"Knowing the path is good but not enough, walking the path with determination leads to destiny"

AS Edexcel
Paper 4 (WMA14)
CLASSIFIED QUESTIONS

Formula list:

Pure Mathematics P4

Candidates sitting Pure Mathematics P4 may also require those formulae listed under Pure Mathematics P1, P2 and P3.

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1\times 2\times \ldots \times r}x^r + \ldots \qquad (|x|<1,\ n\in\mathbb{R})$$

Integration (+ constant)

$\mathbf{f}(x)$	$\int \mathbf{f}(x)\,\mathrm{d}x$
$\operatorname{cosec} x$	$-\ln\lvert\operatorname{cosec} x+\cot x\rvert,\quad \ln\left\lvert\tan\left(\frac{1}{2}x\right)\right\rvert$
$\sec x$	$\ln\lvert\sec x+\tan x\rvert,\quad \ln\left\lvert\tan\left(\frac{1}{2}x+\frac{1}{4}\pi\right)\right\rvert$

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$$

Contents

Algebra

Q1.

Prove by contradiction that, if *a*, *b* are positive real numbers, then $a + b \geqslant 2\sqrt{ab}$

(Total for question = 4 marks)

(**Q06 WMA14/01, Specimen papers**)

Q2.

Given that n is an integer, use algebra, to prove by contradiction, that if n^3 is even then n is even.

(Total for question = 4 marks)

(**Q01 WMA14/01, Oct 2020**)

Q3.

(i) Relative to a fixed origin *O*, the points *A*, *B* and *C* have position vectors **a**, **b** and **c** respectively.

Points *A*, *B* and *C* lie in a straight line, with *B* lying between *A* and *C*.
Given *AB* : *AC* = 1 : 3 show that

$$\mathbf{c} = 3\mathbf{b} - 2\mathbf{a}$$

(3)

(ii) Given that $n \in \mathbb{N}$, prove by contradiction that if n^2 is a multiple of 3 then n is a multiple of 3

(5)

(Total for question = 8 marks)

(**Q09 WMA14/01, June 2021**)

Q4.

Prove by contradiction that there is no greatest odd integer.

(2)

(Total for question = 2 marks)

(**Q03 WMA14/01, Jan 2021**)

Q5.

(a) A student's attempt to answer the question

"Prove by contradiction that if n^3 is even, then n is even"

is shown below. Line 5 of the proof is missing.

Assume that there exists a number n such that n^3 is even, but n is odd.

If n is odd then $n = 2p + 1$ where $p \in \mathbb{Z}$

So $n^3 = (2p+1)^3$

$= 8p^3 + 12p^2 + 6p + 1$

$=$

This contradicts our initial assumption, so if n^3 is even, then n is even.

Complete this proof by filling in line 5.

(1)

(b) Hence, prove by contradiction that $\sqrt[3]{2}$ is irrational.

(5)

(Total for question = 6 marks)

(**Q10 WMA14/01, Oct 2021**)

Extra space for working:

Q6.

Three **consecutive** terms in a sequence of real numbers are given by

$$k, \quad 1 + 2k \quad \text{and} \quad 3 + 3k$$

where k is a constant.

Use proof by contradiction to show that this sequence is not a geometric sequence.

(Total for question = 5 marks)

(**Q06 WMA14/01, Jan 2022**)

Q7.

A student was asked to prove by contradiction that

"there are no positive integers x and y such that $3x^2 + 2xy - y^2 = 25$"

The start of the student's proof is shown in the space below.

Assume that integers x and y exist such that $3x^2 + 2xy - y^2 = 25$

$$\Rightarrow (3x - y)(x + y) = 25$$

If $(3x - y) = 1$ and $(x + y) = 25$

$$\left.\begin{aligned} 3x - y &= 1 \\ x + y &= 25 \end{aligned}\right\} \Rightarrow 4x = 26 \Rightarrow x = 6.5, y = 18.5 \quad \text{Not integers}$$

Show the calculations and statements that are needed to complete the proof.

(4)

(Total for question = 4 marks)

(**Q08 WMA14/01, Oct 2022**)

Extra space for working:

Q8.

A student was asked to prove, for $p \in \mathbb{N}$, that

"if p^3 is a multiple of 3, then p must be a multiple of 3"

The start of the student's proof by contradiction is shown in the space below.

Assumption:

There exists a number p, $p \in \mathbb{N}$, such that p^3 is a multiple of 3, and p is NOT a multiple of 3

Let $p = 3k + 1$, $k \in \mathbb{N}$.

Consider $p^3 = (3k+1)^3 = 27k^3 + 27k^2 + 9k + 1$

$= 3(9k^3 + 9k^2 + 3k) + 1$ which is not a multiple of 3

(a) Show the calculations and statements that are required to complete the proof.

(3)

(b) Hence prove, by contradiction, that $\sqrt[3]{3}$ is an irrational number.

(5)

(Total for question = 8 marks)

(**Q09 WMA14/01, Jan 2023**)

Extra space for working:

Q9.

Use proof by contradiction to prove that $\sqrt{7}$ is irrational.

(You may assume that if k is an integer and k^2 is a multiple of 7 then k is a multiple of 7)

(4)

(Total for question = 4 marks)

(**Q07 WMA14/01, June 2023**)

Q10.

(a) Prove by contradiction that for all positive numbers *k*

$$k + \frac{9}{k} \geqslant 6$$

(4)

(b) Show that the result in part (a) is not true for all real numbers.

(1)

(Total for question = 5 marks)

(**Q04 WMA14/01, Oct 2023**)

Q11.

Use proof by contradiction to prove that the curve with equation

$$y = 2x + x^3 + \cos x$$

has no stationary points.

(Total for question = 4 marks)

(**Q08 WMA14/01, Jan 2024**)

Q12.

Use proof by contradiction to show that, when n is an integer,

$$n^2 - 2$$

is **never** divisible by 4

(Total for question = 4 marks)

(**Q09 WMA14/01, June 2022**)

(**Q09 WMA14/01, Oct 2022**)

Q1.

The curve *C* is defined by the parametric equations

$$x = \frac{1}{t} + 2 \qquad y = \frac{1-2t}{3+t} \qquad t > 0$$

(a) Show that the equation of *C* can be written in the form $y = \mathrm{g}(x)$ where g is the function

$$\mathrm{g}(x) = \frac{ax+b}{cx+d} \qquad x > k$$

where *a*, *b*, *c*, *d* and *k* are integers to be found.

(5)

(b) Hence, or otherwise, state the range of g.

(2)

(Total for question = 7 marks)

(**Q04 WMA14/01, Jan 2021**)

Extra space for working:

Q2.

A curve *C* has parametric equations

$$x = \frac{t}{t-3} \qquad y = \frac{1}{t} + 2 \qquad t \in \mathbb{R} \quad t > 3$$

Show that all points on *C* lie on the curve with Cartesian equation

$$y = \frac{ax-1}{bx}$$

where *a* and *b* are constants to be found.

(3)

(Total for question = 3 marks)

(**Q01 WMA14/01, Oct 2022**)

Q3.

A set of points $P(x, y)$ is defined by the parametric equations

$$x = \frac{t-1}{2t+1} \qquad y = \frac{6}{2t+1} \qquad t \neq -\frac{1}{2}$$

(a) Show that all points $P(x, y)$ lie on a straight line.

(4)

(b) Hence or otherwise, find the x coordinate of the point of intersection of this line and the line with equation $y = x + 12$

(2)

(Total for question = 6 marks)

(**Q02 WMA14/01, Jan 2023**)

Q4.

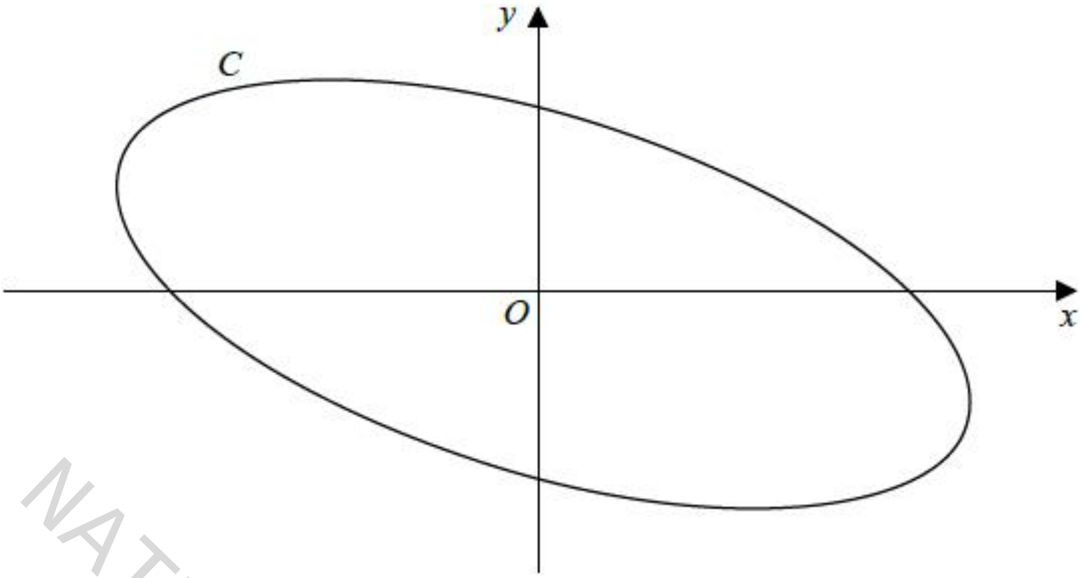

Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 4\cos\left(t + \frac{\pi}{6}\right) \qquad y = 2\sin t \qquad 0 \leqslant t \leqslant 2\pi$$

(a) Show that

$$x + y = 2\sqrt{3}\cos t$$

(3)

(b) Show that a cartesian equation of C is

$$(x + y)^2 + ay^2 = b$$

where a and b are integers to be found.

(2)

(Total for question = 5 marks)

(**Q07 WMA14/01, Specimen papers**)

Extra space for working:

Q5.

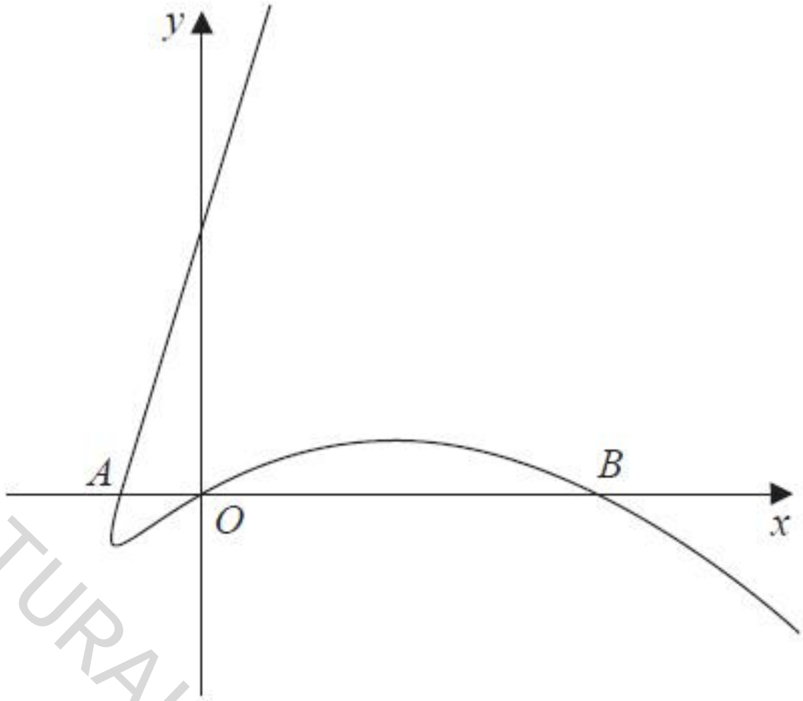

Figure 2

Figure 2 shows a sketch of part of the curve with parametric equations

$$x = 2t^2 - 6t, \qquad y = t^3 - 4t, \qquad t \in$$ ℝ

The curve cuts the x-axis at the origin and at the points A and B, as shown in Figure 2.

(a) Find the coordinates of A and show that B has coordinates (20, 0).

(3)

(b) Show that the equation of the tangent to the curve at B is

$$7y + 4x - 80 = 0$$

(5)

The tangent to the curve at B cuts the curve again at the point P.

(c) Find, using algebra, the x coordinate of P.

(4)

(Total for question = 12 marks)

(**Q04 WMA14/01, Oct 2020**)

Extra space for working:

Q6.

The curve C has parametric equations

$$x = 3 + 2\sin t \qquad y = \frac{6}{7 + \cos 2t} \qquad -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$$

(a) Show that C has Cartesian equation

$$y = \frac{12}{(7 - x)(1 + x)} \qquad p \leqslant x \leqslant q$$

where p and q are constants to be found.

(6)

(b) Hence, find a Cartesian equation for C in the form

$$y = \frac{a}{x + b} + \frac{c}{x + d} \qquad p \leqslant x \leqslant q$$

where a, b, c and d are constants.

(3)

(Total for question = 9 marks)

(**Q03 WMA14/01, Jan 2022**)

Extra space for working:

Q7.

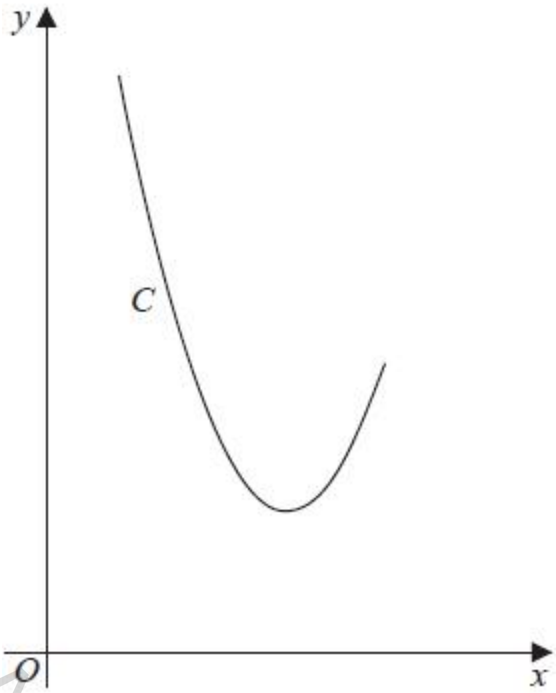

Figure 1

Figure 1 shows a sketch of the curve C with parametric equations

$$x = 5 + 2\tan t \qquad y = 8\sec^2 t \qquad -\frac{\pi}{3} \leqslant t \leqslant \frac{\pi}{4}$$

(a) Use parametric differentiation to find the gradient of C at $x = 3$

(4)

The curve C has equation $y = \mathrm{f}(x)$, where f is a quadratic function.

(b) Find f(x) in the form $a(x + b)^2 + c$, where a, b and c are constants to be found.

(3)

(c) Find the range of f.

(2)

(Total for question = 9 marks)

(**Q05 WMA14/01, Oct 2021**)

Extra space for working:

Q1.

Use the binomial series to find the expansion of

$$\frac{1}{(2+5x)^3} \qquad |x| < \frac{2}{5}$$

in ascending powers of x, up to and including the term in x^3

Give each coefficient as a fraction in its simplest form.

(Total for question = 6 marks)

(**Q01 WMA14/01, Specimen papers**)

Extra space for working:

Q2.

(a) Use the binomial expansion to expand

$$(4-5x)^{-\frac{1}{2}} \qquad |x| < \frac{4}{5}$$

in ascending powers of x, up to and including the term in x^2 giving each coefficient as a fully simplified fraction.

(4)

$$f(x) = \frac{2+kx}{\sqrt{4-5x}} \qquad \text{where } k \text{ is a constant and } |x| < \frac{4}{5}$$

Given that the series expansion of f(x), in ascending powers of x, is

$$1 + \frac{3}{10}x + mx^2 + \ldots \qquad \text{where } m \text{ is a constant}$$

(b) find the value of k,

(2)

(c) find the value of m.

(2)

(Total for question = 6 marks)
(**Q02 WMA14/01, Oct 2020**)

Extra space for working:

Q3.

(a) Find the first 4 terms, in ascending powers of *x*, of the binomial expansion of

$$\left(\frac{1}{4} - 5x\right)^{\frac{1}{2}} \qquad |x| < \frac{1}{20}$$

giving each coefficient in its simplest form.

(5)

By substituting $x = \frac{1}{100}$ into the answer for (a),

(b) find an approximation for $\sqrt{5}$

Give your answer in the form $\frac{a}{b}$ where *a* and *b* are integers to be found.

(2)

(Total for question = 7 marks)

(**Q01 WMA14/01, Jan 2021**)

Q4.

Given that k is a constant and the binomial expansion of

$$\sqrt{1+kx} \qquad |kx| < 1$$

in ascending powers of x up to the term in x^3 is

$$1 + \frac{1}{8}x + Ax^2 + Bx^3$$

(a) (i) find the value of k,

(ii) find the value of the constant A and the constant B.

(5)

(b) Use the expansion to find an approximate value to $\sqrt{1.15}$

Show your working and give your answer to 6 decimal places.

(2)

(Total for question = 7 marks)

(**Q01 WMA14/01, June 2021**)

Extra space for working:

Q5.

$$f(x) = \sqrt{1 - 4x^2} \qquad |x| < \frac{1}{2}$$

(a) Find, in ascending powers of x, the first four non-zero terms of the binomial expansion of f(x) . Give each coefficient in simplest form.

(4)

(b) By substituting $x = \frac{1}{4}$ into the binomial expansion of f(x), obtain an approximation for $\sqrt{3}$

Give your answer to 4 decimal places.

(2)

(Total for question = 6 marks)

(**Q04 WMA14/01, Oct 2021**)

Extra space for working:

Q6.

$$g(x) = \frac{1}{\sqrt{4 - x^2}}$$

(a) Find, in ascending powers of x, the first four non-zero terms of the binomial expansion of g (x). Give each coefficient in simplest form.

(5)

(b) State the range of values of x for which this expansion is valid.

(1)

(c) Use the expansion from part (a) to find a fully simplified rational approximation for $\sqrt{3}$

Show your working and make your method clear.

(2)

(Total for question = 8 marks)

(**Q04 WMA14/01, Oct 2022**)

Extra space for working:

Q7.

(a) Find the first four terms, in ascending powers of *x*, of the binomial expansion of

$$\frac{8}{(2-5x)^2}$$

writing each term in simplest form.

(4)

(b) Find the range of values of *x* for which this expansion is valid.

(1)

(Total for question = 5 marks)

(**Q01 WMA14/01, Oct 2023**)

Q8.

(a) Find the first 4 terms of the binomial expansion, in ascending powers of *x*, of

$$\left(\frac{1}{4}-\frac{1}{2}x\right)^{-\frac{3}{2}} \qquad |x|<\frac{1}{2}$$

giving each term in simplest form.

(5)

Given that

$$\left(\frac{1}{4}-\frac{1}{2}x\right)^{n}\left(\frac{1}{4}-\frac{1}{2}x\right)^{-\frac{3}{2}}=\left(\frac{1}{4}-\frac{1}{2}x\right)^{\frac{1}{2}}$$

(b) write down the value of *n*.

(1)

(c) Hence, or otherwise, find the first 3 terms of the binomial expansion, in ascending powers of *x*, of

$$\left(\frac{1}{4}-\frac{1}{2}x\right)^{\frac{1}{2}} \qquad |x|<\frac{1}{2}$$

giving each term in simplest form.

(3)

(Total for question = 9 marks)

(**Q01 WMA14/01, June 2023**)

Extra space for working:

Q9.

(a) Find, in ascending powers of x, the first three non-zero terms of the binomial series expansion of

$$\sqrt[3]{1+4x^3} \qquad |x| < \frac{1}{\sqrt[3]{4}}$$

giving each coefficient as a simplified fraction.

(4)

(b) Use the expansion from part (a) with $x = \frac{1}{3}$ to find a rational approximation to $\sqrt[3]{31}$

(3)

(Total for question = 7 marks)

(**Q02 WMA14/01, Jan 2022**)

Extra space for working:

Q10.

The binomial expansion of

$$(3 + kx)^{-2} \qquad |kx| < 3$$

where k is a non-zero constant, may be written in the form

$$A + Bx + Cx^2 + Dx^3 + \ldots$$

where A, B, C and D are constants.

(a) Find the value of A

(1)

Given that $C = 3B$

(b) show that

$$k^2 + 6k = 0$$

(3)

(c) Hence (i) find the value of k

(ii) find the value of D

(3)

(Total for question = 7 marks)

(**Q01 WMA14/01, June 2022**)

Extra space for working:

Q11.

Find, in ascending powers of x up to and including the term in x^3, the binomial expansion of

$$(1-4x)^{-3} \qquad |x| < \frac{1}{4}$$

fully simplifying each term.

(Total for question = 4 marks)

(**Q01 WMA14/01, Jan 2024**)

Calculus

Q1.

The curve *C* has equation

$$2x - 4y^2 + 3x^2y = 4x^2 + 8$$

The point *P*(3, 2) lies on *C*.

Find the equation of the normal to *C* at the point *P*, writing your answer in the form $ax + by + c = 0$ where *a*, *b* and *c* are integers to be found.

(7)

(Total for question = 7 marks)

(**Q01 WMA14/01, Oct 2021**)

Q2.

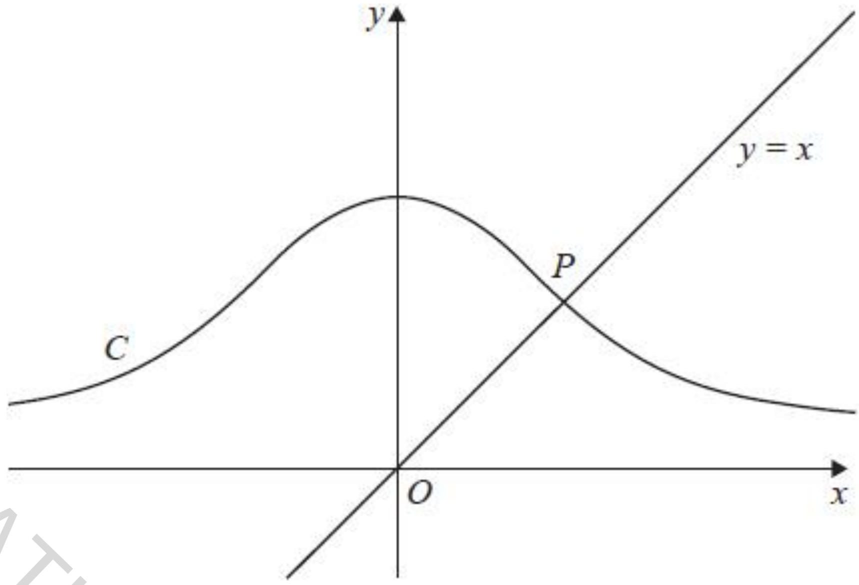

Figure 2

Figure 2 shows a sketch of the curve C with equation

$$y^3 - x^2 + 4x^2y = k$$

where k is a positive constant greater than 1

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

The point P lies on C.

Given that the normal to C at P has equation $y = x$, as shown in Figure 2,

(b) find the value of k.

(5)

(Total for question = 10 marks)

(**Q05 WMA14/01, Oct 2023**)

Extra space for working:

Q3.

A curve *C* has the equation

$$x^3 + 2xy - x - y^3 - 20 = 0$$

(a) Find $\frac{dy}{dx}$ in terms of *x* and *y*.

(5)

(b) Find an equation of the tangent to *C* at the point (3, −2), giving your answer in the form $ax + by + c = 0$, where *a*, *b* and *c* are integers.

(2)

(Total for question = 7 marks)

(**Q02 WMA14/01, Specimen papers**)

Extra space for working:

Q4.

A curve C has equation

$$y = x^{\sin x} \qquad x > 0 \qquad y > 0$$

(a) Find, by firstly taking natural logarithms, an expression for $\frac{dy}{dx}$ in terms of x and y.

(5)

(b) Hence show that the x coordinates of the stationary points of C are solutions of the equation

$$\tan x + x \ln x = 0$$

(2)

(Total for question = 7 marks)

(**Q06 WMA14/01, Oct 2020**)

Extra space for working:

Q5.

A curve has equation

$$4y^2 + 3x = 6ye^{-2x}$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

The curve crosses the y-axis at the origin and at the point P.

(b) Find the equation of the normal to the curve at P, writing your answer in the form $y = mx + c$ where m and c are constants to be found.

(4)

(Total for question = 4 marks)

(**Q06 WMA14/01, Jan 2021**)

Extra space for working:

Q6.

A curve has equation

$$y^2 = y\,e^{-2x} - 3x$$

(a) Show that

$$\frac{dy}{dx} = \frac{2ye^{-2x} + 3}{e^{-2x} - 2y}$$

(4)

The curve crosses the *y*-axis at the origin and at the point *P*.

The tangent to the curve at the origin and the tangent to the curve at *P* meet at the point *R*.

(b) Find the coordinates of *R*.

(5)

(Total for question = 9 marks)

(**Q05 WMA14/01, June 2021**)

Extra space for working:

Q7.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

A curve has equation

$$16x^3 - 9kx^2y + 8y^3 = 875$$

where k is a constant.

(a) Show that

$$\frac{dy}{dx} = \frac{6kxy - 16x^2}{8y^2 - 3kx^2}$$

(4)

Given that the curve has a turning point at $x = \frac{5}{2}$

(b) find the value of k

(4)

(Total for question = 8 marks)

(**Q04 WMA14/01, June 2022**)

Extra space for working:

Q8.

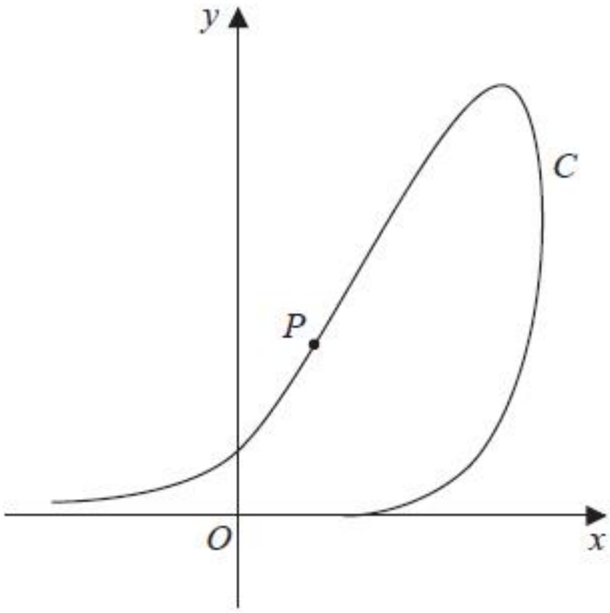

Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$2x - 4xy + y^2 = 13 \qquad y \geq 0$$

The point P lies on C and has x coordinate 2

(2)

(b) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

The tangent to C at P crosses the x-axis at the point Q.

(c) Find the x coordinate of Q, giving your answer in the form $\frac{a\ln 2 + b}{c\ln 2 + d}$ where a, b, c and d are integers to be found.

(3)

(Total for question = 10 marks)

(**Q02 WMA14/01, June 2023**)

Extra space for working:

Q9.

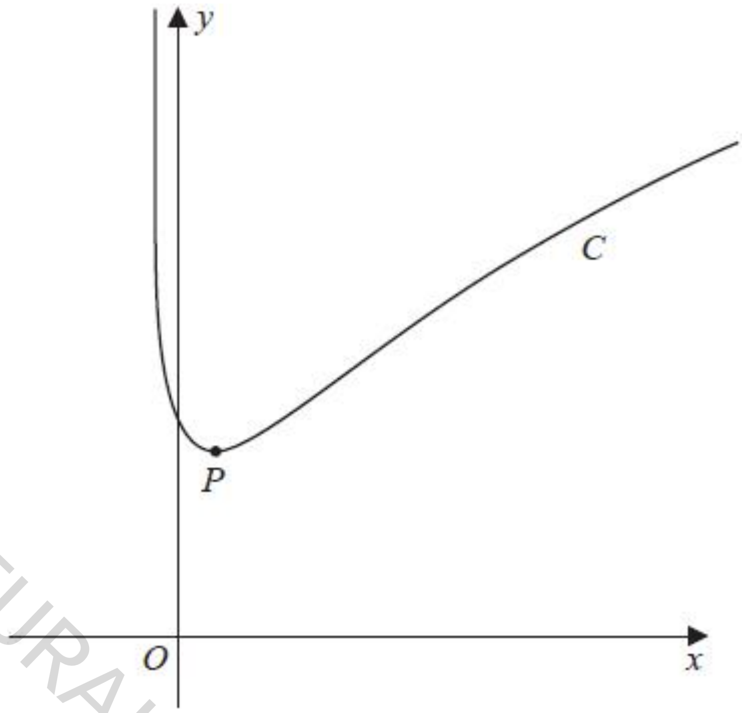

Figure 1

The curve C, shown in Figure 1, has equation

$$y^2x + 3y = 4x^2 + k \qquad y > 0$$

where k is a constant.

(a) Find $\frac{dy}{dx}$ in terms of x and y

(5)

The point P $(p, 2)$, where p is a constant, lies on C.

Given that P is the minimum turning point on C,

(b) find

(i) the value of p

(ii) the value of k

(4)

(Total for question = 9 marks)

(**Q03 WMA14/01, Jan 2024**)

Extra space for working:

Q10.

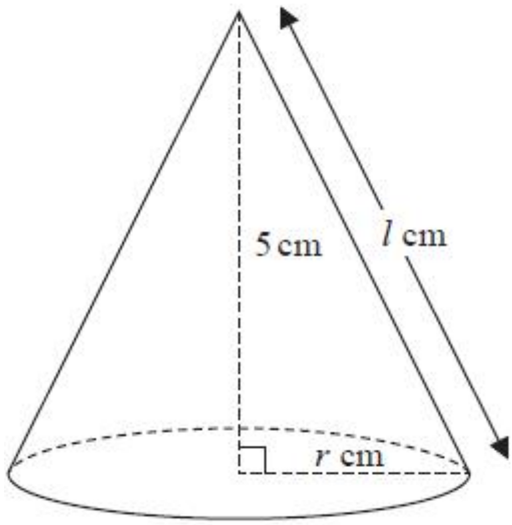

Figure 2

A cone, shown in Figure 2, has

- fixed height 5 cm
- base radius r cm
- slant height l cm

(a) Find an expression for l in terms of r

(1)

Given that the base radius is increasing at a constant rate of 3 cm per minute,

(b) find the rate at which the total surface area of the cone is changing when the radius of the cone is 1.5 cm. Give your answer in cm^2 per minute to one decimal place.

[*The total surface area, S, of a cone is given by the formula* $S = \pi r^2 + \pi r l$]

(4)

(Total for question = 5 marks)

(**Q04 WMA14/01, Jan 2024**)

Extra space for working:

Q11.

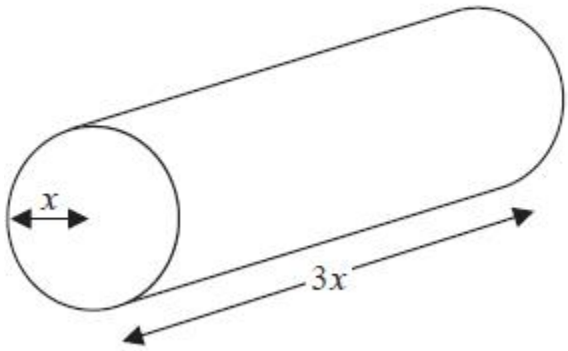

Figure 1

A tablet is dissolving in water.

The tablet is modelled as a cylinder, shown in Figure 1.

At t seconds after the tablet is dropped into the water, the radius of the tablet is x mm and the length of the tablet is $3x$ mm.

The cross-sectional area of the tablet is decreasing at a constant rate of 0.5 mm^2s^{-1}

(a) Find $\frac{dx}{dt}$ when $x = 7$

(4)

(b) Find, according to the model, the rate of decrease of the volume of the tablet when $x = 4$

(4)

(Total for question = 8 marks)

(**Q03 WMA14/01, June 2022**)

Extra space for working:

Q12.

The volume V cm^3 of a spherical balloon with radius r cm is given by the formula

$$V = \frac{4}{3}\pi r^3$$

(a) Find $\frac{dV}{dr}$ giving your answer in simplest form.

(1)

At time t seconds, the volume of the balloon is increasing according to the differential equation

$$\frac{dV}{dt} = \frac{900}{(2t+3)^2} \qquad t \geqslant 0$$

Given that $V = 0$ when $t = 0$

(b) (i) solve this differential equation to show that

$$V = \frac{300t}{2t+3}$$

(ii) Hence find the upper limit to the volume of the balloon.

(5)

(c) Find the radius of the balloon at $t = 3$, giving your answer in cm to 3 significant figures.

(3)

(d) Find the rate of increase of the radius of the balloon at $t = 3$, giving your answer to 2 significant figures. Show your working and state the units of your answer.

(3)

(Total for question = 12 marks)

(**Q07 WMA14/01, Jan 2023**)

Extra space for working:

Q13.

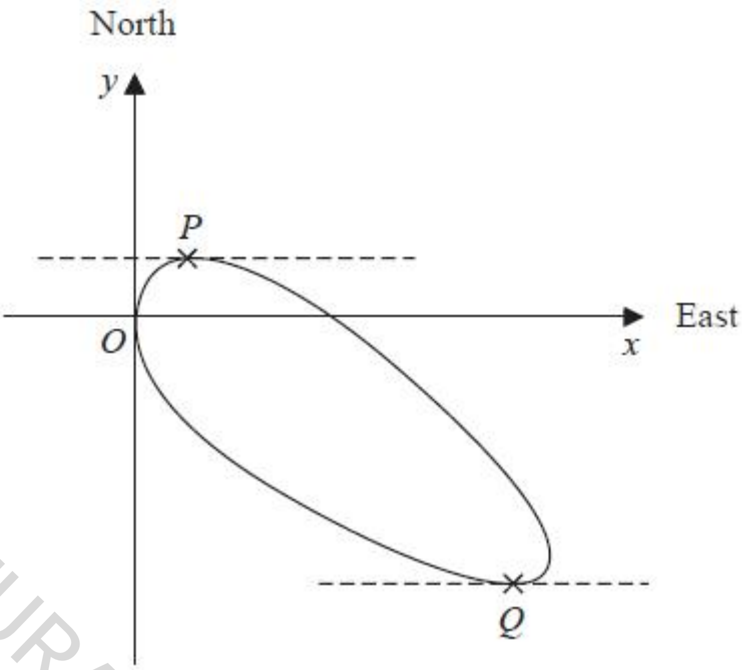

Figure 4

Figure 4 shows a sketch of the closed curve with equation

$$(x+y)^3 + 10y^2 = 108x$$

(a) Show that

$$\frac{dy}{dx} = \frac{108 - 3(x+y)^2}{20y + 3(x+y)^2}$$

(5)

The curve is used to model the shape of a cycle track with both *x* and *y* measured in km.

The points *P* and *Q* represent points that are furthest north and furthest south of the origin *O*, as shown in Figure 4.

Using the result given in part (a),

(b) find how far the point *Q* is south of *O*. Give your answer to the nearest 100 *m*.

(4)

(Total for question = 9 marks)

(**Q11 WMA14/01, Oct 2022**)

Extra space for working:

Q14.

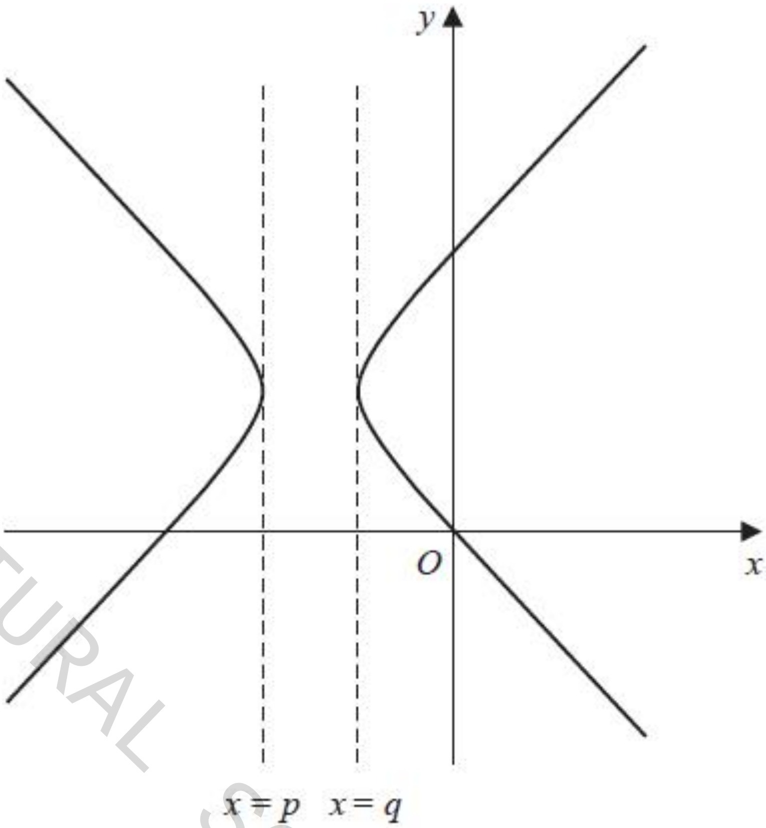

Figure 2

Figure 2 shows a sketch of the curve with equation

$$y^2 = 2x^2 + 15x + 10y$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(4)

The curve is not defined for values of x in the interval (p, q), as shown in Figure 2.

(b) Using your answer to part (a) or otherwise, find the value of p and the value of q.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(3)

(Total for question = 7 marks)

(**Q05 WMA14/01, Jan 2023**)

Extra space for working:

Q15.

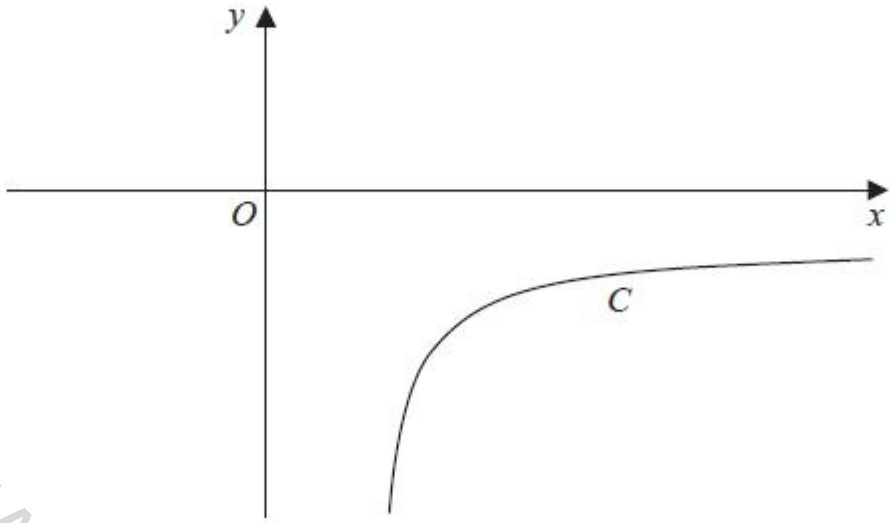

Figure 4

Figure 4 shows a sketch of the curve *C* with parametric equations

$$x = \sec t \qquad y = \sqrt{3}\tan\left(t + \frac{\pi}{3}\right) \qquad \frac{\pi}{6} < t < \frac{\pi}{2}$$

(a) Find $\frac{dy}{dx}$ in terms of *t*.

(3)

(b) Find an equation for the tangent to *C* at the point where $t = \frac{\pi}{3}$

Give your answer in the form $y = mx + c$, where *m* and *c* are constants.

(4)

(c) Show that all points on *C* satisfy the equation

$$y = \frac{Ax^2 + B\sqrt{3x^2 - 3}}{4 - 3x^2}$$

where *A* and *B* are constants to be found.

(5)

(Total for question = 12 marks)

(**Q09 WMA14/01, Jan 2024**)

Extra space for working:

Q16.

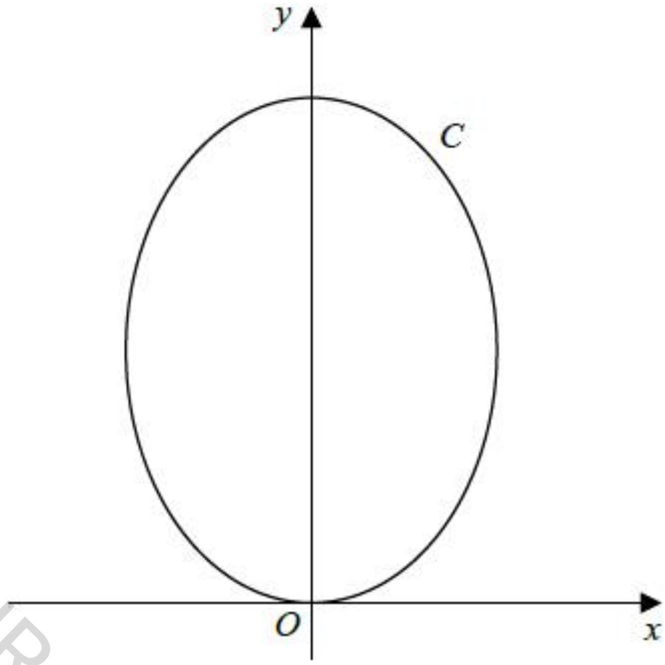

Figure 1

Figure 1 shows a sketch of the curve C with parametric equations

$$x = \sqrt{3}\sin 2t \qquad y = 4\cos^2 t \qquad 0 \leqslant t \leqslant \pi$$

(a) Show that $\dfrac{dy}{dx} = k\sqrt{3}\tan 2t$, where k is a constant to be found.

(5)

(b) Find an equation of the tangent to C at the point where $t = \dfrac{\pi}{3}$

Give your answer in the form $y = ax + b$, where a and b are constants.

(4)

(Total for question = 9 marks)

(**Q04 WMA14/01, Specimen papers**)

Extra space for working:

Q17.

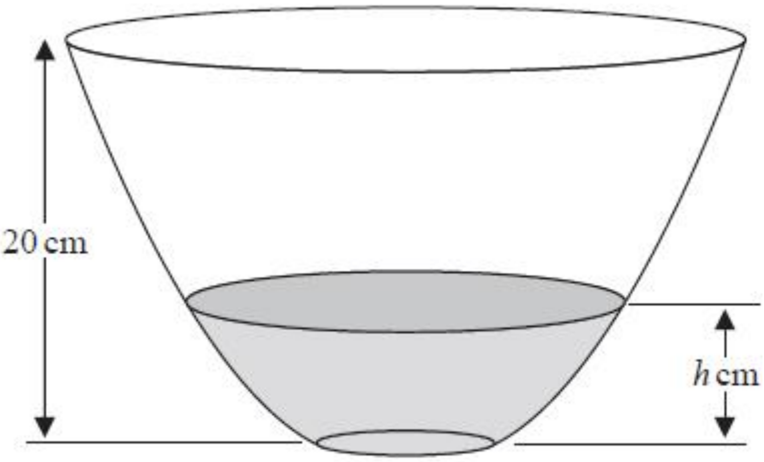

Figure 2

A bowl with circular cross section and height 20 cm is shown in Figure 2.

The bowl is initially empty and water starts flowing into the bowl.

When the depth of water is h cm, the volume of water in the bowl, V cm^3, is modelled by the equation

$$V = \frac{1}{3}h^2(h + 4) \qquad 0 \leq h \leq 20$$

Given that the water flows into the bowl at a constant rate of 160 cm^3 s^{-1}, find, according to the model,

(a) the time taken to fill the bowl,

(2)

(b) the rate of change of the depth of the water, in cm s^{-1}, when $h = 5$

(5)

(Total for question = 7 marks)

(**Q03 WMA14/01, June 2021**)

Extra space for working:

Q18.

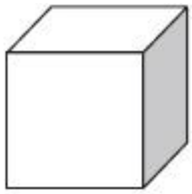

Figure 1

Figure 1 shows a cube which is increasing in size.

At time *t* seconds,

- the length of each edge of the cube is *x* cm
- the surface area of the cube is *S* cm^2
- the volume of the cube is *V* cm^3

Given that the surface area of the cube is increasing at a constant rate of 4 cm^2 s^{-1}

(a) show that $\frac{dx}{dt} = \frac{k}{x}$ where *k* is a constant to be found,

(4)

(b) show that $\frac{dV}{dt} = V^p$ where *p* is a constant to be found.

(3)

(Total for question = 7 marks)

(**Q02 WMA14/01, Oct 2023**)

Extra space for working:

Q19.

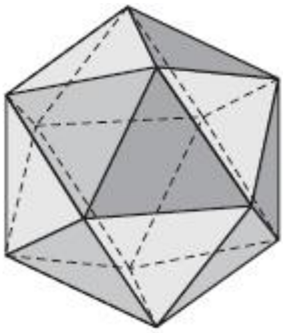

Figure 1

A regular icosahedron of side length x cm, shown in Figure 1, is expanding uniformly.

The icosahedron consists of 20 congruent equilateral triangular faces of side length x cm.

(a) Show that the surface area, A cm^2, of the icosahedron is given by

$$A = 5\sqrt{3}x^2$$

(2)

Given that the volume, V cm^3, of the icosahedron is given by

$$V = \frac{5}{12}\left(3+\sqrt{5}\right)x^3$$

(b) show that $\dfrac{dV}{dA} = \dfrac{\left(3+\sqrt{5}\right)x}{8\sqrt{3}}$

(3)

The surface area of the icosahedron is increasing at a constant rate of 0.025 cm^2 s^{-1}

(c) Find the rate of change of the volume of the icosahedron when $x = 2$, giving your answer to 2 significant figures.

(3)

(Total for question = 8 marks)

(**Q04 WMA14/01, Jan 2022**)

Extra space for working:

Q1.

(a) Using the substitution $u = \sqrt{2x+1}$, show that

$$\int_4^{12} \sqrt{8x+4}\, e^{\sqrt{2x+1}}\, dx$$

may be expressed in the form

$$\int_a^b ku^2 e^u\, du$$

where *a*, *b* and *k* are constants to be found.

(4)

(b) Hence find, by algebraic integration, the exact value of

$$\int_4^{12} \sqrt{8x+4}\, e^{\sqrt{2x+1}}\, dx$$

giving your answer in simplest form.

(5)

(Total for question = 9 marks)

(**Q04 WMA14/01, Jan 2023**)

Extra space for working:

Q2.

$$f(x) = \frac{8x-5}{(2x-1)(4x-3)} \qquad x > 1$$

(a) Express f(x) in partial fractions.

(3)

(b) Hence find $\int f(x)\,dx$

(3)

(c) Use the answer to part (b) to find the value of k for which

$$\int_k^{3k} f(x)\,dx = \frac{1}{2}\ln 20$$

(5)

(Total for question = 11 marks)

(**Q03 WMA14/01, June 2023**)

Extra space for working:

Q3.

Given that

$$\frac{3x+4}{(x-2)(2x+1)^2} \equiv \frac{A}{x-2} + \frac{B}{2x+1} + \frac{C}{(2x+1)^2}$$

(a) find the values of the constants *A*, *B* and *C*.

(4)

(b) Hence find the exact value of

$$\int_7^{12} \frac{3x+4}{(x-2)(2x+1)^2}\,\mathrm{d}x$$

giving your answer in the form $p \ln q + r$ where *p*, *q* and *r* are rational numbers.

(6)

(Total for question = 10 marks)

(**Q02 WMA14/01, Jan 2024**)

Extra space for working:

Q4.

(a) Express $\frac{1}{(1+3x)(1-x)}$ in partial fractions.

(3)

(b) Hence find the solution of the differential equation

$(1 + 3x)(1 - x)\frac{dy}{dx} = \tan y \qquad -\frac{1}{3} < x \leq \frac{1}{2}$

for which $x = \frac{1}{2}$ when $y = \frac{\pi}{2}$

Give your answer in the form $\sin^n y = f(x)$ where n is an integer to be found.

(6)

(Total for question = 9 marks)

(**Q02 WMA14/01, June 2022**)

Extra space for working:

Q5.

$$f(x) = \frac{1}{x(3x-1)^2} = \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(3x-1)^2}$$

(a) Find the values of the constants *A*, *B* and *C*

(4)

(b) (i) Hence find $\int f(x)\,dx$

(ii) Find $\int_1^2 f(x)\,dx$, giving your answer in the form *a* + ln *b*, where *a* and *b* are constants.

(6)

(Total for question = 10 marks)

(**Q03 WMA14/01, Specimen papers**)

Extra space for working:

Q6.

In this question you should show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Using the substitution $u = 3 + \sqrt{2x - 1}$ find the exact value of

$$\int_1^{13} \frac{4}{3 + \sqrt{2x - 1}} \, dx$$

giving your answer in the form $p + q \ln 2$, where p and q are integers to be found.

(8)

(Total for question = 8 marks)

(**Q05 WMA14/01, Jan 2021**)

Extra space for working:

Q7.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

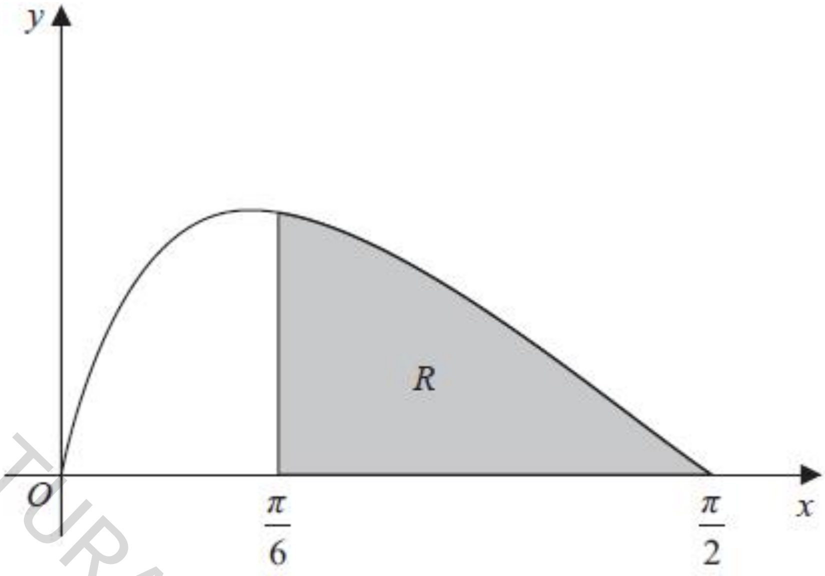

Figure 2

Figure 2 shows a sketch of the curve with equation

$$y = \frac{16\sin 2x}{(3 + 4\sin x)^2} \qquad 0 \leqslant x \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the line with equation $x = \frac{\pi}{6}$

Using the substitution $u = 3 + 4\sin x$, show that the area of R can be written in the form $a + \ln b$, where a and b are rational constants to be found.

(7)

(Total for question = 7 marks)

(**Q06 WMA14/01, Oct 2021**)

Extra space for working:

Q8.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Use the substitution *x* = 2 sin *u* to show that

$$\int_0^1 \frac{3x+2}{(4-x^2)^{\frac{3}{2}}}\,dx = \int_0^p \left(\frac{3}{2}\sec u \tan u + \frac{1}{2}\sec^2 u\right)du$$

where *p* is a constant to be found.

(b) Hence find the exact value of

$$\int_0^1 \frac{3x+2}{(4-x^2)^{\frac{3}{2}}}\,dx$$

(4)

(Total for question = 4 marks)

(**Q05 WMA14/01, June 2022**)

Extra space for working:

Q9.

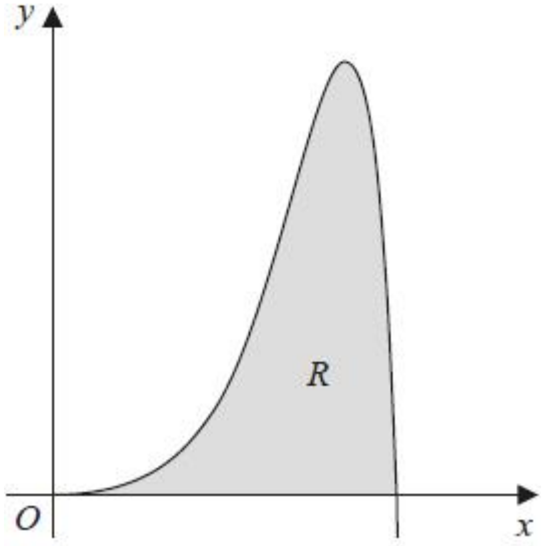

Figure 2

(a) Find $\int e^{2x} \sin x \, dx$

(5)

Figure 2 shows a sketch of part of the curve with equation

$$y = e^{2x} \sin x \qquad x \geq 0$$

The finite region R is bounded by the curve and the x-axis and is shown shaded in Figure 2.

(b) Show that the exact area of R is $\dfrac{e^{2\pi} + 1}{5}$

(Solutions relying on calculator technology are not acceptable.)

(2)

(Total for question = 7 marks)

(**Q07 WMA14/01, Jan 2021**)

Extra space for working:

Q10.

In this question you must show all stages of your working.

Solutions based on calculator technology are not acceptable.

(i) Use integration by parts to find the exact value of

$$\int_0^4 x^2 e^{2x}\,dx$$

giving your answer in simplest form.

(5)

(ii) Use integration by substitution to show that

$$\int_3^{\frac{21}{2}} \frac{4x}{(2x-1)^2}\,dx = a + \ln b$$

where *a* and *b* are constants to be found.

(7)

(Total for question = 12 marks)

(**Q03 WMA14/01, Oct 2023**)

Extra space for working:

Q11.

(i) Find

$$\int x^2 e^x \, dx$$

(4)

(ii) Use the substitution $u = \sqrt{1-3x}$ to show that

$$\int \frac{27x}{\sqrt{1-3x}} \, dx = -2(1-3x)^{\frac{1}{2}}(Ax+B)+k$$

where *A* and *B* are integers to be found and *k* is an arbitrary constant.

(6)

(Total for question = 10 marks)

(**Q05 WMA14/01, June 2023**)

Extra space for working:

Q12.

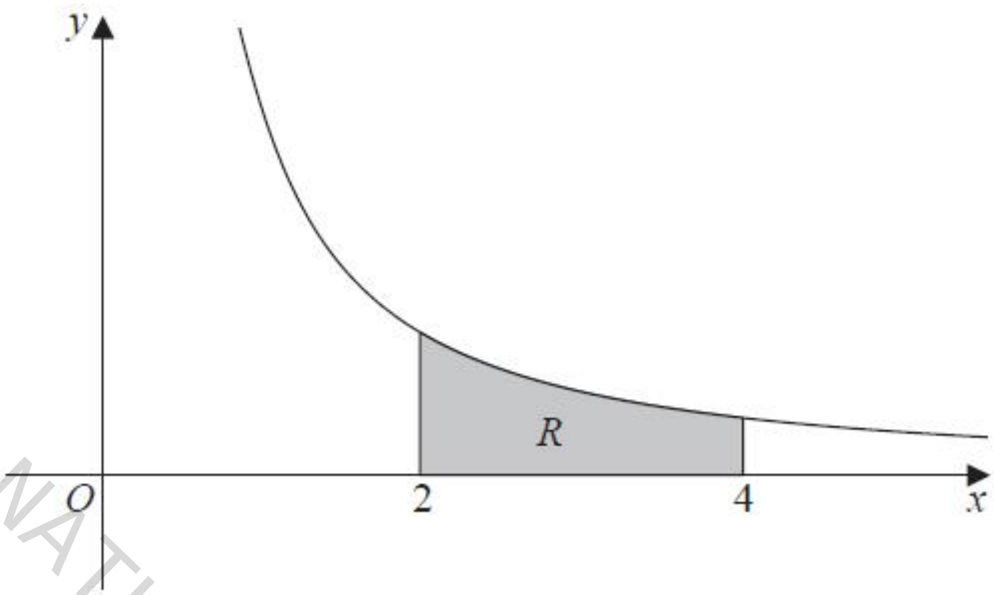

Figure 3

(a) Find $\int \frac{\ln x}{x^2} \mathrm{d}x$

(3)

Figure 3 shows a sketch of part of the curve with equation

$$y = \frac{3 + 2x + \ln x}{x^2} \qquad x > 0.5$$

The finite region R, shown shaded in Figure 3, is bounded by the curve, the line with equation $x = 2$, the x-axis and the line with equation $x = 4$

(b) Use the answer to part (a) to find the exact area of R, writing your answer in simplest form.

(4)

(Total for question = 7 marks)

(**Q05 WMA14/01, Oct 2020**)

Extra space for working:

Q13.

(i) Using a suitable substitution, find, using calculus, the value of

$$\int_1^5 \frac{3x}{\sqrt{2x-1}}\,\mathrm{d}x$$

(*Solutions relying entirely on calculator technology are not acceptable.*)

(6)

(ii) Find

$$\int \frac{6x^2-16}{(x+1)(2x-3)}\,\mathrm{d}x$$

(6)

(Total for question = 12 marks)

(**Q07 WMA14/01, Oct 2020**)

Extra space for working:

Q14.

Use algebraic integration and the substitution $u = \sqrt{x}$ to find the exact value of

$$\int_1^4 \frac{10}{5x + 2x\sqrt{x}}\,dx$$

Write your answer in the form $4\ln\left(\frac{a}{b}\right)$, where *a* and *b* are integers to be found.

(*Solutions relying entirely on calculator technology are not acceptable*.)

(Total for question = 8 marks)

(**Q04 WMA14/01, June 2021**)

Extra space for working:

Q15.

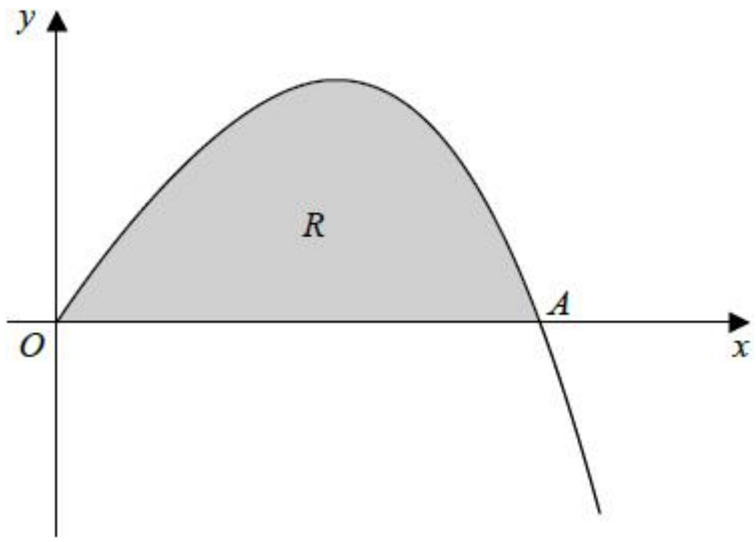

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = 4x - xe^{\frac{1}{2}x}, x \geqslant 0$

The curve meets the x-axis at the origin O and cuts the x-axis at the point A.

(a) Find, in terms of ln 2, the x coordinate of the point A.

(2)

(b) Find $\int xe^{\frac{1}{2}x}\, dx$

(3)

The finite region R, shown shaded in Figure 2, is bounded by the x-axis and the curve with equation

$y = 4x - xe^{\frac{1}{2}x}, x \geqslant 0$

(c) Find, by integration, the exact value for the area of R.

Give your answer in terms of ln 2

(3)

(Total for question = 8 marks)

(**Q05 WMA14/01, Specimen papers**)

Extra space for working:

Q16.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(i) Use the substitution $u = e^x - 3$ to show that

$$\int_{\ln 5}^{\ln 7} \frac{4e^{3x}}{e^x - 3}\,dx = a + b\ln 2$$

where *a* and *b* are constants to be found.

(7)

(ii) Show, by integration, that

$$\int 3e^x \cos 2x\,dx = pe^x \sin 2x + qe^x \cos 2x + c$$

where *p* and *q* are constants to be found and *c* is an arbitrary constant.

(5)

(Total for question = 12 marks)

(**Q07 WMA14/01, Oct 2022**)

Extra space for working:

Q17.

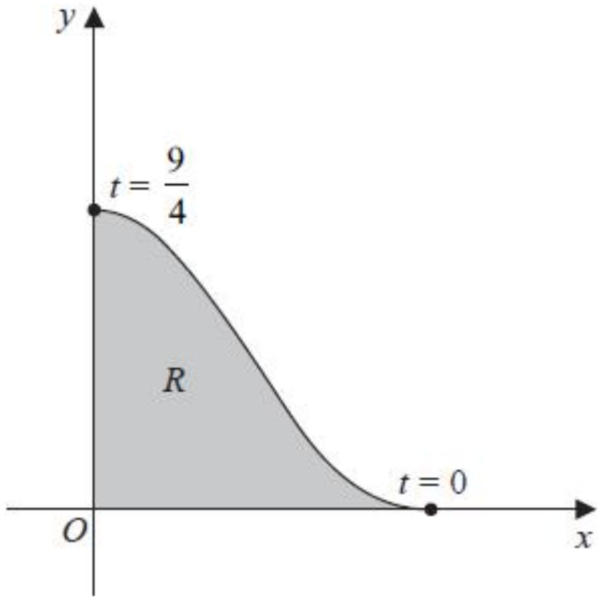

Figure 2

Figure 2 shows a sketch of the curve with parametric equations

$$x = \sqrt{9 - 4t} \qquad y = \frac{t^3}{\sqrt{9 + 4t}} \qquad 0 \leqslant t \leqslant \frac{9}{4}$$

The curve touches the x-axis when $t = 0$ and meets the y-axis when $t = \frac{9}{4}$

The region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the y-axis.

(a) Show that the area of R is given by

$$K \int_0^{\frac{9}{4}} \frac{t^3}{\sqrt{81 - 16t^2}} \, dt$$

where K is a constant to be found.

(4)

(b) Using the substitution $u = 81 - 16t^2$, or otherwise, find the exact area of R.

(*Solutions relying on calculator technology are not acceptable*.)

(6)

(Total for question = 10 marks)

(**Q05 WMA14/01, Jan 2022**)

Q1.

Water is being heated in a kettle. At time t seconds, the temperature of the water is θ °C.

The rate of increase of the temperature of the water at time t is modelled by the differential equation

$$\frac{d\theta}{dt} = \lambda(120 - \theta) \qquad \theta \leqslant 100$$

where λ is a positive constant.

Given that $\theta = 20$ when $t = 0$

(a) solve this differential equation to show that

$$\theta = 120 - 100e^{-\lambda t}$$

(8)

When the temperature of the water reaches 100 °C, the kettle switches off.

(b) Given that $\lambda = 0.01$, find the time, to the nearest second, when the kettle switches off.

(3)

(Total for question = 11 marks)

(**Q08 WMA14/01, Specimen papers**)

Extra space for working:

Q2.

Bacteria are growing on the surface of a dish in a laboratory.

The area of the dish, A cm^2, covered by the bacteria, t days after the bacteria start to grow, is modelled by the differential equation

$$\frac{dA}{dt} = \frac{A^{\frac{3}{2}}}{5t^2} \qquad t > 0$$

Given that $A = 2.25$ when $t = 3$

(a) show that

$$A = \left(\frac{pt}{qt + r}\right)^2$$

where p, q and r are integers to be found.

(7)

According to the model, there is a limit to the area that will be covered by the bacteria.

(b) Find the value of this limit.

(2)

(Total for question = 9 marks)

(**Q09 WMA14/01, Oct 2020**)

Extra space for working:

Q3.

(a) Given that $y = 1$ at $x = 0$, solve the differential equation

$$\frac{dy}{dx} = \frac{6xy^{\frac{1}{3}}}{e^{2x}} \qquad y \geqslant 0$$

giving your answer in the form $y^2 = g(x)$.

(7)

(b) Hence find the equation of the horizontal asymptote to the curve with equation $y^2 = g(x)$.

(2)

(Total for question = 9 marks)

(**Q08 WMA14/01, June 2021**)

Extra space for working:

Q4.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

The temperature, θ °C, of a car engine, t minutes after the engine is turned off, is modelled by the differential equation

$$\frac{d\theta}{dt} = -k(\theta - 15)^2$$

where k is a constant.

Given that the temperature of the car engine

- is 85 °C at the instant the engine is turned off
- is 40 °C exactly 10 minutes after the engine is turned off

(a) solve the differential equation to show that, according to the model

$$\theta = \frac{at + b}{ct + d}$$

where a, b, c and d are integers to be found.

(7)

(b) Hence find, according to the model, the time taken for the temperature of the car engine to reach 20 °C. Give your answer to the nearest minute.

(2)

(Total for question = 9 marks)

(**Q06 WMA14/01, June 2023**)

Extra space for working:

Q5.

Find the particular solution of the differential equation

$$\frac{dy}{dx} = \frac{4y^2}{\sqrt{4x+5}} \qquad x > -\frac{5}{4}$$

for which $y = \frac{1}{3}$ at $x = -\frac{1}{4}$ giving your answer in the form $y = f(x)$

(6)

(Total for question = 6 marks)

(**Q02 WMA14/01, Oct 2021**)

Extra space for working:

Q6.

The number of goats on an island is being monitored.

When monitoring began there were 3000 goats on the island.

In a simple model, the number of goats, x, in thousands, is modelled by the equation

$$x = \frac{k(9t+5)}{4t+3}$$

where k is a constant and t is the number of years after monitoring began.

(a) Show that $k = 1.8$

(2)

(b) Hence calculate the long-term population of goats predicted by this model.

(1)

In a **second** model, the number of goats, x, in thousands, is modelled by the differential equation

$$3\frac{dx}{dt} = x(9-2x)$$

(c) Write $\frac{3}{x(9-2x)}$ in partial fraction form.

(3)

(d) Solve the differential equation with the initial condition to show that

$$x = \frac{9}{2+e^{-3t}}$$

(5)

(e) Find the long-term population of goats predicted by this **second** model.

(1)

(Total for question = 12 marks)

(**Q07 WMA14/01, Oct 2023**)

Extra space for working:

Q7.

(a) Write $\dfrac{1}{(H-5)(H+3)}$ in partial fraction form.

(3)

The depth of water in a storage tank is being monitored.

The depth of water in the tank, *H* metres, is modelled by the differential equation

$$\frac{dH}{dt} = -\frac{(H-5)(H+3)}{40}$$

where *t* is the time, in days, from when monitoring began.

Given that the initial depth of water in the tank was 13 m,

(b) solve the differential equation to show that

$$H = \frac{10+3e^{-0.2t}}{2-e^{-0.2t}}$$

(7)

(c) Hence find the time taken for the depth of water in the tank to fall to 8 m.

(Solutions relying entirely on calculator technology are not acceptable.)

(3)

According to the model, the depth of water in the tank will eventually fall to *k* metres.

(d) State the value of the constant *k*.

(1)

(Total for question = 14 marks)

(**Q10 WMA14/01, Jan 2021**)

Extra space for working:

Q8.

A spherical ball of ice of radius 12 cm is placed in a bucket of water.

In a model of the situation,

- the ball remains spherical as it melts
- t minutes after the ball of ice is placed in the bucket, its radius is r cm
- the rate of decrease of the radius of the ball of ice is inversely proportional to the square of the radius
- the radius of the ball of ice is 6 cm after 15 minutes

Using the model and the information given,

(a) find an equation linking r and t,

(5)

(b) find the time taken for the ball of ice to melt completely.

(2)

(c) On Diagram 1, sketch a graph of r against t.

(1)

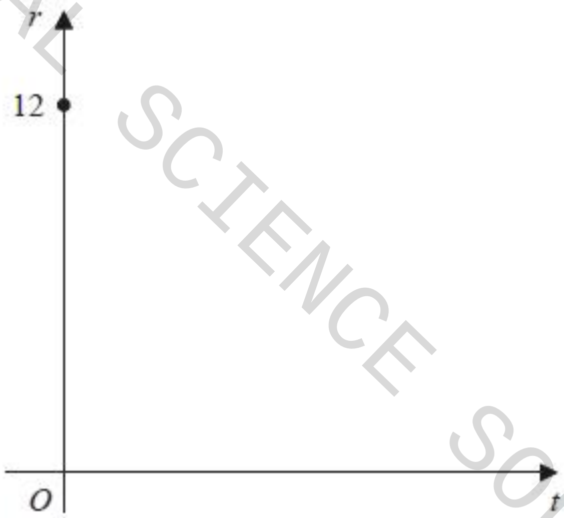

Diagram 1

(Total for question = 8 marks)

(**Q10 WMA14/01, Oct 2022**)

Extra space for working:

Q9.

(a) Find the derivative with respect to y of

$$\frac{1}{(1+2\ln y)^2}$$

(2)

(b) Hence find a general solution to the differential equation

$$3\operatorname{cosec}(2x)\frac{dy}{dx} = y(1+2\ln y)^3 \qquad y > 0 \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

(4)

(c) Show that the particular solution of this differential equation for which $y = 1$ at $x = \frac{\pi}{6}$ is given by

$$y = e^{\frac{1}{\sqrt{A\sec x - \frac{1}{2}}}}$$

where A is an irrational number to be found.

(5)

(Total for question = 11 marks)

(**Q09 WMA14/01, Jan 2022**)

Extra space for working:

Q1.

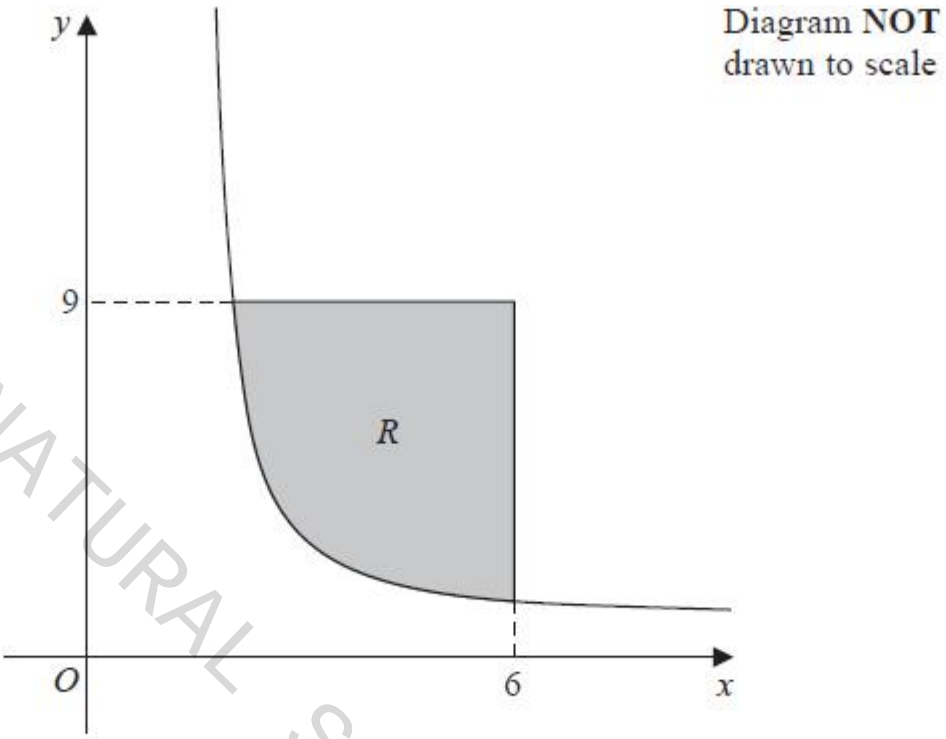

Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = \frac{9}{(2x-3)^{1.25}} \qquad x > \frac{3}{2}$$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the line with equation $y = 9$ and the line with equation $x = 6$

This region is rotated through 2π radians about the x-axis to form a solid of revolution.

Find, by algebraic integration, the exact volume of the solid generated.

(Total for question = 7 marks)

(**Q02 WMA14/01, June 2021**)

Extra space for working:

Q2.

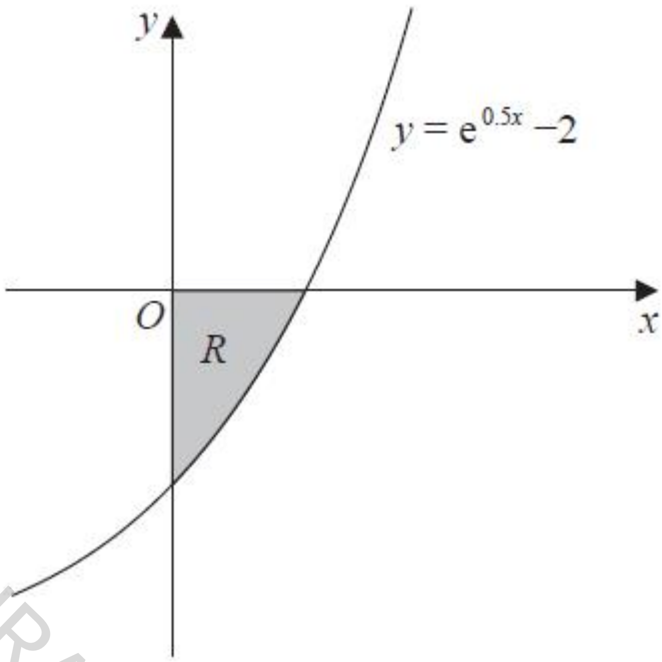

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = e^{0.5x} - 2$

The region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the y-axis.

The region R is rotated 360° about the x-axis to form a solid of revolution.

Show that the volume of this solid can be written in the form $a \ln 2 + b$, where a and b are constants to be found.

(Total for question = 6 marks)

(**Q03 WMA14/01, Oct 2020**)

Extra space for working:

Q3.

(a) Using the substitution $u = 4x + 2\sin 2x$, or otherwise, show that

$$\int_0^{\frac{\pi}{2}} e^{4x+2\sin 2x}\cos^2 x \, dx = \frac{1}{8}(e^{2\pi} - 1)$$

(5)

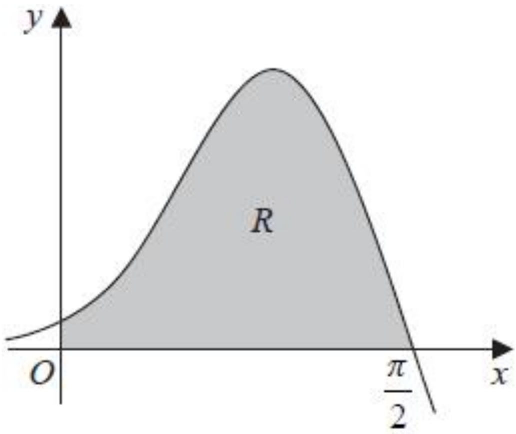

Figure 3

The curve shown in Figure 3, has equation

$$y = 6e^{2x + \sin 2x}\cos x$$

The region R, shown shaded in Figure 3, is bounded by the positive x-axis, the positive y-axis and the curve.

The region R is rotated through 2π radians about the x-axis to form a solid.

(b) Use the answer to part (a) to find the volume of the solid formed, giving the answer in simplest form.

(3)

(Total for question = 8 marks)

(**Q07 WMA14/01, Jan 2024**)

Extra space for working:

Q4.

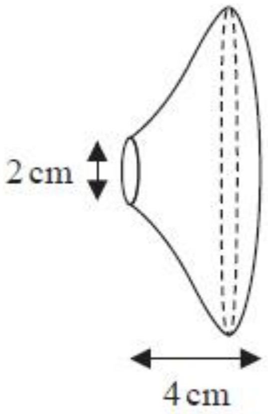

Figure 3

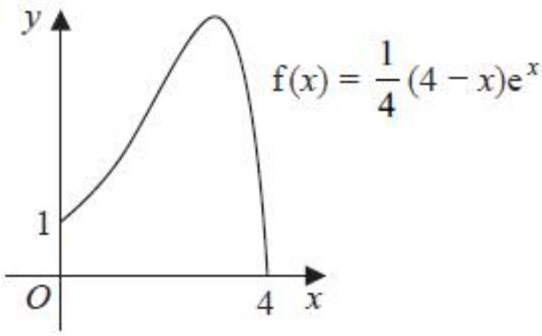

Figure 4

Figure 3 shows the design of a doorknob.

The shape of the doorknob is formed by rotating the curve shown in Figure 4 through 360° about the x-axis, where the units are centimetres.

The equation of the curve is given by

$$f(x) = \frac{1}{4}(4 - x)e^x \qquad 0 \leqslant x \leqslant 4$$

(a) Show that the volume, V cm^3, of the doorknob is given by

$$V = K\int_0^4 (x^2 - 8x + 16)e^{2x}\,dx$$

where K is a constant to be found.

(3)

(b) Hence, find the exact value of the volume of the doorknob.

Give your answer in the form $p\pi(e^q + r)$cm^3 where p, q and r are simplified rational numbers to be found.

(5)

(Total for question = 8 marks)

(**Q07 WMA14/01, Jan 2022**)

Extra space for working:

Q5.

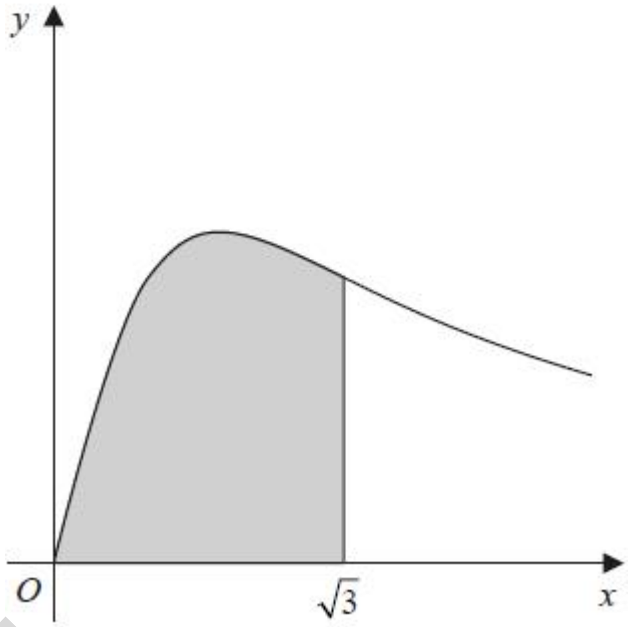

Figure 3

Figure 3 shows a sketch of part of the curve with parametric equations

$$x = \tan\theta \qquad y = 2\sin 2\theta \qquad \theta \geq 0$$

The finite region, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line with equation $x = \sqrt{3}$

The region is rotated through 2π radians about the x-axis to form a solid of revolution.

(a) Show that the exact volume of this solid of revolution is given by

$$\int_0^k p(1 - \cos 2\theta)\,d\theta$$

where p and k are constants to be found.

(7)

(b) Hence find, by algebraic integration, the exact volume of this solid of revolution.

(3)

(Total for question = 10 marks)

(**Q09 WMA14/01, Jan 2021**)

Extra space for working:

Q6.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

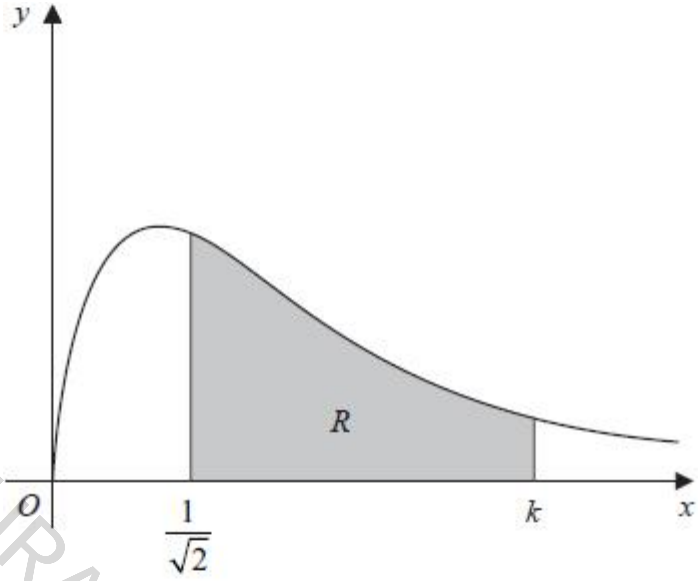

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = \frac{12\sqrt{x}}{(2x^2 + 3)^{1.5}}$$

The region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = \frac{1}{\sqrt{2}}$, the x-axis and the line with equation $x = k$.

This region is rotated through 360° about the x-axis to form a solid of revolution.

Given that the volume of this solid is $\frac{713}{648}\pi$, use algebraic integration to find the exact value of the constant k.

(6)

(Total for question = 6 marks)

(**Q05 WMA14/01, Oct 2022**)

Extra space for working:

Q7.

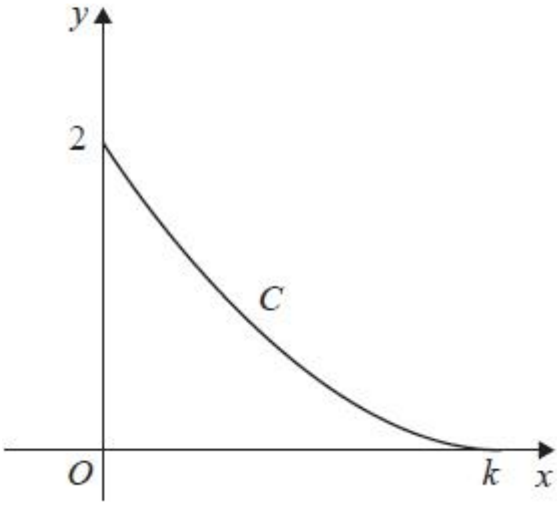

Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 6t - 3\sin 2t \qquad y = 2\cos t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The curve meets the y-axis at 2 and the x-axis at k, where k is a constant.

(a) State the value of k.

(1)

(b) Use parametric differentiation to show that

$$\frac{dy}{dx} = \lambda \operatorname{cosec} t$$

where λ is a constant to be found.

(4)

The point P with parameter $t = \frac{\pi}{4}$ lies on C.

The tangent to C at the point P cuts the y-axis at the point N.

(c) Find the exact y coordinate of N, giving your answer in simplest form.

(3)

The region bounded by the curve, the x-axis and the y-axis is rotated through 2π radians about the x-axis to form a solid of revolution.

(d) (i) Show that the volume of this solid is given by

$$\int_0^{\alpha} \beta(1 - \cos 4t)\, dt$$

where α and β are constants to be found.

(ii) Hence, using algebraic integration, find the exact volume of this solid.

(6)

(Total for question = 14 marks)
(**Q08 WMA14/01, Oct 2023**)

Extra space for working:

Extra space for working:

Q8.

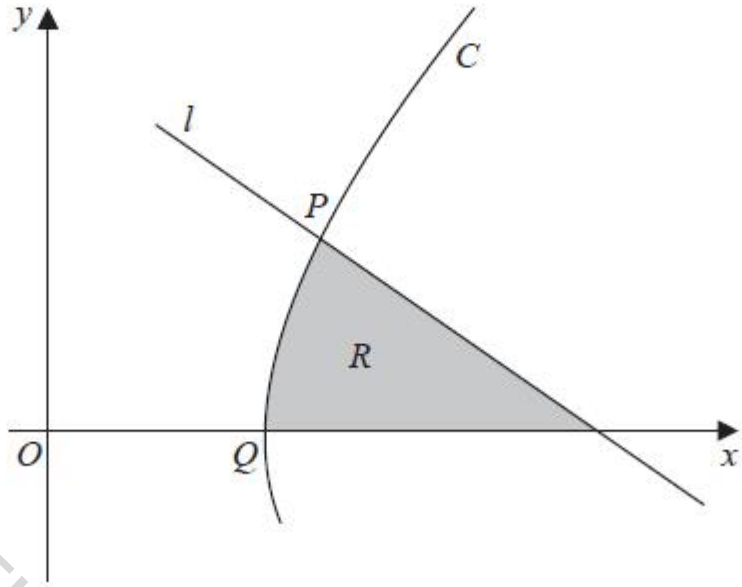

Figure 2

Figure 2 shows a sketch of part of the curve *C* with parametric equations

$$x = t + \frac{1}{t} \qquad y = t - \frac{1}{t} \qquad t > 0.7$$

The curve *C* intersects the *x*-axis at the point *Q*.

(a) Find the *x* coordinate of *Q*.

(1)

The line *l* is the normal to *C* at the point *P* as shown in Figure 2.

Given that $t = 2$ at *P*

(b) write down the coordinates of *P*

(1)

(c) Using calculus, show that an equation of *l* is

$$3x + 5y = 15$$

(3)

The region, *R*, shown shaded in Figure 2 is bounded by the curve *C*, the line *l* and the *x*-axis.

(d) Using algebraic integration, find the exact volume of the solid of revolution formed when the region *R* is rotated through 2π radians about the *x*-axis.

(7)

(Total for question = 12 marks)

(**Q08 WMA14/01, June 2023**)

Extra space for working:

Extra space for working:

Q9.

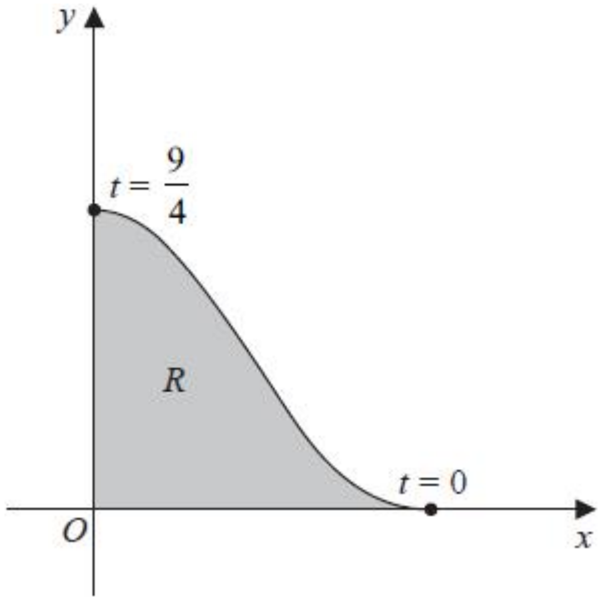

Figure 2

Figure 2 shows a sketch of the curve with parametric equations

$$x = \sqrt{9 - 4t} \qquad y = \frac{t^3}{\sqrt{9 + 4t}} \qquad 0 \leqslant t \leqslant \frac{9}{4}$$

The curve touches the x-axis when $t = 0$ and meets the y-axis when $t = \frac{9}{4}$

The region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the y-axis.

(a) Show that the area of R is given by

$$K \int_0^{\frac{9}{4}} \frac{t^3}{\sqrt{81 - 16t^2}} \, dt$$

where K is a constant to be found.

(4)

(b) Using the substitution $u = 81 - 16t^2$, or otherwise, find the exact area of R.

(*Solutions relying on calculator technology are not acceptable*.)

(6)

(Total for question = 10 marks)

(**Q05 WMA14/01, Jan 2022**)

Extra space for working:

Extra space for working:

Q10.

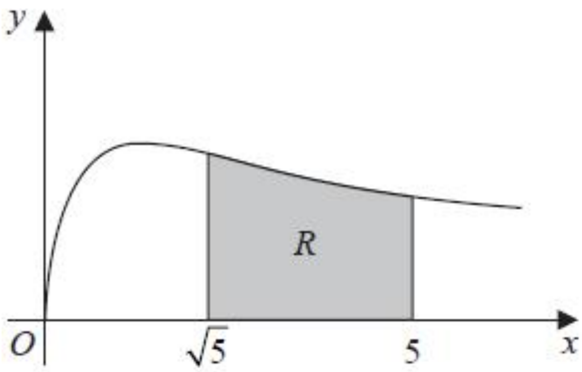

Figure 1

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of the curve with equation

$$y = \sqrt{\frac{3x}{3x^2 + 5}} \qquad x \geqslant 0$$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the lines with equations
$x = 5$ and $x = 5$

The region R is rotated through 360° about the x-axis.

Use integration to find the exact volume of the solid generated. Give your answer in the form $a \ln b$, where a is an irrational number and b is a prime number.

(Total for question = 5 marks)

(**Q03 WMA14/01, Jan 2023**)

Extra space for working:

Q11.

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

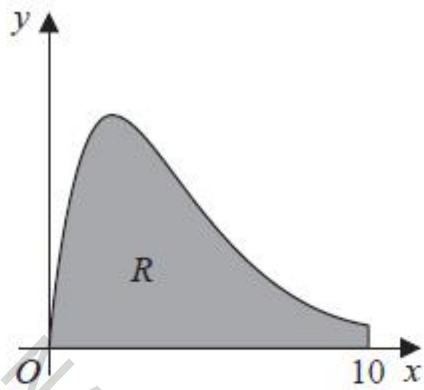

Figure 2

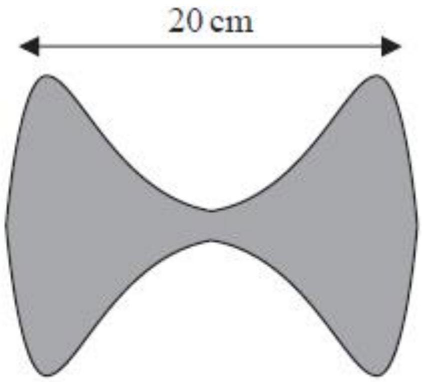

Figure 3

Figure 2 shows the curve with equation

$$y = 10xe^{-\frac{1}{2}x} \qquad 0 \leqslant x \leqslant 10$$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the line with equation $x = 10$

The region R is rotated through 2π radians about the x-axis to form a solid of revolution.

(a) Show that the volume, V, of this solid is given by

$$V = k\int_0^{10} x^2 e^{-x}\,dx$$

where k is a constant to be found.

(2)

(b) Find $\int x^2 e^{-x}\,dx$

(3)

Figure 3 represents an exercise weight formed by joining two of these solids together.

The exercise weight has mass 5 kg and is 20 cm long.

Given that

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

and using your answers to part (a) and part (b),

(c) find the density of this exercise weight. Give your answer in grams per cm^3 to 3 significant figures.

(5)

(Total for question = 10 marks)

(**Q08 WMA14/01, June 2022**)

Extra space for working:

Extra space for working:

Q12.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

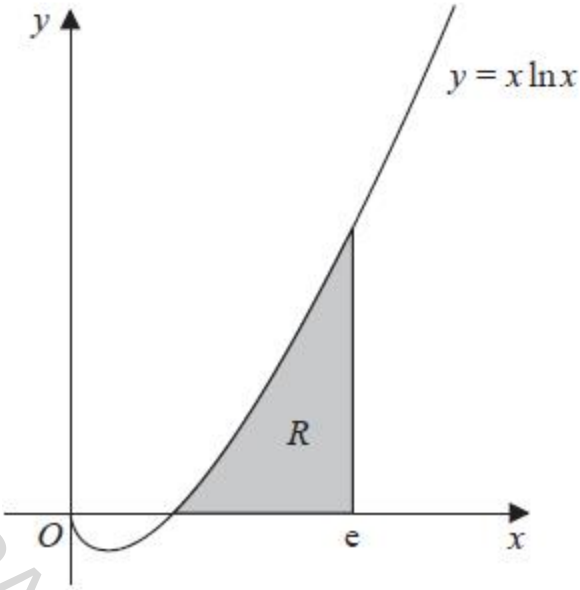

Figure 3

(a) Find $\int x^2 \ln x \, dx$

(3)

Figure 3 shows a sketch of part of the curve with equation

$$y = x\ln x \qquad x > 0$$

The region R, shown shaded in Figure 3, lies entirely above the x-axis and is bounded by the curve, the x-axis and the line with equation $x = e$.

This region is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Find the exact volume of the solid formed, giving your answer in simplest form.

(4)

(Total for question = 7 marks)

(**Q08 WMA14/01, Oct 2021**)

Extra space for working:

Vectors

Q1.

Relative to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = (3\mathbf{i} + p\mathbf{j} + 7\mathbf{k}) + \lambda\,(2\mathbf{i} - 5\mathbf{j} + 4\mathbf{k})$$

$$l_2 : \mathbf{r} = (8\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}) + \mu\,(4\mathbf{i} + \mathbf{j} + 2\mathbf{k})$$

where λ and μ are scalar parameters and p is a constant.

Given that l_1 and l_2 intersect,

(a) find the value of p,

(4)

(b) find the position vector of the point of intersection.

(2)

(c) Find the acute angle between l_1 and l_2

Give your answer in degrees to one decimal place.

(3)

The point A lies on l_1 with parameter $\lambda = 2$

The point B lies on l_2 with $\overrightarrow{AB}$ perpendicular to l_2

(d) Find the coordinates of B

(5)

(Total for question = 14 marks)

(**Q06 WMA14/01, Jan 2024**)

Extra space for working:

Q2.

Relative to a fixed origin O,

- the point A has position vector $4\mathbf{i} + 8\mathbf{j} + \mathbf{k}$
- the point B has position vector $5\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$
- the point P has position vector $2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

The straight line l passes through A and B.

(a) Find a vector equation for l.

(2)

The point C lies on l so that PC is perpendicular to l.

(b) Find the coordinates of C.

(4)

The point P' is the reflection of P in the line l.

(c) Find the coordinates of P'

(2)

(d) Hence find $|\overrightarrow{PP'}|$, giving your answer as a simplified surd.

(2)

(Total for question = 10 marks)

(**Q04 WMA14/01, June 2023**)

Extra space for working:

Q3.

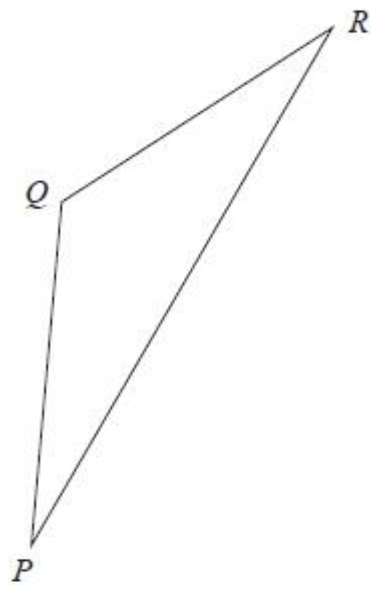

Figure 1

Figure 1 shows a sketch of triangle *PQR*.

Given that

- $\overrightarrow{PQ} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$
- $\overrightarrow{PR} = 8\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$

(a) Find $\overrightarrow{RQ}$

(2)

(b) Find the size of angle *PQR*, in degrees, to three significant figures.

(3)

(Total for question = 5 marks)

(**Q03 WMA14/01, Oct 2022**)

Extra space for working:

Q4.

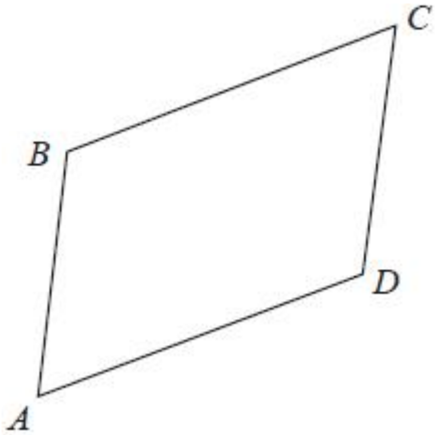

Figure 1

Figure 1 shows a sketch of parallelogram *ABCD*.

Given that $\overrightarrow{AB} = 6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $\overrightarrow{BC} = 2\mathbf{i} + 5\mathbf{j} + 8\mathbf{k}$

(a) find the size of angle *ABC*, giving your answer in degrees, to 2 decimal places.

(3)

(b) Find the area of parallelogram *ABCD*, giving your answer to one decimal place.

(2)

(Total for question = 5 marks)

(**Q02 WMA14/01, Jan 2021**)

Extra space for working:

Q5.

With respect to a fixed origin O,

- the line l has equation $\mathbf{r} = \begin{pmatrix} 4 \\ 2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} -4 \\ -3 \\ 5 \end{pmatrix}$ where λ is a scalar constant

- the point A has position vector $9\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$

Given that X is the point on l nearest to A,

(a) find

(i) the coordinates of X
(ii) the shortest distance from A to l.

Give your answer in the form $\sqrt{d}$, where d is an integer.

(7)

The point B is the image of A after reflection in l.

(b) Find the position vector of B.

(2)

(Total for question = 9 marks)

(**Q07 WMA14/01, Oct 2021**)

Extra space for working:

Q6.

Relative to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1: \quad \mathbf{r} = \begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} \qquad \text{where } \lambda \text{ is a scalar parameter}$$

$$l_2: \quad \mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ -9 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} \qquad \text{where } \mu \text{ is a scalar parameter}$$

Given that l_1 and l_2 meet at the point X,

(a) find the position vector of X.

(5)

The point P (10, −7, 0) lies on l_1

The point Q lies on l_2

Given that $\overrightarrow{PQ}$ is perpendicular to l_2

(b) calculate the coordinates of Q.

(5)

(Total for question = 10 marks)

(**Q08 WMA14/01, Oct 2020**)

Extra space for working:

Q7.

With respect to a fixed origin O, the line l_1 is given by the equation

$$\mathbf{r} = \begin{pmatrix} 8 \\ 1 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$$

where μ is a scalar parameter.

The point A lies on l_1 where $\mu = 1$

(a) Find the coordinates of A.

(1)

The point P has position vector $\begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$

The line l_2 passes through the point P and is parallel to the line l_1

(b) Write down a vector equation for the line l_2

(2)

(c) Find the exact value of the distance AP.

Give your answer in the form $k\sqrt{2}$, where k is a constant to be found.

(2)

The acute angle between AP and l_2 is θ

(d) Find the value of $\cos\theta$

(3)

A point E lies on the line l_2

Given that $AP = PE$,

(e) find the area of triangle APE,

(2)

(f) find the coordinates of the two possible positions of E.

(5)

(Total for question = 15 marks)

(**Q09 WMA14/01, Specimen papers**)

Extra space for working:

Q8.

The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ where λ is a scalar parameter.

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -7 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix}$ where μ is a scalar parameter.

Given that l_1 and l_2 meet at the point P

(a) state the coordinates of P

(1)

Given that the angle between lines l_1 and l_2 is θ

(b) find the value of cos θ, giving the answer as a fully simplified fraction.

(3)

The point Q lies on l_1 where $\lambda = 6$

Given that point R lies on l_2 such that triangle QPR is an isosceles triangle with $PQ = PR$

(c) find the exact area of triangle QPR

(3)

(d) find the coordinates of the possible positions of point R

(3)

(Total for question = 10 marks)

(**Q06 WMA14/01, Oct 2023**)

Extra space for working:

Q9.

Relative to a fixed origin *O*, the line *l* has equation

r = $\begin{pmatrix} 1 \\ -10 \\ -9 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix}$ where λ is a scalar parameter

Given that $\overrightarrow{OA}$ is a unit vector parallel to *l*,

(a) find $\overrightarrow{OA}$

(2)

The point *X* lies on *l*.

Given that *X* is the point on *l* that is closest to the origin,

(b) find the coordinates of *X*.

(5)

The points *O*, *X* and *A* form the triangle *OXA*.

(c) Find the exact area of triangle *OXA*.

(3)

(Total for question = 10 marks)

(**Q07 WMA14/01, June 2021**)

Extra space for working:

Q10.

With respect to a fixed origin O, the equations of lines l_1 and l_2 are given by

$$l_1 : \mathbf{r} = \begin{pmatrix} 2 \\ 8 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$$

$$l_2 : \mathbf{r} = \begin{pmatrix} -4 \\ -1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 4 \\ 8 \end{pmatrix}$$

where λ and μ are scalar parameters.

Prove that lines l_1 and l_2 are skew.

(5)

(Total for question = 5 marks)

(**Q09 WMA14/01, Oct 2022**)

Extra space for working:

Q11.

With respect to a fixed origin *O* the points *A* and *B* have position vectors

$$\begin{pmatrix} 6 \\ 6 \\ 2 \end{pmatrix} \text{ and } \begin{pmatrix} 6 \\ 0 \\ 7 \end{pmatrix}$$

respectively.

The line l_1 passes through the points *A* and *B*.

(a) Write down an equation for l_1

Give your answer in the form $\mathbf{r} = \mathbf{p} + \lambda\mathbf{q}$, where λ is a scalar parameter.

(2)

The line l_2 has equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 5 \\ 9 \end{pmatrix}$$

where μ is a scalar parameter.

(b) Show that l_1 and l_2 do **not** meet.

(4)

The point *C* is on l_2 where $\mu = -1$

(c) Find the acute angle between *AC* and l_2

Give your answer in degrees to one decimal place.

(5)

(Total for question = 11 marks)

(**Q08 WMA14/01, Jan 2022**)

Extra space for working: