16)$ instead of $C(11,4)$. |
| | Checks $D(5, 16)$ **or** checks gradient of $CD$ and then states<br>$D$ lies on the line **or** $CD$ has gradient –2→ hence shown or tangent | **A1** | Clear check and reasoning needed. Checks that the other point lies on the line or checks gradient. |
| 9(b) | **Alternative method for question 9(b)** | | |
| | $C$ has coordinates (11, 4) **or** Gradient of $AB$, $BC$ or $AC$ = 0.5 | **B1** | Only one of $AB$, $BC$ or $AC$ needed. |
| | Equation of the perpendicular is $y-4=-2(x-11)$ | **B1** | Finding equation of $CD$. |
| | $(x-5)^2+(-2x+26-1)^2=45 \rightarrow (x^2-22x+121=0)$ | **M1** | Solving simultaneously with the equation of the circle. |
| | $(x-11)^2=0$ or $b^2-4ac=0$ → repeated root → hence shown or tangent | **A1** | Must state repeated root. |
| | **Alternative method for question 9(b)** | | |
| | $C$ has coordinates (11, 4) | **B1** | |
| | Finding $CD=\sqrt{180}$ and $BD=\sqrt{225}$ | **B1** | OE. Calculated from the co-ordinates of $B$, $C$ & $D$ without using $r$. |
| | Checking (their BD)$^2$ – (their CD)$^2$ is the same as (their r)$^2$ | **M1** | |
| | ∴ Pythagoras valid ∴ perpendicular → hence shown or tangent | **A1** | Triangle $ACD$ could be used instead. |
| | **Alternative method for question 9(b)** | | |
| | $C$ has coordinates (11, 4) | **B1** | |
| | Finding vectors $\overrightarrow{AC}$ and $\overrightarrow{CD}$ **or** $\overrightarrow{BC}$ and $\overrightarrow{CD}$<br>$(=\begin{pmatrix}6\\3\end{pmatrix} and \begin{pmatrix}-6\\12\end{pmatrix}$ **or** $\begin{pmatrix}12\\6\end{pmatrix} and \begin{pmatrix}-6\\12\end{pmatrix})$ | **B1** | Must be correct pairing. |
| | Applying the scalar product to one of these pairs of vectors | **M1** | Accept *their* $\overrightarrow{AC}$ and $\overrightarrow{CD}$ or *their* $\overrightarrow{BC}$ and $\overrightarrow{CD}$ |
| | Scalar product = 0 then states ∴ perpendicular → hence shown or tangent | **A1** | |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 9(c) | $E(-1, 4)$ | **B1 B1** | WWW<br>B1 for each coordinate<br>Note: Equation of DE which is $y=2x+6$ may be used to find $E$ |
| | | **2** | |

14.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4 | $3x^2-4x+4=mx+m-1 \rightarrow 3x^2-(4+m)x+(5-m)\ (=0)$ | **M1** | 3-term quadratic |
| | $b^2-4ac=(4+m)^2-4\times3\times(5-m)$ | **M1** | Find $b^2-4ac$ for *their* quadratic |
| | $m^2+20m-44$ | **A1** | |
| | $(m+22)(m-2)$ | **A1** | Or use of formula or completing square. This step must be seen |
| | $m>2$ , $m<-22$ | **A1** | Allow $x>2$ , $x<-22$ |
| | | **5** | |

15.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 11(a) | $(-6-8)^2+(6-4)^2$ | M1 | OE |
| | $=200$ | A1 | |
| | $\sqrt{200}>10$, hence outside circle | A1 | AG ('Shown' not sufficient). Accept equivalents of $\sqrt{200}>10$ |
| | **Alternative method for question 11(a)** | | |
| | Radius = 10 and $C=(8, 4)$ | B1 | |
| | Min($x$) on circle = 8 – 10 = –2 | M1 | |
| | Hence outside circle | A1 | AG |
| | | 3 | |
| 11(b) | angle $=\sin^{-1}\left(\frac{their\,10}{their\,10\sqrt{2}}\right)$ | M1 | Allow decimals for 10√2 at this stage. If cosine used, angle *ACT* or *BCT* must be identified, or implied by use of 90°– 45°. |
| | angle = $\sin^{-1}\left(\frac{1}{\sqrt{2}}\text{ or }\frac{\sqrt{2}}{2}\text{ or }\frac{10}{10\sqrt{2}}\text{ or }\frac{10}{\sqrt{200}}\right)=45°$ | A1 | AG Do not allow decimals |
| | **Alternative method for question 11(b)** | | |
| | $(10\sqrt{2})^2=10^2+TA^2$ | M1 | |
| | $TA=10\rightarrow 45°$ | A1 | AG |
| | | 2 | |
| 11(c) | Gradient, $m$, of $CT=-\frac{1}{7}$ | B1 | OE |
| | Attempt to find mid-point (M) of *CT* | *M1 | Expect (1, 5) |
| | Equation of *AB* is $y-5=7(x-1)$ | DM1 | Through *their* (1, 5) with gradient $-\frac{1}{m}$ |
| | $y=7x-2$ | A1 | |
| | | 4 | |
| 11(d) | $(x-8)^2+(7x-2-4)^2=100$ or equivalent in terms of $y$ | M1 | Substitute *their* equation of *AB* into equation of circle. |
| | $50x^2-100x\ (=0)$ | A1 | |
| | $x=0$ and 2 | A1 | WWW |
| | **Alternative method for question 11(d)** | | |
| | **MC** = $\begin{pmatrix}7\\-1\end{pmatrix}$ | M1 | |
| | $\begin{pmatrix}1\\5\end{pmatrix}+\begin{pmatrix}-1\\-7\end{pmatrix}=\begin{pmatrix}0\\-2\end{pmatrix}$, $\begin{pmatrix}1\\5\end{pmatrix}+\begin{pmatrix}1\\7\end{pmatrix}=\begin{pmatrix}2\\12\end{pmatrix}$ | A1 | |
| | $x=0$ and 2 | A1 | |
| | | 3 | |

16.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4 | $x^2+kx+6=3x+k$ leading to $x^2+x(k-3)+(6-k)$ [$=0$] | M1 | Eliminate $y$ and form 3-term quadratic. |
| | $(k-3)^2-4(6-k)[>0]$ | M1 | OE. Apply $b^2-4ac$. |
| | $k^2-2k-15[>0]$ | A1 | Form 3-term quadratic. |
| | $(k+3)(k-5)[>0]$ | A1 | Or $k=-3,\ 5$ from use of formula or completing square. |
| | $k<-3, \quad k>5$ | A1 FT | Or any correct alternative notation, do not allow $\leqslant, \geqslant$.<br>FT for *their* outside regions. |
| | | **5** | |

17.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(a) | Centre of circle is (4, 5) | B1 B1 | |
| | $r^2=(7-4)^2+(1-5)^2$ | M1 | OE. Either using *their* centre and $A$ or $C$ **or** using $A$ and $C$ and dividing by 2. |
| | $r=5$ | A1 FT | FT on *their* (4, 5) if used. |
| | Equation is $(x-4)^2+(y-5)^2=25$ | A1 | OE. Allow $5^2$ for 25. |
| | | **5** | |
| 8(b) | Gradient of radius $=\frac{9-5}{7-4}=\frac{4}{3}$ | B1 FT | FT for use of *their* centre. |
| | Equation of tangent is $y-9=-\frac{3}{4}(x-7)$ | B1 | or $y=\frac{-3x}{4}+\frac{57}{4}$ |
| | | **2** | |

18.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 11(a) | $9\left(x^{-\frac{1}{2}}-4x^{-\frac{3}{2}}\right)=0$ leading to $9x^{-\frac{3}{2}}(x-4)=0$ | M1 | OE. Set $y$ to zero and attempt to solve. |
| | $x=4$ **only** | A1 | From use of a correct method. |
| | | **2** | |
| 11(b) | $\frac{dy}{dx}=9\left(-\frac{1}{2}x^{-\frac{3}{2}}+6x^{-\frac{5}{2}}\right)$ | B2, 1, 0 | B2; all 3 terms correct: 9, $-\frac{1}{2}x^{-\frac{3}{2}}$ and $6x^{-\frac{5}{2}}$<br>B1; 2 of the 3 terms correct |
| | At $x=4$ gradient $=9\left(-\frac{1}{16}+\frac{6}{32}\right)=\frac{9}{8}$ | M1 | Using *their* $x=4$ in *their* differentiated expression and attempt to find equation of the tangent. |
| | Equation is $y=\frac{9}{8}(x-4)$ | A1 | or $y=\frac{9x}{8}-\frac{9}{2}$ OE |
| | | **4** | |

19.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6 | $(2k-3)x^2-kx-(k-2)=3x-4$ | *M1 | Equating curve and line |
| | $(2k-3)x^2-(k+3)x-(k-6)[=0]$ | DM1 | Forming a 3-term quadratic |
| | $(k+3)^2+4(2k-3)(k-6)[=0]$ | DM1 | Use of discriminant (dependent on **both** previous M marks) |
| | $9k^2-54k+81[=0]$ [leading to $k^2-6k+9=0$] | M1 | Simplifying and solving *their* 3-term quadratic in $k$ |
| | $k=3$ | A1 | |
| | **Alternative method for Question 6** | | |
| | $(2k-3)x^2-kx-(k-2)=3x-4$ | *M1 | Equating curve and line |
| | $2(2k-3)x-k=3 \Rightarrow x=\frac{k+3}{4k-6}$ or $k=\frac{3+6x}{4x-1}$ | DM1 | Differentiating and solving for $x$ or $k$ |
| | **Either** $(2k-3)\left(\frac{k+3}{4k-6}\right)^2-k\left(\frac{k+3}{4k-6}\right)-(k-2)=3\left(\frac{k+3}{4k-6}\right)-4$ <br> **Or** $4x\left(\frac{3x^2+3x-6}{2x^2-x-1}\right)-6x-\left(\frac{3x^2+3x-6}{2x^2-x-1}\right)=3$ | DM1 | Substituting *their* $x$ into equation or *their* $k=\frac{3x^2+3x-6}{2x^2-x-1}$ or $k=\frac{3x+6}{2x+1}$ into derivative equation (dependent on **both** previous M marks) |
| | $9k^2-54k+81[=0]$ [leading to $k^2-6k+9=0$] | M1 | Simplifying and solving *their* 3-term quadratic in $k$ (or solving for $x$) |
| | $k=3$ | A1 | |
| | | | **SC** If M0, B1 for differentiating, equating to 3 and solving for $x$ or $k$ |
| | | 5 | |

20.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 10(a) | When $y=0$ $\quad x^2-4x-77=0$ [$\Rightarrow (x+7)(x-11)=0$ or $(x-2)^2=81$] | M1 | Substituting $y=0$ |
| | So $x$-coordinates are $-7$ and $11$ | A1 | |
| | | 2 | |

| | | | |
|---|---|---|---|
| 10(b) | Centre of circle $C$ is $(2, -3)$ | B1 | |
| | Gradient of $AC$ is $-\frac{1}{3}$ or Gradient of $BC$ is $\frac{1}{3}$ | M1 | For either gradient (M1 sign error, M0 if $x$-coordinate(s) in numerator) |
| | Gradient of tangent at $A$ is 3 or Gradient of tangent at $B$ is $-3$ | M1 | For either perpendicular gradient |
| | Equations of tangents are $y = 3x + 21$, $y = -3x + 33$ | A1 | For either equation |
| | Meet when $3x + 21 = -3x + 33$ | M1 | OR: (centre of circle has $x$ coordinate 2) so $x$ coordinate of point of intersection is 2 |
| | Coordinates of point of intersection (2, 27) | A1 | |
| | **Alternative method for Question 10(b)** | | |
| | Implicit differentiation: $2y\frac{dy}{dx}$ seen | B1 | |
| | $2x-4+2y\frac{dy}{dx}+6\frac{dy}{dx}=0$ | M1 | Fully differentiated $=0$ with at least one term involving $y$ differentiated correctly |
| | Gradient of tangent at $A$ is 3 or Gradient of tangent at $B$ is $-3$ | M1 | For either gradient |
| | Equations of tangents are $y = 3x + 21$, $y = -3x + 33$ | A1 | For either equation |
| | Meet when $3x + 21 = -3x + 33$ | M1 | OR: (centre of circle has $x$ coordinate 2) so $x$ coordinate of point of intersection is 2 |
| | Coordinates of point of intersection (2, 27) | A1 | |
| | | **6** | |

21.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | 1.2679 | B1 | AWRT. ISW if correct answer seen. $3-\sqrt{3}$ scores B0 |
| | | **1** | |
| 3(b) | 1.7321 | B1 | AWRT. ISW if correct answer seen. |
| | | **1** | |
| 3(c) | Sight of 2 or 2.0000 or two in reference to the gradient | *B1 | |
| | This is because the gradient at $E$ is the limit of the gradients of the chords as the $x$-value tends to 3 or $\delta x$ tends to 0. | DB1 | Allow it gets nearer/approaches/tends/almost/approximately 2 |
| | | **2** | |

22.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6 | Gradient AB = $\frac{1}{2}$ | B1 | SOI |
| | Lines meet when $-2x+4=\frac{1}{2}(x-8)+3$<br>Solving as far as $x=$ | *M1 | Equating given perpendicular bisector with the line through (8, 3) using *their* gradient of *AB* (but not -2) and solving. Expect $x=2$, $y=0$. |
| | Using mid-point to get as far as $p=$ or $q=$ | DM1 | Expect $\frac{8+p}{2}=2$ or $\frac{3+q}{2}=0$ |
| | $p=-4$, $q=-3$ | A1 | Allow coordinates of *B* are $(-4, -3)$. |
| | **Alternative method for Question 6** | | |
| | Gradient AB = $\frac{1}{2}$ | B1 | SOI |
| | $\frac{q-3}{p-8}=\frac{1}{2}$ [leading to $2q=p-2$],<br>$\frac{q+3}{2}=-2\left(\frac{8+p}{2}\right)+4$ [leading to $q=-11-2p$] | *M1 | Equating gradient of *AB* with *their* gradient of *AB* (but not -2) and using mid-point in equation of perpendicular bisector. |
| | Solving simultaneously *their* 2 linear equations | DM1 | Equating and solving 2 correct equations as far as $p=$ or $q=$ . |
| | $p=-4$, $q=-3$ | A1 | Allow coordinates of *B* are $(-4, -3)$. |
| 6 | **Alternative method for Question 6** | | |
| | Gradient AB = $\frac{1}{2}$ | B1 | |
| | $\frac{q-3}{p-8}=\frac{1}{2}$ [leading to $p=2q+2$],<br>$y-\frac{q+3}{2}=-2(x-(q+5))$ $\left[\text{leading to } y=-2x+\frac{5q+23}{2}\right]$ | *M1 | Equating gradient of *AB* with *their* gradient of *AB* (but not -2) and using mid-point in equation of perpendicular bisector. |
| | *their* $\frac{5q+23}{2}=4\Rightarrow q=$ | DM1 | Equating and solving as far as $q$ or $p=$ |
| | $p=-4$, $q=-3$ | A1 | Allow coordinates of *B* are $(-4, -3)$. |
| | | **4** | |

23.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | $(5-1)^2 + (11-5)^2 = 52$ or $\frac{11-5}{5-1}$ | M1 | For substituting (1,5) into circle equation or showing gradient $=\frac{3}{2}$. |
| | For both circle equation and gradient, and proving line is perpendicular and stating that *A* lies on the circle | A1 | Clear reasoning. |
| | **Alternative method for Question 7(a)** | | |
| | $(x-5)^2+(y-11)^2=52$ and $y-5=-\frac{2}{3}(x-1)$ | M1 | Both equations seen and attempt to solve. May see $y=-\frac{2}{3}x+\frac{17}{3}$ |
| | Solving simultaneously to obtain $(y-5)^2 = 0$ or $(x-1)^2 = 0 \Rightarrow 1$ root or tangent or discriminant $= 0 \Rightarrow 1$ root or tangent | A1 | Clear reasoning. |
| | **Alternative method for Question 7(a)** | | |
| | $\frac{dy}{dx}=\frac{10-2x}{2y-22}=\frac{10-2}{10-22}$ | M1 | Attempting implicit differentiation of circle equation and substitute $x = 1$ and $y = 5$. |
| | Showing gradient of circle at A is $-\frac{2}{3}$ | A1 | Clear reasoning. |
| | | **2** | |
| 7(b) | Centre is $(-3, -1)$ | B1 B1 | B1 for each correct co-ordinate. |
| | Equation is $(x+3)^2 + (y+1)^2 = 52$ | B1 FT | FT *their* centre, but not if either (1, 5) or (5, 11). Do not accept $\sqrt{52^2}$ . |
| | | **3** | |

24.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | $x^2-4x+3=mx-6$ leading to $x^2-x(4+m)+9$ | ***M1** | Equating and gathering terms. May be implied on the next line. |
| | $b^2-4ac$ leading to $(4+m)^2-4\times9$ | **DM1** | SOI. Use of the discriminant with *their a*, *b* and *c* |
| | $4+m=\pm6$ or $(m-2)(m+10)=0$ leading to $m=2$ or $-10$ | **A1** | Must come from $b^2-4ac=0$ SOI |
| | Substitute both *their m* values into *their* equation in line 1 | **DM1** | |
| | $m = 2$ leading to $x = 3$ ; $m = -10$ leading to $x = -3$ | **A1** | |
| | (3, 0), (–3, 24) | **A1** | Accept 'when $x = 3$, $y = 0$; when $x = -3$, $y = 24$' If final A0A0 scored, **SC B1** for one point correct WWW |
| | **Alternative method for Question 3** | | |
| | $\frac{dy}{dx}=2x-4 \to 2x-4=m$ | ***M1** | |
| | $x^2-4x+3=(2x-4)x-6$ | **DM1** | |
| | $x^2-4x+3=2x^2-4x-6 \to 9=x^2 \to x=\pm3$ | **A1** | |
| | $y=0,\ 24$ or (3, 0), (–3, 24) | **A1** | |
| | Substitute both *their x* values into *their* equation in line 1 | **DM1** | Or substitute both *their* $(x, y)$ into $y=mx-6$ |
| | When $x = 3$, $m = 2$; when $x = -3$, $m = -10$ | **A1** | If A0, DM1, A0 scored, **SC B1** for one point correct WWW |
| | | **6** | |

25.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 10(a) | Gradient of $AB = -\frac{3}{5}$, gradient of $BC = \frac{5}{3}$ or lengths of all 3 sides or vectors | M1 | Attempting to find required gradients, sides or vectors |
| | $m_{ab}m_{bc} = -1$ or Pythagoras or $\overline{AB}.\overline{BC} = 0$ or $\cos ABC = 0$ from cosine rule | A1 | WWW |
| | | 2 | |
| 10(b) | Centre = mid-point of $AC = (2,4)$ | B1 | |
| | | 1 | |
| 10(c) | $(x - their\ x_c)^2 + (y - their\ y_c)^2 [= r^2]$ or $(their\ x_c - x)^2 + (their\ y_c - y)^2 = [r^2]$ | M1 | Use of circle equation with *their* centre |
| | $(x-2)^2 + (y-4)^2 = 17$ | A1 | Accept $x^2 - 4x + y^2 - 8y + 3 = 0$ OE |
| | | 2 | |
| 10(d) | $\left(\frac{x+3}{2}, \frac{y+0}{2}\right) = (2,4)$ or **BE** = 2**BD** = $2\begin{pmatrix}-1\\4\end{pmatrix}$<br>Or Equation of *BE* is $y = -4(x-3)$or $y - 4 = -4(x-2)$ leading to $y = -4x + 12$<br>Substitute equation of *BE* into circle and form a 3-term quadratic. | M1 | Use of mid-point formula, vectors, steps on a diagram<br><br>May be seen to find *x* coordinate at *E* |
| | $(x, y) = (1,8)$ or **OE** = $\begin{pmatrix}3\\0\end{pmatrix} + \begin{pmatrix}-2\\8\end{pmatrix} = \begin{pmatrix}1\\8\end{pmatrix}$ | A1 | $E = (1, 8)$<br>Accept without working for both marks **SC B2** |
| | Gradient of *BD*, *m*, = –4 or gradient $AC = \frac{1}{4}$ = gradient of tangent | B1 | Or gradient of *BE* = -4 |
| | Equation of tangent is $y - 8 = ¼(x-1)$ OE | M1 A1 | For M1, equation through *their* E or (1, 8) (not, *A*, *B* or *C*) and with gradient $\frac{-1}{their\ -4}$ |
| | | 5 | |

26.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | $r^2\left[=(5-2)^2 + (7-5)^2\right] = 13$ | B1 | $r^2 = 13$ or $r = \sqrt{13}$ |
| | Equation of circle is $(x-5)^2 + (y-2)^2 = 13$ | B1 FT | OE. FT on *their* 13 but LHS must be correct. |
| | | 2 | |
| 7(b) | $(x-5)^2 + (5x - 10 - 2)^2 = 13$ | M1 | Substitute $y = 5x - 10$ into *their* equation. |
| | $26x^2 - 130x + 156\ [=0]$ | A1 FT | OE 3-term quadratic with all terms on one side.<br>FT on *their* circle equation. |
| | $[26](x-2)(x-3)\ [=0]$ | M1 | Solve 3-term quadratic in *x* by factorising, using formula or completing the square. Factors must expand to give *their* coefficient of $x^2$. |
| | (2, 0), (3, 5) | A1 A1 | Coordinates must be clearly paired; A1 for each correct point. A1 A0 available if two *x* or *y* values only.<br>If M0 for solving quadratic, **SC B2** can be awarded for correct coordinates, **SC B1** if two *x* or *y* values only. |
| | $(AB)^2 = (3-2)^2 + (5-0)^2$ | M1 | SOI. Using *their* points to find length of *AB*. |
| | $AB = \sqrt{26}$ | A1 | ISW. Dependent on final M1 only. |

| | | | |
|---|---|---|---|
| 7(b) | **Alternative method for question 7(b)** | | |
| | $\left(\frac{y+10}{5}-5\right)^2+(y-2)^2=13$ | **M1** | Substitute $x=\frac{y+10}{5}$ into *their* equation. |
| | $\frac{26y^2}{25}-\frac{26y}{5}\ [=0]$ | **A1 FT** | OE 2-term quadratic with all terms on one side. FT on *their* circle equation. |
| | $[26]y(y-5)\ [=0]$ | **M1** | Solve 2-term quadratic in $y$ by factorising, using formula or completing the square. Factors must expand to give *their* coefficient of $y^2$. |
| | (2, 0), (3, 5) | **A1 A1** | Coordinates must be clearly paired; A1 for each correct point. A1 A0 available if two $x$ or $y$ values only.<br>If M0 for solving quadratic, **SC B2** can be awarded for correct coordinates, **SC B1** if two $x$ or $y$ values only. |
| | $(AB)^2=(3-2)^2+(5-0)^2$ | **M1** | SOI. Using *their* points to find length of $AB$. |
| | $AB=\sqrt{26}$ | **A1** | ISW. Dependent on final M1 only. |
| | | **7** | |

1.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7 | $OC = 6\cos 0.8 = 4.18(0)$ | M1A1 | SOI |
| | Area sector $OCD = \frac{1}{2}(their\,4.18)^2 \times 0.8$ | *M1 | OE |
| | $\Delta OCA = \frac{1}{2} \times 6 \times their\,4.18 \times \sin 0.8$ | M1 | OE |
| | Required area = *their* $\Delta OCA$ – *their* sector$OCD$ | DM1 | SOI. If not seen *their* areas of sector and triangle must be seen |
| | 2.01 | A1 | CWO. Allow or better e.g. 2.0064 |
| | | 6 | |

2.

| Question | Answer | Marks |
|---|---|---|
| 8 | Angle $AOB = 15 \div 6 = 2.5$ radians | B1 |
| | Angle $BOC = \pi - 2.5$ (**FT** on angle AOB) | B1FT |
| | $BC = 6(\pi - 2.5)$ $(BC = 3.850)$ | M1 |
| | $\sin(\pi - 2.5) = BX \div 6$ $(BX = 3.59)$ | M1 |
| | **Either** $OX = 6\cos(\pi - 2.5)$ **or** Pythagoras $(OX = 4.807)$ | M1 |
| | $XC = 6 - OX$ $(XC = 1.193) \rightarrow P = 8.63$ | A1 |
| | | 6 |

3.

| Question | Answer | Marks |
|---|---|---|
| 7(a) | $BC^2 = r^2 + 4r^2 - 2r.2r \times \cos\left(\frac{\pi}{6}\right) = 5r^2 - 2r^2\sqrt{3}$ | M1 |
| | $BC = r\sqrt{\left(5 - 2\sqrt{3}\right)}$ | A1 |
| | | 2 |
| 7(b) | Perimeter $= \frac{2\pi r}{6} + r + r\sqrt{\left(5 - 2\sqrt{3}\right)}$ | M1 A1 |
| | | 2 |
| 7(c) | Area = sector – triangle | |
| | Sector area $= \frac{1}{2}4r^2\frac{\pi}{6}$ | M1 |
| | Triangle area = ½ $r$. $2r \sin\frac{\pi}{6}$ | M1 |
| | Shaded area $= r^2\left(\frac{\pi}{3} - \frac{1}{2}\right)$ | A1 |
| | | 3 |

4.

| Question | Answer | Marks |
|---|---|---|
| 5 | $\cos POA = \frac{5}{13} \to POA = 1.17(6)$ Allow 67.4° <br> **or** $\sin = \frac{12}{13}$ or $\tan = \frac{12}{5}$ | M1 A1 |
| | Reflex $AOB = 2\pi - 2\times$ *their* 1.17(6) OE in degrees <br> **or** minor arc AB = $5\times2\times$*their*1.17(6) | M1 |
| | Major arc = $5 \times$ *their* 3.93(1) <br> **or** $2\pi\times5$ - *their* 11.7(6) | M1 |
| | $AP$ (or $BP$) = $\sqrt{13^2 - 5^2} = 12$ | B1 |
| | Cord length = 43.7 | A1 |
| | | 6 |

5.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 10(a) | $\left(\sin\theta = \frac{r}{OC} \to\right) OC = \frac{r}{\sin\theta}$ | M1 A1 | |
| | $CD = r + \frac{r}{\sin\theta}$ | A1 | |
| | | 3 | |
| 10(b) | Radius of arc $AB = 4 + \frac{4}{\sin\frac{\pi}{6}} = 4+8=12$ | B1 | SOI |
| | (Arc $AB$ =) *their* $12\times\frac{2\pi}{6}$ or $\left(\frac{1}{2}AB=\right)\left(\textit{their } 12\times\frac{\pi}{6}\right)$ | M1 | Expect 4π, must use *their* CD, not 4 |
| | Perimeter = $24+4\pi$ | A1 | |
| | | 3 | |
| 10(c) | Area $FOC = \frac{1}{2}\times4\times\textit{their } OC\times\sin\frac{\pi}{3}$ | M1 | |
| | $8\sqrt{3}$ | A1 | |
| | Area sector $FOE = \frac{1}{2}\times\frac{2\pi}{3}\times4^2 = \frac{16\pi}{3}$ | B1 | |
| | Shaded area = $16\sqrt{3} - \frac{16\pi}{3}$ | A1 | |
| | **Alternative method for question 10(c)** | | |
| | $FC = \sqrt{(\textit{their } OC)^2 - 4^2}$ | M1 | $\sqrt{48}$ or $4\sqrt{3}$ |
| | Area $FOC = \frac{1}{2}\times4\times4\sqrt{3} = 8\sqrt{3}$ | A1 | |
| | Area of half sector $FOE = \frac{1}{2}\times\frac{\pi}{3}\times4^2 = \frac{8\pi}{3}$ | B1 | |
| | Shaded area = $16\sqrt{3} - \frac{16\pi}{3}$ | A1 | |
| | | 4 | |

6.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(a) | Use of correct formula for the area of triangle *ABC* | M1 | Use of $180-2\theta$ scores M0. Condone $2\pi-2\theta$ |
| | $\frac{1}{2}r^2\sin(\pi-2\theta)$ **or** $\frac{1}{2}r^2\sin 2\theta$ **or** $2\times\frac{1}{2}r\times r\cos\theta\times\sin\theta$ **or** $2\times\frac{1}{2}r\cos\theta\times r\sin\theta$ | A1 | OE |
| | [Shaded area = triangle – sector] = *their* triangle area $-\frac{1}{2}r^2\theta$ | B1 FT | FT for *their* triangle area $-\frac{1}{2}r^2\theta$ (Condone use of 180 degrees for triangle area for B1) |
| | | 3 | |
| 8(b) | Arc $BD = r\theta = 6$ cm | B1 | SOI |
| | $AC = 2r\cos\theta = (2\times10\cos0.6 = 20\cos0.6 = 16.506)$ or $\sqrt{(2r^2-2r^2\cos(\pi-2\theta))}$ or $\frac{r\times\sin(\pi-2\theta)}{\sin\theta}$ | *M1 | Finding *AC* or $\frac{1}{2}AC$ (= 8.25) |
| | $DC = 2r\cos\theta - r$ or $\sqrt{(2r^2-2r^2\cos(\pi-2\theta))} - r$ (= 6.506) | DM1 | Subtracting *r* from *their AC* or $r$-$r\cos\theta$ from *their* half *AC* (8.25-1.75) |
| | (Perimeter = 10 + 6 + 6.506 =) 22.5 | A1 | AWRT |
| | | 4 | |

7.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 9(a) | $\cos BAO = \frac{6}{8}$ or $\frac{8^2+12^2-8^2}{2\times8\times12}$ | M1 | Or other correct method |
| | $BAO = 0.723$ | A1 | |
| | | 2 | |
| 9(b) | Sector $ABC$ = ½×12² × *their* 0.7227 | *M1 | Accept 52.1 |
| | Triangle $AOB$ = ½×8×12sin(*their* 0.7227) or ½×12×√28 | *M1 | or ½×8×8sin($\pi$ – 2× *their* 0.7227) . Expect 31.7 or 31.8 |
| | Shaded area = *their* 52.0 – *their* 31.7 = 20.3 | DM1<br>A1 | M1 dependent on both previous M marks |
| | | 4 | |
| 9(c) | Arc $BC$ = 12× *their* 0.7227 | *M1 | Expect 8.67 |
| | Perimeter = 8 + 4 + *their* 8.67 = 20.7 | DM1<br>A1 | |
| | | 3 | |

8.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 10(a) | $\Delta ADE = \frac{1}{2}(ka)^2 \sin\frac{\pi}{6}$ | M1 | Attempt to find the area of $\Delta ADE$. |
| | $\frac{1}{4}k^2a^2$ | A1 | OE. |
| | Sector $ABC = \frac{1}{2}a^2\frac{\pi}{6}$ | B1 | |
| | $2\times\frac{1}{4}k^2a^2 = \frac{1}{2}a^2\frac{\pi}{6}$ | M1 | OE. For $2\times\Delta ADE = \text{sector}ABC$ with at least one correct area. |
| | $k = \left(\sqrt{\frac{\pi}{6}}\right) = 0.7236$ | A1 | |
| | | 5 | |
| 10(b) | $2\times\frac{1}{2}(ka)^2\sin\theta = \frac{1}{2}a^2\theta$ | M1 | Condone omission of '2' or '1/2' on LHS for M1 only. |
| | $k^2 = \frac{\theta}{2\sin\theta}$ | A1 | |
| | $k^2 > \frac{1}{2}$ leading to $\frac{1}{\sqrt{2}} < k < 1$ | A1 | OE. Accept $k > \frac{1}{\sqrt{2}}$ or $k > 0.707$ (AWRT) or $0.707\text{(AWRT)} < k < 1$ or $k > \sqrt{\frac{1}{2}}$ OE |
| | | 3 | |

9.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(a) | **Either** Let midpoint of $PQ$ be $H$: $\sin HCP = \frac{2}{4} \Rightarrow$ Angle $HCP = \frac{\pi}{6}$<br>**Or** $\sin PSQ = \frac{4}{8} \Rightarrow$ Angle $PSQ = \frac{\pi}{6}$<br>**Or** using cosine rule: angle $PCQ = \frac{\pi}{3}$<br>**Or** by inspection: triangle $PCQ$ or $PCT$ is equilateral so angle $PCQ = \frac{\pi}{3}$ | M1 | |
| | Angle $PCS = \pi - \frac{\pi}{6} - \frac{\pi}{6} = \frac{2}{3}\pi$ | A1 | AG |
| | | 2 | |
| 8(b) | Perimeter $= 2\times 4\times\frac{2\pi}{3}$ or $8\pi - \frac{8\pi}{3}$ | M1 | Length of two arcs $PS$ and $QR$ |
| | $+2\pi\times 2$ | M1 | Adding circumference of two semicircles |
| | $\frac{28\pi}{3}$ | A1 | Must be a single term |
| | | 3 | |

| | | | |
|---|---|---|---|
| 8(c) | Area sector $CPQ = \frac{1}{2}\times 4^2 \times \frac{\pi}{3} = \frac{8\pi}{3}$ | M1 | Uses correct formula for sector |
| | Area of segment of large circle beyond $CPQ$ $= \frac{8\pi}{3} - \frac{1}{2}\times 4^2 \times \sin\left(\frac{\pi}{3}\right) = \frac{8\pi}{3} - 4\sqrt{3}$ | M1 | Attempts to find area of segment |
| | Area of small semicircle = $\pi \times 2$ or area of small circle = $\pi \times 2^2$ | M1 | |
| | Area of plate = Large circle – [2 ×] small semicircle – [2 ×] segment area | M1 | |
| | $\pi\times 4^2 - \pi\times 2^2 - 2\times\left(\frac{8\pi}{3} - 4\sqrt{3}\right) = \frac{20\pi}{3} + 8\sqrt{3}$ | A1 | AG |
| | **Alternative method for Question 8(c)** | | |
| | Area of sector $PCS = \frac{1}{2}\times 4^2 \times \frac{2\pi}{3} = \frac{16\pi}{3}$ | M1 | Uses correct formula for sector |
| | Area of triangle $PCQ = \frac{1}{2}\times 4^2 \times \sin\frac{\pi}{3} = 4\sqrt{3}$ | M1 | Uses correct formula for triangle |
| | Area of small semicircle = $\pi \times 2$ or area of circle = $\pi \times 2^2$ | M1 | |
| | Area of plate = [2 ×] large sector + [2 ×] triangle – [2 ×] small semicircle | M1 | |
| | $2\left(\frac{16\pi}{3}\right) + 2\left(4\sqrt{3}\right) - \pi\times 2^2 = \frac{20\pi}{3} + 8\sqrt{3}$ | A1 | AG |
| | | 5 | |

10.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 12(a) | [By symmetry] [6 × $P\hat{A}Q$ = 2π], [ $P\hat{A}Q$ =] 2π÷6, | M1 | |
| | Explaining that there are six sectors around the diagram that make up a complete circle. | A1 | AG |
| | **Alternative method for Question 12(a)** | | |
| | Using area or circumference of circle centre $A$ ÷ 6 | M1 | $\frac{400\pi}{6}$ or $\frac{40\pi}{6}$ |
| | Justification for dividing by 6 followed by comparison with the sector area or arc length. | A1 | AG |
| | **Alternative method for Question 12(a)** | | |
| | Explain why $\Delta PAQ$ is an equilateral triangle | M1 | Assumption of this scores M0 |
| | Using $\Delta PAQ$ is an equilateral triangle ∴ $P\hat{A}Q = \frac{\pi}{3}$ | A1 | AG |
| | **Alternative method for Question 12(a)** | | |
| | Using the internal angle of a regular hexagon = $\frac{2\pi}{3}$<br>Or $F\hat{A}O + O\hat{A}B = \frac{2\pi}{3}$ , equilateral triangles | M1 | |
| | $P\hat{A}Q = 2\pi - \left(\frac{\pi}{2} + \frac{2\pi}{3} + \frac{\pi}{2}\right) = \frac{\pi}{3}$ | A1 | AG |
| 12(a) | **Alternative method for Question 12(a)** | | |
| | $Sin\theta = \frac{20}{40}$, with $\theta$ clearly identified | M1 | |
| | $\theta = \frac{\pi}{6}, 2\theta = \frac{\pi}{3} = F\hat{A}O$ and by similar triangles = $P\hat{A}Q$ | A1 | AG |
| | | 2 | |

| | | | |
|---|---|---|---|
| 12(b) | Each straight section of rope has length 40 cm | B1 | SOI |
| | Each curved section round each pipe has length $r\theta = 20\times\frac{\pi}{3}$ | *M1 | Use of $r\theta$ with $r = 20$ and $\theta$ in radians |
| | Total length = $6\times((their\ 40)+k\pi)$ | DM1 | $6\times$(*their* straight section + *their* curved section). *Their* curved section must be from acceptable use of $r\theta$ – this could now be numeric. |
| | $240+40\pi$ or 366 (AWRT) (cm) | A1 | Or directly: (6× diameter) + circumference |
| | | **4** | |

| | | | |
|---|---|---|---|
| 12(c) | [Triangle area =] $\frac{1}{2}\times40\times40\times\sin\left(\frac{\pi}{3}\right)$ or $\frac{1}{2}\times40\times20\sqrt{3}$ or $400\sqrt{3}$ or 693(AWRT) | B1 | |
| | [Total area of hexagon = 6 × $400\sqrt{3}$ =] $2400\sqrt{3}$ | B1 | Condone $4800\frac{\sqrt{3}}{2}$ |
| | **Alternative method for Question 12(c)** | | |
| | [Trapezium area =] $\frac{1}{2}\times(40+80)\times40\sin\left(\frac{\pi}{3}\right)$ or $1200\sqrt{3}$ or 2080 (AWRT) | B1 | |
| | [Total area of hexagon = 2 × $1200\sqrt{3}$ =] $2400\sqrt{3}$ | B1 | Condone $4800\frac{\sqrt{3}}{2}$ |
| | **Alternative method for Question 12(c)** | | |
| | Area of triangle *ABC* = $400\sqrt{3}$ or 693 (AWRT) or 4 × Area of half of triangle *ABC* = $4\times200\sqrt{3}$ or 1390 (AWRT) or Area of rectangle *ABDE* = $1600\sqrt{3}$ or 2770 (AWRT) | B1 | |
| | [Total area of hexagon = $2\times400\sqrt{3}+1600\sqrt{3}$ =] $2400\sqrt{3}$ Or [= $4\times200\sqrt{3}+1600$ =] $2400\sqrt{3}$ | B1 | Condone $4800\frac{\sqrt{3}}{2}$ |
| | | | If B0B0, **SC B1** can be scored for sight of 4160 (AWRT) as final answer. |
| | | **2** | |

| | | | |
|---|---|---|---|
| 12(d) | Each rectangle area = 40 × 20 (= 800) | B1 | SOI, e.g. by sight of 4800 |
| | Each sector area = $\frac{1}{2}r^2\theta=\frac{1}{2}\times20^2\times\frac{\pi}{3}\left[=\frac{200\pi}{3}\right]$ | B1 | SOI. |
| | Total area = $2400\sqrt{3}+4800+400\pi$ or 10 200 (cm²) (AWRT) | B1 | Or directly: part **(c)** + 6800 + area circle radius 20. |
| | | **3** | |

11.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | $½\times4^2\times$ angle BAD = 10 | **M1** | Use of sector area formula |
| | Angle BAD = 1.25 | **A1** | OE. Accept $0.398\pi$, $71.6°$ for **SC B1** only |
| | | **2** | |
| 5(b) | Arc $BD = 4\times their 1.25$ | **M1** | Use of arc length formula. Expect 5. |
| | $BC = 4\tan(their 1.25)$ | **M1** | Expect 12.0(4). May use *ACB*=0.321 or 18.4° |
| | $CD = \frac{4}{\cos(their 1.25)}-4$ or $\sqrt{4^2+(their\ BC)^2}-4$ | **M1** | Expect 12.69 – 4 = 8.69. May use *ACB*. |
| | Perimeter = 5 + 12.0(4) + 8.69 = 25.7 (cm) | **A1** | AWRT |
| | | **4** | |

12.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | Recognise that at least one of angles $A$, $B$, $C$ is $\frac{\pi}{3}$ | B1 | SOI; allow 60°. |
| | One arc $6 \times \textit{their}\ \frac{\pi}{3}$ leading to two arcs $2 \times 6 \times \textit{their}\ \frac{\pi}{3}$ | M1 | SOI e.g. may see $2\pi$ or $4\pi$.<br>Use of correct formula for length of arc and multiply by 2. |
| | Perimeter = $6 + 4\pi$ | A1 | Must be exact value. |
| | **Alternative method for question 6(a)** | | |
| | Calculate circumference of whole circle = $12\pi$ | B1 | |
| | One arc $\frac{1}{6} \times 12\pi$ leading to two arcs $2 \times \frac{1}{6} \times 12\pi$ | M1 | SOI e.g. may see $2\pi$ or $4\pi$. |
| | Perimeter = $6 + 4\pi$ | A1 | Must be exact value. |
| | | 3 | |
| 6(b) | Sector = $\frac{1}{2} \times 6^2 \times \textit{their}\left(\frac{\pi}{3}\right)$ | M1 | Use of correct formula for area of sector.<br>SOI e.g. may see $6\pi$ or $12\pi$. |
| | $\frac{1}{2}\times(6^2)\times \textit{their}\left(\frac{\pi}{3}\right) - \frac{1}{2}\times(6^2)\times\sin\left(\textit{their}\left(\frac{\pi}{3}\right)\right) + 6\pi\ \left[= 6\pi - 9\sqrt{3} + 6\pi\right]$ | M1 A1 | M1 for attempt at strategy with values substituted:<br>**area of segment + area of sector**<br>A1 if correct (unsimplified). |
| | Area = $12\pi - 9\sqrt{3}$ | A1 | Must be simplified exact value. |
| | **Alternative method for question 6(b)** | | |
| | Sector = $\frac{1}{2} \times 6^2 \times \textit{their}\left(\frac{\pi}{3}\right)$ | M1 | Use of correct formula for area of sector.<br>SOI e.g. may see $6\pi$ or $12\pi$. |
| | $2\times\left(\frac{1}{2}\times 6^2 \times \textit{their}\left(\frac{\pi}{3}\right)\right) - \frac{1}{2}\times(6^2)\times\sin\left(\textit{their}\left(\frac{\pi}{3}\right)\right)$ | M1 A1 | M1 for attempt at strategy with values substituted:<br>**2 × sector – triangle**<br>A1 if correct (unsimplified). |
| | Area = $12\pi - 9\sqrt{3}$ | A1 | Must be simplified exact value. |
| | **Alternative method for question 6(b)** | | |
| | Sector = $\frac{1}{2} \times 6^2 \times \textit{their}\left(\frac{\pi}{3}\right)$ | M1 | Use of correct formula for area of sector.<br>SOI e.g. may see $6\pi$ or $12\pi$. |
| | $2\times\left(\frac{1}{2}\times(6^2)\times \textit{their}\left(\frac{\pi}{3}\right) - \frac{1}{2}\times(6^2)\times\sin\left(\textit{their}\left(\frac{\pi}{3}\right)\right)\right) +$<br>$\frac{1}{2}\times(6^2)\times\sin\left(\textit{their}\left(\frac{\pi}{3}\right)\right)\ \left[= 12\pi - 18\sqrt{3} + 9\sqrt{3}\right]$ | M1 A1 | M1 for attempt at strategy with values substituted:<br>**2 × segment + triangle**<br>A1 if correct (unsimplified). |
| | Area $\left[= 6\pi - 9\sqrt{3} + 6\pi\right]$ = $12\pi - 9\sqrt{3}$ | A1 | Must be simplified exact value. |
| | | 4 | |

# Trigonometry

1.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5 | $2\tan\theta - 6\sin\theta + 2 = \tan\theta + 3\sin\theta + 2 \rightarrow \tan\theta - 9\sin\theta\ (=0)$ | M1 | Multiply by denominator and simplify |
| | $\sin\theta - 9\sin\theta\cos\theta\ (=0)$ | M1 | Multiply by $\cos\theta$ |
| | $\sin\theta(1-9\cos\theta)\ (=0) \rightarrow \sin\theta = 0,\quad \cos\theta = \frac{1}{9}$ | M1 | Factorise and attempt to solve at least one of the factors = 0 |
| | $\theta = 0$ or $83.6°$ (only answers in the given range) | A1A1 | |
| | | 5 | |

2.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 11(a) | $(\tan x - 2)(3\tan x + 1)\ (=0)$. **or** formula **or** completing square | M1 | Allow reversal of signs in the factors. Must see a method |
| | $\tan x = 2$ or $-\frac{1}{3}$ | A1 | |
| | $x = 63.4°$(only value in range) or $161.6°$(only value in range ) | B1FT<br>B1FT | |
| | | 4 | |
| 11(b) | Apply $b^2 - 4ac < 0$ | M1 | SOI. Expect $25 - 4(3)(k) < 0$, $\tan x$ must not be in coefficients |
| | $k > \frac{25}{12}$ | A1 | Allow $b^2 - 4ac = 0$ leading to correct $k > \frac{25}{12}$ for M1A1 |
| | | 2 | |
| 11(c) | $k = 0$ | M1 | SOI |
| | $\tan x = 0$ or $\frac{5}{3}$ | A1 | |
| | $x = 0°$ or $180°$ or $59.0°$ | A1 | All three required |
| | | 3 | |

3.

| Question | Answer | Marks |
|---|---|---|
| 7(a) | $\frac{(1+\sin\theta)^2 + \cos^2\theta}{\cos\theta(1+\sin\theta)}$ | M1 |
| | Use of $\sin^2\theta + \cos^2\theta = 1 \rightarrow \frac{2+2\sin\theta}{\cos\theta(1+\sin\theta)} \rightarrow \frac{2}{\cos\theta}$. | M1A1 |
| | | 3 |
| 7(b) | $\frac{2}{\cos\theta} = \frac{3}{\sin\theta} \rightarrow \tan\theta = 1.5$ | M1 |
| | $\theta = 0.983$ or $4.12$<br>(**FT** on second value for 1st value $+ \pi$) | A1<br>A1FT |
| | | 3 |

4.

| Question | Answer | Marks |
|---|---|---|
| 2(a) | $3\cos\theta = 8\tan\theta \rightarrow 3\cos\theta = \dfrac{8\sin\theta}{\cos\theta}$ | M1 |
| | $3(1 - \sin^2\theta) = 8\sin\theta$ | M1 |
| | $3\sin^2\theta + 8\sin\theta - 3 = 0$ | A1 |
| | | 3 |
| 2(b) | $(3\sin\theta - 1)(\sin\theta + 3) = 0 \rightarrow \sin\theta = ⅓$ | M1 |
| | $\theta = 19.5°$ | A1 |
| | | 2 |

5.

| Question | Answer | Marks |
|---|---|---|
| 7(a) | $\dfrac{\tan\theta}{1+\cos\theta} + \dfrac{\tan\theta}{1-\cos\theta} = \dfrac{\tan\theta(1-\cos\theta)+\tan\theta(1+\cos\theta)}{1-\cos^2\theta}$ | M1 |
| | $= \dfrac{2\tan\theta}{\sin^2\theta}$ | M1 |
| | $= \dfrac{2\sin\theta}{\cos\theta\sin^2\theta}$ | M1 |
| | $= \dfrac{2}{\sin\theta\cos\theta}$ **AG** | A1 |
| | | 4 |
| 7(b) | $\dfrac{2}{\sin\theta\cos\theta} = \dfrac{6\cos\theta}{\sin\theta}$ | M1 |
| | $\cos^2\theta = \dfrac{1}{3} \rightarrow \cos\theta = (\pm)0.5774$ | A1 |
| | 54.7º, 125.3º<br>(**FT** for 180º – 1st solution) | A1<br>A1FT |
| | | 4 |

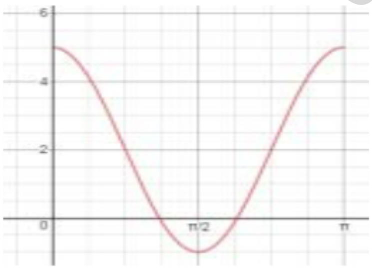
6.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | $\left(\dfrac{\sin\theta}{1-\sin\theta} - \dfrac{\sin\theta}{1+\sin\theta} =\right)\dfrac{\sin\theta(1+\sin\theta)-\sin\theta(1-\sin\theta)}{1-\sin^2\theta}$ | *M1 | Put over a single common denominator |
| | $\dfrac{2\sin^2\theta}{\cos^2\theta}$ | DM1 | Replace $1-\sin^2\theta$ by $\cos^2\theta$ and simplify numerator |
| | $2\tan^2\theta$ | A1 | AG |
| | | 3 | |
| 7(b) | $2\tan^2\theta = 8 \rightarrow \tan\theta = (\pm)2$ | B1 | SOI |
| | $(\theta =)$ 63.4°, 116.6° | B1<br>B1 FT | FT on 180 – 1st solution (with justification) |
| | | 3 | |

7.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | $\left(\frac{1}{\cos x}-\frac{\sin x}{\cos x}\right)\left(\frac{1}{\sin x}+1\right)$ | B1 | Uses "tan*x* = sin*x* ÷ cos*x*" throughout |
| | $\left(\frac{1-\sin x}{\cos x}\right)\left(\frac{1+\sin x}{\sin x}\right)$ or $\left(\frac{1-\sin^2 x}{\cos x\sin x}\right)$ | M1 | Correct algebra leading to two or four terms |
| | $\left(\frac{\cos^2 x}{\cos x\sin x}\right)$ | A1 | OE. A correct expression which can be cancelled directly to $\frac{\cos x}{\sin x}$ e.g. $\frac{\cos x\left(1-\sin^2 x\right)}{\sin x\left(1-\sin^2 x\right)}$ |
| | $\left(\frac{\cos^2 x}{\cos x\sin x}\right)=\left(\frac{\cos x}{\sin x}\right)=\frac{1}{\tan x}$ | A1 | AG. Must show cancelling. If *x* is missing throughout their working withhold this mark. |
| | | 4 | |
| 6(b) | Uses **(a)** → $\frac{1}{\tan x}=2\tan^2 x$ $\tan^3 x=\frac{1}{2}$ | M1 | Reducing to $\tan^3 x = k$. |
| | $(x =)$ 38.4° | A1 | AWRT.<br>Ignore extra answers outside the range 0 to 180° but A0 if within. |
| | | 2 | |

8.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 11(a) | 5, –1 | B1 B1 | Sight of each value |
| | | 2 | |
| 11(b) |  | *B1 | Needs to be a curve, not straight lines.<br>One complete cycle starting and finishing at *their* largest value. |
| | | DB1 | One complete cycle starting and finishing at $y = 5$ and going down to $y = -1$ and starting to level off at least one end. |
| | | 2 | |
| 11(c)(i) | 0 solution | B1 | |
| | | 1 | |
| 11(c)(ii) | 2 solutions | B1 | |
| | | 1 | |
| 11(c)(iii) | 1 solution | B1 | |
| | | 1 | |

9.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | $3\tan^4\theta+\tan^2\theta-2\ (=0)$ | M1 | SOI 3-term quartic, condone sign errors for this mark only |
| | $\left(3\tan^2\theta-2\right)\left(\tan^2\theta+1\right)\ (=0)$ | M1 | Attempt to factorise or solve 3-term quadratic in $\tan^2\theta$. |
| | $\tan\theta=(\pm)\sqrt{\frac{2}{3}}$ or $(\pm)0.816$ or $(\pm)0.817$ | A1 | SOI Implied by final answer = 39.2° after 1st M1 scored |
| | 39.2°, 140.8° | A1<br>A1 FT | FT for 2nd solution =180° – 1st solution |
| | | 5 | |

10.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | $\tan\theta + 2\sin\theta = 3\tan\theta - 6\sin\theta$ leading to $2\tan\theta - 8\sin\theta\ [=0]$ | M1 | OE |
| | $2\sin\theta - 8\sin\theta\cos\theta\ (=0)$ leading to $[2]\sin\theta(1-4\cos\theta)\ [=0]$ | M1 | |
| | $\cos\theta = \frac{1}{4}$ | A1 | Ignore $\sin\theta = 0$ |
| | $\theta = 75.5°$ **only** | A1 | |
| | | 4 | |

11.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4 | $a = 2$ | B1 | |
| | $b = \frac{\pi}{4}$ | B1 | or $\frac{2\pi}{8}$ |
| | $c = 1$ | B1 | |
| | | 3 | |

12.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | Reach $\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}$ **or** $\frac{1-\sin^2\theta}{1-\sin^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}$ **or** $\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta} - 2\tan^2\theta$ **or** $\sec^2\theta - \frac{2\sin^2\theta}{\cos^2\theta}$ **or** $2 - \sec^2\theta$ **or** $\frac{\cos 2\theta}{\cos^2\theta}$ | M1 | May start with $1 - \tan^2\theta$ |
| | $1 - \tan^2\theta$ | A1 | AG, must show sufficient stages |
| | | 2 | |
| 7(b) | $1 - \tan^2\theta = 2\tan^4\theta \Rightarrow 2\tan^4\theta + \tan^2\theta - 1\ [=0]$ | M1 | Forming a 3-term quadratic in $\tan^2\theta$ or e.g. $u$ |
| | $\tan^2\theta = 0.5$ or $-1$ leading to $\tan\theta = [\pm]\sqrt{0.5}$ | M1 | |
| | $\theta = 35.3°$ and $144.7°$ (AWRT) | A1 | Both correct. Radians 0.615, 2.53 scores A0. |
| | | 3 | |

13.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 10(a) | $\frac{1+\sin x}{1-\sin x} - \frac{1-\sin x}{1+\sin x} \equiv \frac{(1+\sin x)^2 - (1-\sin x)^2}{(1-\sin x)(1+\sin x)}$ | *M1 | For using a common denominator of $(1-\sin x)(1+\sin x)$ and reasonable attempt at the numerator(s). |
| | $\equiv \frac{1+2\sin x+\sin^2 x - (1-2\sin x+\sin^2 x)}{(1-\sin x)(1+\sin x)}$ | DM1 | For multiplying out the numerators correctly. Condone sign errors for this mark. |
| | $\equiv \frac{4\sin x}{1-\sin^2 x} \equiv \frac{4\sin x}{\cos^2 x}$ | DM1 | For simplifying denominator to $\cos^2 x$. |
| | $\equiv \frac{4\sin x}{\cos x\cos x} \equiv \frac{4\tan x}{\cos x}$ | A1 | AG.<br>Do not award A1 if undefined notation such as s, c, t or missing $x$'s used throughout or brackets are missing. |

**Alternative method for Question 10(a)**

| | | | |
|---|---|---|---|
| | $\frac{4\tan x}{\cos x} \equiv \frac{4\sin x}{\cos^2 x} \equiv \frac{4\sin x}{1-\sin^2 x}$ | *M1 | Using $\tan x = \frac{\sin x}{\cos x}$ and $\cos^2 x = 1-\sin^2 x$ |
| | $\equiv \frac{-2}{1+\sin x} + \frac{2}{1-\sin x}$ | DM1 | Separating into partial fractions. |
| | $\equiv 1 + \frac{-2}{1+\sin x} + \frac{2}{1-\sin x} - 1$ | DM1 | Use of 1-1 or similar |
| | $\equiv -\frac{1-\sin x}{1+\sin x} + \frac{1+\sin x}{1-\sin x}$ | A1 | |
| | | **4** | |
| 10(b) | $\cos x = \frac{1}{2}$ | *B1 | OE. WWW. |
| | $x = \frac{\pi}{3}$ | DB1 | Or AWRT 1.05 |
| | $x = 0$ from $\tan x = 0$ or $\sin x = 0$ | B1 | WWW. Condone extra solutions outside the domain 0 to $\frac{\pi}{2}$ but B0 if any inside. |
| | | **3** | |

14.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | $\frac{\tan x + \sin x}{\tan x - \sin x}[=k]$ leading to $\frac{\sin x + \sin x\cos x}{\sin x - \sin x\cos x}[=k]$ or $\frac{\frac{1}{\cos x}+1}{\frac{1}{\cos x}-1}$ [=k] or $\frac{\tan x + \tan x\cos x}{\tan x - \tan x\cos x}$ [=k] | **M1** | Multiply numerator and denominator by $\cos x$, or divide numerator and denominator by $\tan x$ or $\sin x$ |
| | $\frac{\sin x(1+\cos x)}{\sin x(1-\cos x)}$ or $\frac{\frac{1}{\cos x}+1}{\frac{1}{\cos x}-1} \cdot \frac{\cos x}{\cos x}$ or $\frac{\tan x(1+\cos x)}{\tan x(1-\cos x)}$ leading to $\frac{1+\cos x}{1-\cos x}[=k]$ | **A1** | AG, WWW |
| | | **2** | |
| 4(b) | $k - k\cos x = 1 + \cos x$ leading to $k - 1 = k\cos x + \cos x$ | **M1** | Gather like terms on LHS and RHS |
| | $k - 1 = (k+1)\cos x$ leading to $\cos x = \frac{k-1}{k+1}$ | **A1** | WWW, OE |
| | | **2** | |
| 4(c) | Obtaining $\cos x$ from *their* **(b)** or **(a)** | **M1** | Expect $\cos x = \frac{3}{5}$ |
| | $\pm 0.927$ (only solutions in the given range) | **A1** | AWRT. Accept $\pm 0.295\pi$ |
| | | **2** | |

15.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | $3\cos\theta(2\tan\theta - 1) + 2(2\tan\theta - 1)\ [=0]$ | **M1** | Or similar partial factorisation; condone sign errors. |
| | $(2\tan\theta - 1)(3\cos\theta + 2)\ [=0]$<br>[leading to $\tan\theta = \frac{1}{2}$, $\cos\theta = -\frac{2}{3}$ ] | **M1** | OE. At least 2 out of 4 products correct. |
| | 26.6°, 131.8° | **A1 A1** | WWW. Must be 1 d.p. or better.<br>Final A0 if extra solution within the interval.<br>**SC B1** No factorisation: Division by $2\tan\theta - 1$ leading to 131.8° or division by $3\cos\theta + 2$ or similar leading to 26.6°. |

| | | | |
|---|---|---|---|
| **Alternative method for question 3** | | | |
| $6\cos\theta\left(\frac{\sin\theta}{\cos\theta}\right)-3\cos\theta+4\left(\frac{\sin\theta}{\cos\theta}\right)-2\ [=0]$<br>$6\cos\theta\sin\theta-3\cos^2\theta+4\sin\theta-2\cos\theta\ [=0]$<br>$2\sin\theta(3\cos\theta+2)-\cos\theta(3\cos\theta+2)\ \ [=0]$ | **M1** | Using $\tan\theta=\frac{\sin\theta}{\cos\theta}$ and reaching a partial factorisation; condone sign errors. |
| $(2\sin\theta-\cos\theta)(3\cos\theta+2)\ \ [=0]$<br>[leading to $\tan\theta=\frac{1}{2}$, $\cos\theta=-\frac{2}{3}$] | **M1** | At least 2 out of 4 products correct. |
| 26.6°, 131.8° | **A1 A1** | WWW. Must be 1 d.p. or better.<br>Final A0 if extra solution within the interval.<br>**SC B1** No factorisation: Division by $2\tan\theta-1$ leading to 131.8° or division by $3\cos\theta+2$ or similar leading to 26.6°. |
| | **4** | |

16.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | $a=5$ | **B1** | |
| | $b=2$ | **B1** | |
| | $c=3$ | **B1** | |
| | | **3** | |
| 5(b)(i) | 3 | **B1** | |
| | | **1** | |
| 5(b)(ii) | 2 | **B1** | |
| | | **1** | |

# Calculus

1.

| Question | Answer | Marks |
| --- | --- | --- |
| 9(a) | $\frac{dy}{dx} = 3(3-2x)^2 \times -2 + 24 = -6(3-2x)^2 + 24$ (**B1** without ×−2. **B1** for ×−2) | B1B1 |
| | $\frac{d^2y}{dx^2} = -12(3-2x)\times -2 = 24(3-2x)$ (**B1FT** from $\frac{dy}{dx}$ without − 2) | B1FT B1 |
| | | 4 |

2.

| Question | Answer | Marks | Guidance |
| --- | --- | --- | --- |
| 4 | $\frac{dy}{dx} = 2x - 2$ | B1 | |
| | $\frac{dy}{dx} = \frac{4}{6}$ | B1 | OE, SOI |
| | $their(2x-2) = their\frac{4}{6}$ | M1 | LHS and RHS must be *their* $\frac{dy}{dx}$ expression and value |
| | $x = \frac{4}{3}$ oe | A1 | |
| | | 4 | |

3.

| Question | Answer | Marks |
| --- | --- | --- |
| 10(a) | $\frac{dy}{dx} = 54 - 6(2x-7)^2$ | B2,1 |
| | $\frac{d^2y}{dx^2} = -24(2x-7)$ (**FT** only for omission of '×2' from the bracket) | B2,1 FT |
| | | 4 |

4.

| Question | Answer | Marks | Guidance |
| --- | --- | --- | --- |
| 6 | $\frac{dy}{dx} = \left[\frac{1}{2}(25-x^2)^{-1/2}\right] \times [-2x]$ | B1 B1 | |
| | $\frac{-x}{(25-x^2)^{1/2}} = \frac{4}{3} \rightarrow \frac{x^2}{25-x^2} = \frac{16}{9}$ | M1 | Set $= \frac{4}{3}$ and square both sides |
| | $16(25-x^2) = 9x^2 \rightarrow 25x^2 = 400 \rightarrow x = (\pm)4$ | A1 | |
| | When $x = -4$, $y = 5 \rightarrow (-4, 5)$ | A1 | |
| | | 5 | |

5.

| Question | Answer | Marks | Guidance |
| --- | --- | --- | --- |
| 8(a) | $\frac{dy}{dx} = [2] \quad [-2(2x+1)^{-2}]$ | B1 B1 | |
| | $\frac{d^2y}{dx^2} = 8(2x+1)^{-3}$ | B1 | |
| | | 3 | |

6.

| Question | Answer | Marks | Guidance |
| --- | --- | --- | --- |
| 10(a) | $\frac{dy}{dx}=\left[\frac{x^{-1/2}}{2k}\right]-\left[\frac{x^{-3/2}}{2}\right]+([0])$ | B2, 1, 0 | $([0])$ implies that more than 2 terms counts as an error |
| | Sub $\frac{dy}{dx}=3$ when $x=\frac{1}{4}$ Expect $3=\frac{1}{k}-4$ | M1 | |
| | $k=\frac{1}{7}$ (or 0.143) | A1 | |
| | | 4 | |

1.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | $f'(x)=\left[-(3x+2)^{-2}\right]\times[3]+[2x]$ | B2, 1, 0 | |
| | $<0$ hence decreasing | B1 | Dependent on at least B1 for $f'(x)$ and must include $<0$ or '(always) neg' |
| | | 3 | |

2.

| Question | Answer | Marks |
|---|---|---|
| 9(a) | $\frac{dy}{dx}=3(3-2x)^2\times-2+24=-6(3-2x)^2+24$ (**B1** without $\times-2$. **B1** for $\times-2$) | B1B1 |
| | $\frac{d^2y}{dx^2}=-12(3-2x)\times-2=24(3-2x)$ (**B1FT** from $\frac{dy}{dx}$ without $-2$) | B1FT B1 |
| | | 4 |
| 9(b) | $\frac{dy}{dx}=0$ when $6(3-2x)^2=24\rightarrow 3-2x=\pm2$ | M1 |
| | $x=½, y=20$ or $x=2½, y=52$ (**A1** for both $x$ values or a correct pair) | A1A1 |
| | | 3 |
| 9(c) | If $x=½$, $\frac{d^2y}{dx^2}=48$ Minimum | B1FT |
| | If $x=2½$, $\frac{d^2y}{dx^2}=-48$ Maximum | B1FT |
| | | 2 |

3.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | $[f'(x)=]\left((2x-1)^{1/2}\right)\times\left(\frac{1}{3}\times2\times\frac{3}{2}\right)(-2)$ | B2, 1, 0 | Expect $(2x-1)^{1/2}-2$ |
| | $(2x-1)^{1/2}-2\leqslant0\rightarrow 2x-1\leqslant4$ or $2x-1<4$ | M1 | SOI. Rearranging and then squaring, must have power of ½ not present<br>Allow '=0' at this stage but do not allow '$\geq 0$' or '$>0$'<br>If '$-2$' missed then must see $\leqslant$ or $<$ for the M1 |
| | Value [of $a$] is 2½ or $a=2½$ | A1 | WWW, OE e.g. $\frac{5}{2}$, 2.5<br>Do not allow from '=0' unless some reference to negative gradient. |
| | | 4 | |

1.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 10(a) | $2(a+3)^{\frac{1}{2}}-a=0$ | M1 | SOI. Set $\frac{dy}{dx}=0$ when $x=a$. Can be implied by an answer in terms of $a$ |
| | $4(a+3)=a^2 \to a^2-4a-12=0$ | M1 | Take $a$ to RHS and square. Form 3-term quadratic |
| | $(a-6)(a+2) \to a=6$ | A1 | Must show factors, or formula or completing square. Ignore $a=-2$ **SC** If $a$ is never used maximum of M1A1 for $x=6$ ,with visible solution |
| | | 3 | |
| 10(b) | $\frac{d^2y}{dx^2}=(x+3)^{-\frac{1}{2}}-1$ | B1 | |
| | Sub *their a* $\to \frac{d^2y}{dx^2}=\frac{1}{3}-1=-\frac{2}{3}$ $(or<0) \to$ MAX | M1A1 | A mark only if completely correct. If the second differential is not $-\frac{2}{3}$ correct conclusion must be drawn to award the M1 |
| | | 3 | |
| 10(c) | $(y=)\frac{2(x+3)^{\frac{3}{2}}}{\frac{3}{2}}-\frac{1}{2}x^2$ $(+c)$ | B1B1 | |
| | Sub $x$ = *their a* and $y=14$ $\to 14=\frac{4}{3}(9)^{\frac{3}{2}}-18+c$ | M1 | Substitute into an integrated expression. $c$ must be present. Expect $c=-4$ |
| | $y=\frac{4}{3}(x+3)^{\frac{3}{2}}-\frac{1}{2}x^2-4$ | A1 | Allow $f(x)=\ldots$ |
| | | 4 | |

2.

| Question | Answer | Marks |
|---|---|---|
| 10(a) | $\frac{dy}{dx}=54-6(2x-7)^2$ | B2,1 |
| | $\frac{d^2y}{dx^2}=-24(2x-7)$ (**FT** only for omission of '×2' from the bracket) | B2,1 FT |
| | | 4 |
| 10(b) | $\frac{dy}{dx}=0 \to (2x-7)^2=9$ | M1 |
| | $x=5, y=243$ or $x=2, y=135$ | A1 A1 |
| | | 3 |
| 10(c) | $x=5$ $\frac{d^2y}{dx^2}=-72 \to$ Maximum (**FT** only for omission of '×2' from the bracket) | B1FT |
| | $x=2$ $\frac{d^2y}{dx^2}=72 \to$ Minimum (**FT** only for omission of '×2' from the bracket) | B1FT |
| | | 2 |

3.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 12(a) | $4x^{\frac{1}{2}}-2x=3-x \rightarrow x-4x^{\frac{1}{2}}+3(=0)$ | *M1 | 3-term quadratic. Can be expressed as e.g. $u^2-4u+3$ (=0) |
| | $\left(x^{\frac{1}{2}}-1\right)\left(x^{\frac{1}{2}}-3\right)(=0)$ or $(u-1)(u-3)(=0)$ | DM1 | Or quadratic formula or completing square |
| | $x^{\frac{1}{2}}=1,\ 3$ | A1 | SOI |
| | $x=1,\ 9$ | A1 | |
| | **Alternative method for question 12(a)** | | |
| | $\left(4x^{\frac{1}{2}}\right)^2=(3+x)^2$ | *M1 | Isolate $x^{\frac{1}{2}}$ |
| | $16x=9+6x+x^2 \rightarrow x^2-10x+9\ (=0)$ | A1 | 3-term quadratic |
| | $(x-1)(x-9)\ (=0)$ | DM1 | Or formula or completing square on a quadratic obtained by a correct method |
| | $x=1,\ 9$ | A1 | |
| | | 4 | |
| 12(b) | $\frac{dy}{dx}=2x^{1/2}-2$ | *B1 | |
| | $\frac{dy}{dx}$ or $2x^{1/2}-2=0$ when $x=1$ hence $B$ is a stationary point | DB1 | |
| | | 2 | |

4.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(a) | $\frac{dy}{dx}=[2]\quad[-2(2x+1)^{-2}]$ | B1 B1 | |
| | $\frac{d^2y}{dx^2}=8(2x+1)^{-3}$ | B1 | |
| | | 3 | |
| 8(b) | Set *their* $\frac{dy}{dx}=0$ and attempt solution | M1 | |
| | $(2x+1)^2=1 \rightarrow 2x+1=(\pm)1$ or $4x^2+4x=0 \rightarrow (4)x(x+1)=0$ | M1 | Solving as far as $x=$ .... |
| | $x=0$ | A1 | WWW. Ignore other solution. |
| | (0, 2) | A1 | One solution only. Accept $x=0,\ y=2$ only. |
| | $\frac{d^2y}{dx^2}>0$ from a solution $x>-\frac{1}{2}$ hence minimum | B1 | Ignore other solution. Condone arithmetic slip in value of $\frac{d^2y}{dx^2}$. *Their* $\frac{d^2y}{dx^2}$ must be of the form $k(2x+1)^{-3}$ |
| | | 5 | |

5.

| | | | |
|---|---|---|---|
| 11(c) | $9x^{-\frac{5}{2}}\left(-\frac{1}{2}x+6\right)=0$ | M1 | Set *their* $\frac{dy}{dx}$ to zero and an attempt to solve. |
| | $x=12$ | A1 | Condone $(\pm)12$ from use of a correct method. |
| | | 2 | |

6.

| Question | Answer | Marks | Guidance |
| --- | --- | --- | --- |
| 11(a) | At stationary point $\frac{dy}{dx}=0$ so $6(3\times2-5)^3-k\times2^2=0$ | M1 | Setting given $\frac{dy}{dx}=0$ and substituting $x=2$ into it. |
| | $[k=]\frac{3}{2}$ | A1 | OE |
| | | 2 | |
| 11(b) | $[y=]\frac{6}{4\times3}(3x-5)^4-\frac{1}{3}kx^3$ $[+c]$. | *M1 A1FT | Integrating (increase of power by 1 in at least one term) given $\frac{dy}{dx}$. Expect $\frac{1}{2}(3x-5)^4-\frac{1}{2}x^3$. FT *their* non zero $k$. |
| | $-\frac{7}{2}=\frac{1}{2}(3\times2-5)^4-\frac{1}{3}\times\frac{3}{2}\times2^3+c$ [leading to $-3.5+c=-3.5$] | DM1 | Using (2,-3.5) in an integrated expression. + $c$ needed. Substitution needs to be seen, simply stating $c=0$ is DM0. |
| | $y=\frac{1}{2}(3x-5)^4-\frac{1}{2}x^3$ | A1 | $y=$ or f($x$)= must be seen somewhere in solution. |
| 11(b) | **Alternative method for Question 11(b)** | | |
| | $[y=]\frac{81}{2}x^4-\frac{541}{2}x^3+675x^2-750x(+c)$ or $-270x^3-k\frac{x^3}{3}$ | *M1 A1 FT | From $\frac{dy}{dx}=162x^3-810x^2-kx^2-1350x-750$. FT *their* $k$ |
| | $-\frac{7}{2}=\frac{81}{2}\times2^4-\frac{541}{2}\times2^3+675\times2^2-750\times2+c$ | DM1 | Using (2, -3.5) in an integrated expression. + $c$ needed |
| | $y=\frac{81}{2}x^4-\frac{541}{2}x^3+675x^2-750x+\frac{625}{2}$ | A1 | $y=$ or f($x$)= must be seen somewhere in solution. |
| | | 4 | |
| 11(c) | $[3\times]\left[18(3x-5)^2\right][-2kx]$ | B2,1,0 FT | FT *their* $k$. Square brackets indicate each required component. B2 for fully correct, B1 for one error or one missing component, B0 for 2 or more errors. |
| | **Alternative method for Question 11(c)** | | |
| | $486x^2-1623x+1350$ or $-1620x-2kx$ | B2,1,0 FT | FT *their* $k$. B2 for fully correct, B1 for one error, B0 for 2 or more errors. |
| | | 2 | |
| 11(d) | [At $x=2$] $\left[\frac{d^2y}{dx^2}=\right]54(3\times2-5)^2-4k$ or 48 | M1 | OE. Substituting $x=2$ into *their* second differential or other valid method. |
| | [> 0] Minimum | A1 | WWW |
| | | 2 | |



7.

| Question | Answer | Marks | Guidance |
| --- | --- | --- | --- |
| 11(a) | $\frac{dy}{dx}=\frac{1}{2}x^{-1/2}-\frac{1}{2}k^2x^{-3/2}$ | B1 B1 | Allow any correct unsimplified form |
| | $\frac{1}{2}x^{-1/2}-\frac{1}{2}k^2x^{-3/2}=0$ leading to $\frac{1}{2}x^{-1/2}=\frac{1}{2}k^2x^{-3/2}$ | M1 | OE. Set to zero and one correct algebraic step towards the solutions. $\frac{dy}{dx}$ must only have 2 terms. |
| | $(k^2,2k)$ | A1 | |
| | | 4 | |

| | | | |
|---|---|---|---|
| 11(b) | When $x = 4k^2$, $\frac{dy}{dx} = \left[\frac{1}{4k} - \frac{1}{16k} = \right]\frac{3}{16k}$ | B1 | OE |
| | $y = \left[2k + k^2 \times \frac{1}{2k}\right] = \frac{5k}{2}$ | B1 | OE. Accept $2k + \frac{k}{2}$ |
| | Equation of tangent is $y - \frac{5k}{2} = \frac{3}{16k}\left(x - 4k^2\right)$ or $y = mx + c \rightarrow \frac{5k}{2} = \frac{3}{16k}\left(4k^2\right) + c$ | M1 | Use of line equation with *their* gradient and $(4k^2, \textit{their } y)$, |
| | When $x = 0$, $y = \left[\frac{5k}{2} - \frac{3k}{4} = \right]\frac{7k}{4}$ or from $y = mx + c$, $c = \frac{7k}{4}$ | A1 | OE |
| | | 4 | |

| | | | |
|---|---|---|---|
| 11(c) | $\int\left(x^{\frac{1}{2}} + k^2x^{-\frac{1}{2}}\right)dx = \frac{2x^{\frac{3}{2}}}{3} + 2k^2x^{\frac{1}{2}}$ | B1 | Any unsimplified form |
| | $\left(\frac{16k^3}{3} + 4k^3\right) - \left(\frac{9k^3}{4} + 3k^3\right)$ | M1 | Apply limits $\frac{9}{4}k^2 \rightarrow 4k^2$ to an integration of $y$. M0 if volume attempted. |
| | $\frac{49k^3}{12}$ | A1 | OE. Accept $4.08\,k^3$ |
| | | 3 | |

8.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 9(a) | $f(x) = \frac{2}{3}x^3 - 7x + 4x^{-1}$ $[+c]$ | B2, 1, 0 | Allow terms on different lines; allow unsimplified. |
| | $-\frac{1}{3} = \frac{2}{3} - 7 + 4 + c$ leading to $c = [2]$ | M1 | Substitute $f(1) = -\frac{1}{3}$ into an integrated expression and evaluate $c$. |
| | $f(x) = \frac{2}{3}x^3 - 7x + 4x^{-1} + 2$ | A1 | OE. |
| | | 4 | |
| 9(b) | $2x^4 - 7x^2 - 4\ [=0]$ | M1 | Forms 3-term quadratic in $x^2$ with all terms on one side. Accept use of substitution e.g. $2y^2 - 7y - 4[=0]$. |
| | $(2x^2 + 1)(x^2 - 4)\ [=0]$ | M1 | Attempt factors or use formula or complete the square. Allow $\pm$ sign errors. Factors must expand to give *their* coefficient of $x^2$ or e.g. $y$. Must be quartic equation. Accept use of substitution e.g. $(2y+1)(y-4)$. |
| | $x = [\pm]2$ | A1 | If M0 for solving quadratic, **SC B1** can be awarded for $[\pm]2$. |
| | $\left[\frac{2}{3}(2)^3 - 7(2) + \frac{4}{2} + 2 \text{ leading to}\right]\left(2, -\frac{14}{3}\right)$ <br> $\left[\frac{2}{3}(-2)^3 - 7(-2) + \frac{4}{-2} + 2 \text{ leading to}\right]\left(-2, \frac{26}{3}\right)$ | B1 B1 | B1 B1 for correct coordinates clearly paired; B1 for each correct point; B1 B0 if additional point. |
| | | 5 | |
| 9(c) | $f''(x) = 4x + 8x^{-3}$ | B1 | OE |
| | | 1 | |

| | | | |
|---|---|---|---|
| 9(d) | $f''(2)=9>0$ MINIMUM at $x=$ *their* 2 | B1 FT | FT on *their* $x=[\pm]2$ provided $f''(x)$ is correct. Must have correct value of $f''(x)$ if $x=2$. |
| | $f''(-2)=-9<0$ MAXIMUM at $x=$ *their* $-2$ | B1 FT | FT on *their* $x=[\pm]2$ provided $f''(x)$ is correct. Must have correct value of $f''(x)$ if $x=-2$. **Special case:** If values not shown and B0B0 scored, **SC B1** for $f''(2)>0$ MIN and $f''(-2)<0$ MAX |
| | **Alternative method for question 9(d)** | | |
| | Evaluate $f'(x)$ for *x*-values either side of 2 and –2 | M1 | FT on *their* $x=[\pm]2$ |
| | MINIMUM at $x=$ *their* 2, MAXIMUM at $x=$ *their* 2 | A1 FT | FT on *their* $x=[\pm]2$. Must have correct values of $f'(x)$ if shown. **Special case:** If values not shown and M0A0 scored **SC B1** $f'(2)$ $-/0/+$ MIN and $f'(-2)$ $+/0/-$ MAX |
| | **Alternative method for question 9(d)** | | |
| | Justify maximum and minimum using correct sketch graph | B1 B1 | Need correct coordinates in **(b)** for this method. |
| | | 2 | |

1.

| Question | Answer | Marks |
|---|---|---|
| 2 | $(y)=\frac{3x^{\frac{3}{2}}}{\frac{3}{2}}-\frac{3x^{\frac{1}{2}}}{\frac{1}{2}}\ (+c)$ | B1 B1 |
| | $7 = 16 - 12 + c$ <br>(**M1** for subsituting $x = 4$, $y = 7$ into *their* integrated expansion) | M1 |
| | $y=2x^{\frac{3}{2}}-6x^{\frac{1}{2}}+3$ | A1 |
| | | 4 |

2.

| Question | Answer | Marks |
|---|---|---|
| 11(a) | $\frac{dy}{dx}=3x^2-4bx+b^2$ | B1 |
| | $3x^2-4bx+b^2=0 \rightarrow (3x-b)(x-b)\ (=0)$ | M1 |
| | $x=\frac{b}{3}$ or $b$ | A1 |
| | $a=\frac{b}{3} \rightarrow b=3a$ **AG** | A1 |
| | **Alternative method for question 11(a)** | |
| | $\frac{dy}{dx}=3x^2-4bx+b^2$ | B1 |
| | Sub $b = 3a$ & obtain $\frac{dy}{dx}=0$ when $x = a$ and when $x = 3a$ | M1 |
| | $\frac{d^2y}{dx^2}=6x-12a$ | A1 |
| | $< 0$ Max at $x = a$ and $> 0$ Min at $x = 3a$. Hence $b = 3a$ **AG** | A1 |
| | | 4 |
| 11(b) | Area under curve $= \int (x^3-6ax^2+9a^2x)dx$ | M1 |
| | $\frac{x^4}{4}-2ax^3+\frac{9a^2x^2}{2}$ | B2,1,0 |
| | $\frac{a^4}{4}-2a^4+\frac{9a^4}{2}\ \left(=\frac{11a^4}{4}\right)$ <br>(**M1** for applying limits $0 \rightarrow a$) | M1 |
| | When $x = a$, $y=a^3-6a^3+9a^3=4a^3$ | B1 |
| | Area under line $=\frac{1}{2}a\times$ *their* $4a^3$ | M1 |
| | Shaded area $=\frac{11a^4}{4}-2a^4=\frac{3}{4}a^4$ | A1 |
| | | 7 |

3.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | $(y=)\left[-(x-3)^{-1}\right]\left[+\frac{1}{2}x^2\right]\ (+c)$ | B1 B1 | |
| | $7=1+2+c$ | M1 | Substitute $x=2$, $y=7$ into an integrated expansion ($c$ present). Expect $c=4$ |
| | $y=-(x-3)^{-1}+\frac{1}{2}x^2\ +4$ | A1 | OE |
| | | 4 | |

4.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 12(a) | $4x^{\frac{1}{2}}-2x=3-x\ \rightarrow x-4x^{\frac{1}{2}}+3(=0)$ | *M1 | 3-term quadratic. Can be expressed as e.g. $u^2-4u+3\ \ (=0)$ |
| | $\left(x^{\frac{1}{2}}-1\right)\left(x^{\frac{1}{2}}-3\right)(=0)$ or $(u-1)(u-3)(=0)$ | DM1 | Or quadratic formula or completing square |
| | $x^{\frac{1}{2}}=1,\ 3$ | A1 | SOI |
| | $x=1,\ 9$ | A1 | |
| | **Alternative method for question 12(a)** | | |
| | $\left(4x^{\frac{1}{2}}\right)^2=(3+x)^2$ | *M1 | Isolate $x^{\frac{1}{2}}$ |
| | $16x=9+6x+x^2\ \rightarrow\ x^2-10x+9\ (=0)$ | A1 | 3-term quadratic |
| | $(x-1)(x-9)\ (=0)$ | DM1 | Or formula or completing square on a quadratic obtained by a correct method |
| | $x=1,\ 9$ | A1 | |
| | | 4 | |
| 12(b) | $\frac{dy}{dx}=2x^{1/2}-2$ | *B1 | |
| | $\frac{dy}{dx}$ or $2x^{1/2}-2=0$ when $x=1$ hence $B$ is a stationary point | DB1 | |
| | | 2 | |
| 12(c) | Area of correct triangle $=\frac{1}{2}\ (9-3)\times 6$ | M1 | or $\int_3^9(3-x)(dx)=\left[3x-\frac{1}{2}x^2\right]\rightarrow\ -18$ |
| | $\int(4x^{\frac{1}{2}}-2x)(dx)=\left[\frac{4x^{\frac{3}{2}}}{\frac{3}{2}}-x^2\right]$ | B1 B1 | |
| | $(72-81)-\left(\frac{64}{3}-16\right)$ | M1 | Apply limits $4\rightarrow$ *their* 9 to an integrated expression |
| | $-14\frac{1}{3}$ | A1 | OE |
| | Shaded region $=18-14\frac{1}{3}=3\frac{2}{3}$ | A1 | OE |
| | | 6 | |

5.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | $f'(4)\left(=\frac{5}{2}\right)$ | *M1 | Substituting 4 into $f'(x)$ |
| | $\left(\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}\right) \to \left(\frac{dy}{dt}\right) = \frac{5}{2}\times 0.12$ | DM1 | Multiplies *their* $f'(4)$ by 0.12 |
| | $\left(\frac{dy}{dt}=\right)0.3$ | A1 | OE |
| | | 3 | |
| 7(b) | $\frac{6x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{4x^{-\frac{1}{2}}}{-\frac{1}{2}}(+c)$ | B1 B1 | B1 for each unsimplified integral. |
| | Uses (4, 7) leading to $c$ = (-21) | M1 | Uses (4, 7) to find a $c$ value |
| | $y$ or $f(x)=12x^{\frac{1}{2}}+8x^{-\frac{1}{2}}-21$ or $12\sqrt{x}+\frac{8}{\sqrt{x}}-21$ | A1 | Need to see $y$ or f($x$) = somewhere in *their* solution and 12 and 8 |
| | | 4 | |

6.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 10(a) | $\left(\frac{dy}{dx}\right) = [8] \times [(3-2x)^{-3}] + [-1]$ $\left(=\frac{8}{(3-2x)^3}-1\right)$ | B2, 1, 0 | B2 for all three elements correct, B1 for two elements correct, B0 for only one or no elements correct. |
| | $\frac{d^2y}{dx^2} = -3\times8\times(3-2x)^{-4}\times(-2)$ $\left(=\frac{48}{(3-2x)^4}\right)$ | B1 FT | FT providing *their* bracket is to a negative power |
| | $\int y\,dx = [(3-2x)^{-1}]\ [2 \div (-1\times -2)]\ [-\frac{1}{2}x^2]\ (+c)$ $\left(=\frac{1}{3-2x}-\frac{1}{2}x^2+c\right)$ | B1 B1 B1 | Simplification not needed, B1 for each correct element |
| | | 6 | |
| 10(b) | $\frac{dy}{dx} = 0 \to (3-2x)^3 = 8 \to 3-2x = k \to x =$ | M1 | Setting *their* 2-term differential to 0 and attempts to solve as far as $x$ = |
| | $\frac{1}{2}$ | A1 | |
| | **Alternative method for question 10(b)** | | |
| | $y=0 \to \frac{2}{(3-2x)^2} - x = 0 \to (x-2)(2x-1)^2 = 0 \to x =$ | M1 | Setting $y$ to 0 and attempts to solve a cubic as far as $x$ = (3 factors needed) |
| | $\frac{1}{2}$ | A1 | |
| | | 2 | |
| 10(c) | Area under curve = *their* $\left[\frac{1}{3-2\times\left(\frac{1}{2}\right)} - \frac{\left(\frac{1}{2}\right)^2}{2}\right] - \left[\frac{1}{3-2\times0} - 0\right]$ | M1 | Using *their* integral, *their* positive $x$ limit from **part (b)** and 0 correctly. |
| | $\frac{1}{24}$ | A1 | |
| | | 2 | |

7.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(a) | $\frac{-2}{x+2}$ | B1 | Integrate f(x). Accept $-2(x+2)^{-1}$ . Can be unsimplified. |
| | $0-\left(-\frac{2}{3}\right) = \frac{2}{3}$ | M1 A1 | Apply limit(s) to an integrated expansion. CAO for A1 |
| | | 3 | |
| 2(b) | $-1=-2+c$ | M1 | Substitute $x=-1, y=-1$ into *their* integrated expression (*c* present) |
| | $y=\frac{-2}{x+2}+1$ | A1 | Accept $y=-2(x+2)^{-1}+1$. $-2$ must be resolved. |
| | | 2 | |

8.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 10(b) | $\int \frac{1}{k}x^{1/2}+x^{-1/2}+\frac{1}{k^2} = \left[\frac{2x^{3/2}}{3k}\right]+\left[2x^{1/2}\right]+\left[\frac{x}{k^2}\right]$ | B2, 1, 0 | OE |
| | $\left(\frac{2k^2}{3}+2k+1\right)-\left(\frac{k^2}{12}+k+\frac{1}{4}\right)$ | M1 | Apply limits $\frac{k^2}{4}\rightarrow k^2$ to an integrated expression. Expect $\frac{7}{12}k^2+k+\frac{3}{4}$ |
| | $\frac{7}{12}k^2+k+\frac{3}{4}=\frac{13}{12}$ | M1 | Equate to $\frac{13}{12}$ and simplify to quadratic. OE, expect $7k^2+12k-4\ (=0)$ |
| | $k=\frac{2}{7}$ only (or 0.286) | A1 | Dependent on $(7k-2)(k+2)\ (=0)$ or formula or completing square. |
| | | 5 | |



9.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 11(d) | $\int 9\left(x^{-\frac{1}{2}}-4x^{-\frac{3}{2}}\right)dx = 9\left(\frac{x^{\frac{1}{2}}}{\frac{1}{2}}-\frac{4x^{-\frac{1}{2}}}{-\frac{1}{2}}\right)$ | B2, 1, 0 | B2; all 3 terms correct: 9, $\frac{x^{\frac{1}{2}}}{\frac{1}{2}}$, $\frac{-4x^{-\frac{1}{2}}}{-\frac{1}{2}}$ B1; 2 of the 3 terms correct |
| | $9\left[\left(6+\frac{8}{3}\right)-(4+4)\right]$ | M1 | Apply limits *their* 4 $\rightarrow$ 9 to an integrated expression with no consideration of other areas. |
| | 6 | A1 | Use of π scores A0 |
| | | 4 | |

10.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | $[\mathrm{f}(x)=]\ 2x^3+\frac{8}{x}\ [+c]$ | B1 | Allow any correct form |
| | $7=16+4+c$ | M1 | Substitute f(2) = 7 into an integral. *c* must be present. Expect $c=-13$ |
| | $\mathrm{f}(x)=\ 2x^3+\frac{8}{x}-13$ | A1 | Allow $y=$, $\mathrm{f}(x)$ or $y$ can appear earlier in answer |
| | | 3 | |

11.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 9(a) | $f(x)=\frac{2}{3}x^3-7x+4x^{-1}\ [+c]$ | **B2, 1, 0** | Allow terms on different lines; allow unsimplified. |
| | $-\frac{1}{3}=\frac{2}{3}-7+4+c$ leading to $c=[2]$ | **M1** | Substitute $f(1)=-\frac{1}{3}$ into an integrated expression and evaluate $c$. |
| | $f(x)=\frac{2}{3}x^3-7x+4x^{-1}+2$ | **A1** | OE. |
| | | **4** | |

12.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 10(a) | $\left\{\frac{(3x-2)^{-\frac{1}{2}}}{-1/2}\right\}\div\{3\}$ | **B2, 1, 0** | Attempt to integrate |
| | $-\frac{2}{3}[0-1]$ | **M1** | M1 for applying limits $1\to\infty$ to an integrated expression (either correct power or dividing by their power). |
| | $\frac{2}{3}$ | **A1** | |
| | | **4** | |

1.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | $(\pi)\int(y-1)\,dy$ | *M1 | SOI<br>Attempt to integrate $x^2$ or $(y-1)$ |
| | $(\pi)\left[\frac{y^2}{2}-y\right]$ | A1 | |
| | $(\pi)\left[\left(\frac{25}{2}-5\right)-\left(\frac{1}{2}-1\right)\right]$ | DM1 | Apply limits $1 \to 5$ to an integrated expression |
| | $8\pi$ or AWRT 25.1 | A1 | |
| | | 4 | |

2.

| Question | Answer | Marks |
|---|---|---|
| 11(a) | Simultaneous equations $\frac{8}{x+2} = 4 - ½x$ | M1 |
| | $x = 0$ or $x = 6 \to A\ (0, 4)$ and $B\ (6, 1)$ | B1A1 |
| | At $C$ $\frac{-8}{(x+2)^2} = -\frac{1}{2} \to C\ (2, 2)$<br>(**B1** for the differentiation. **M1** for equating and solving) | B1<br>M1A1 |
| | | 6 |
| 11(b) | Volume under line $= \pi\int(-\frac{1}{2}x+4)^2dx = \pi\left[\frac{x^3}{12}-2x^2+16x\right] = (42\pi)$<br>(**M1** for volume formula. **A2,1** for integration) | M1<br>A2,1 |
| | Volume under curve $= \pi\int\left(\frac{8}{x+2}\right)^2 dx = \pi\left[\frac{-64}{x+2}\right] = (24\pi)$ | A1 |
| | Subtracts and uses 0 to 6 $\to 18\pi$ | M1A1 |
| | | 6 |

3.

| Question | Answer | Marks |
|---|---|---|
| 8(a) | Volume $= \pi\int x^2dy = \pi\int\frac{36}{y^2}dy$ | *M1 |
| | $= \pi\left[\frac{-36}{y}\right]$ | A1 |
| | Uses limits 2 to 6 correctly $\to (12\pi)$ | DM1 |
| | Vol of cylinder $= \pi.1^2.4$ or $\int 1^2.dy$ $= [y]$ from 2 to 6 | M1 |
| | Vol $= 12\pi - 4\pi = 8\pi$ | A1 |
| | | 5 |
| 8(b) | $\frac{dy}{dx} = \frac{-6}{x^2}$ | B1 |
| | $\frac{-6}{x^2} = -2 \to x = \sqrt{3}$ | M1 |
| | $y = \frac{6}{\sqrt{3}} = 2\sqrt{3}$ Lies on $y = 2x$ | A1 |
| | | 3 |

4.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 9 | Curve intersects $y = 1$ at (3, 1) | B1 | **Throughout Question 9: 1 < *their* 3 < 5**<br>Sight of $x = 3$ |
| | Volume $= [\pi]\int (x-2)[dx]$ | M1 | M1 for showing the intention to integrate $(x-2)$.<br>Condone missing $\pi$ or using $2\pi$. |
| | $[\pi]\left[\frac{1}{2}x^2 - 2x\right]$ or $[\pi]\left[\frac{1}{2}(x-2)^2\right]$ | A1 | Correct integral. Condone missing $\pi$ or using $2\pi$. |
| | $= [\pi]\left[\left(\frac{5^2}{2} - 2\times 5\right) - \left(\frac{their\ 3^2}{2} - 2\times their\ 3\right)\right]$<br>$= [\pi]\left[\frac{5}{2} + \frac{3}{2}\right]$ as a minimum requirement for *their* values | M1 | Correct use of '*their* 3' and 5 in an integrated expression.<br>Condone missing $\pi$ or using $2\pi$. Condone +c.<br>Can be obtained by integrating and substituting between 5 and 2 and then 3 and 2 then subtracting. |
| | Volume of cylinder $= \pi \times 1^2 \times (5 - their\ 3)[= 2\pi]$ | B1 FT | Or by integrating 1 to obtain $x$ (condone $y$ if 5 and *their* 3 used). |
| | [Volume of solid $= 4\pi - 2\pi =$]$2\pi$ or 6.28 | A1 | AWRT |
| 9 | **Alternative method for Question 9** | | |
| | Curve intersects $y = 1$ at (3, 1) | B1 | Sight of $x = 3$ |
| | Volume of solid $= \pi\int (x-2) - 1[dx]$ | M1 B1 | M1 for showing the intention to integrate $(x-2)$<br>B1 for correct integration of –1.<br>Condone missing $\pi$ or $2\pi$ for M1 but not for B1. |
| | $[\pi]\left[\frac{1}{2}x^2 - 3x\right]$ or $[\pi]\left[\frac{1}{2}(x-3)^2\right]$ | A1 | Correct integral, allow as two integrals.<br>Condone missing $\pi$ or using $2\pi$. |
| | $= [\pi]\left[\left(\frac{5^2}{2} - 3\times 5\right) - \left(\frac{their\ 3^2}{2} - 3\times their\ 3\right)\right]$ | M1 | Correct use of '*their* 3' and 5 in an integrated expression.<br>Condone missing $\pi$ or using $2\pi$. Condone +c.<br>Can be obtained by integrating and substituting between 5 and 2 and then 3 and 2 then subtracting. |
| | [Volume of solid $= 4\pi - 2\pi =$]$2\pi$ or 6.28 | A1 | AWRT |
| | | **6** | |

5.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 10(a) | $\left\{\frac{(3x-2)^{-\frac{1}{2}}}{-1/2}\right\} \div \{3\}$ | **B2, 1, 0** | Attempt to integrate |
| | $-\frac{2}{3}[0-1]$ | **M1** | M1 for applying limits $1 \rightarrow \infty$ to an integrated expression (either correct power or dividing by their power). |
| | $\frac{2}{3}$ | **A1** | |
| | | **4** | |
| 10(b) | $[\pi]\int y^2 dx = [\pi]\int (3x-2)^{-3} dx = [\pi]\frac{(3x-2)^{-2}}{-2\times 3}$ | ***M1 A1** | M1 for attempt to integrate $y^2$ (power increases); allow 1 error. A1 for correct result in any form. |
| | $[\pi]\left[-\frac{1}{6}\right]\left[\frac{1}{16} - 1\right]$ | **DM1** | Apply limits 1 and 2 to an integrated expression and subtract correctly; allow 1 error. |
| | $\frac{5\pi}{32}$ | **A1** | OE |
| | | **4** | |

| | | | |
|---|---|---|---|
| 10(c) | $\frac{dy}{dx} = -\frac{3}{2} \times 3(3x-2)^{-\frac{5}{2}}$ | **M1** | M1 for attempt to differentiate (power decreases), allow 1 error. |
| | At $x = 1$, $\frac{dy}{dx} = -\frac{9}{2}$ | ***M1** | Substitute $x = 1$ into *their* differentiated expression; allow 1 error. |
| | [Equation of normal is] $y - 1 = \frac{2}{9}(x-1)$ OR evaluates $c$ | **DM1** | Forms equation of line or evaluates $c$ using (1, 1) and gradient $\frac{-1}{their\ \frac{dy}{dx}}$. |
| | At $A$, $y = \frac{7}{9}$ | **A1** | OE e.g. AWRT 0.778; must clearly identify $y$-intercept |
| | | **4** | |