# A2 Edexcel Math paper-3 (WMA13) CLASSIFIED QUESTIONS

# 2019 - 2023



*"Knowing the path is good but not enough, walking the path with determination leads to destiny"*

# TABLE OF CONTENTS

# Algebra_Functions_Transformation

**Q1.**

$$f(x) = 3 - \frac{x-2}{x+1} + \frac{5x+26}{2x^2-3x-5} \qquad x > 4$$

(a) Show that

$$f(x) = \frac{ax+b}{cx+d} \qquad x > 4$$

where $a$, $b$, $c$ and $d$ are integers to be found.

**(4)**

(b) Hence find $f^{-1}(x)$

**(2)**

(c) Find the domain of $f^{-1}$

**(2)**

**(Total for question = 8 marks)**

(**Q03 WMA13/01, Jan 2021**)

**Extra space for working:**

**Q2.**

The function f is defined by

$$f(x) = \frac{x+3}{x-4} \qquad x \in \mathbb{R}, x \neq 4$$

(a) Find ff(6)

**(2)**

(b) Find $f^{-1}$

**(3)**

The function g is defined by

$$g(x) = x^2 + 5 \qquad x \in \mathbb{R}, x > 0$$

(c) Find the exact value of *a* for which

$$gf(a) = 7$$

**(3)**

**(Total for question = 8 marks)**

(**Q02 WMA13/01, Oct 2023**)

**Extra space for working:**

**Q3.**

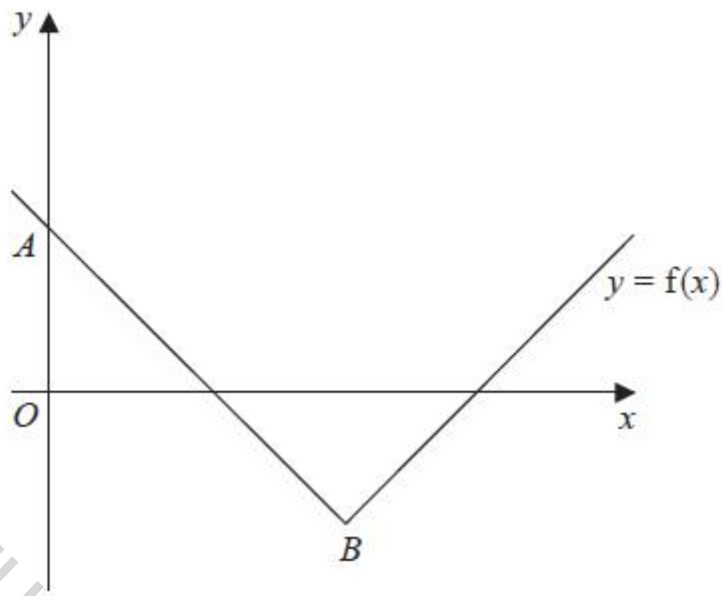


**Figure 2**

Figure 2 shows part of the graph with equation $y = \text{f}(x)$, where

$\text{f}(x) = \tfrac{1}{2}kx - 9\tfrac{1}{2} - 2 \qquad x \in \mathbb{R}$

and $k$ is a positive constant.

The graph intersects the $y$-axis at the point $A$ and has a minimum point at $B$ as shown.

(a) (i) Find the $y$ coordinate of $A$

(ii) Find, in terms of $k$, the $x$ coordinate of $B$

**(2)**

(b) Find, in terms of $k$, the range of values of $x$ that satisfy the inequality

$$|kx - 9| - 2 < 0$$

**(3)**

Given that the line $y = 3 - 2x$ intersects the graph $y = \text{f}(x)$ at two distinct points,

(c) find the range of possible values of $k$

**(3)**

**(Total for question = 8 marks)**

(**Q05 WMA13/01, June 2022**)

**Extra space for working:**

**Q4.**

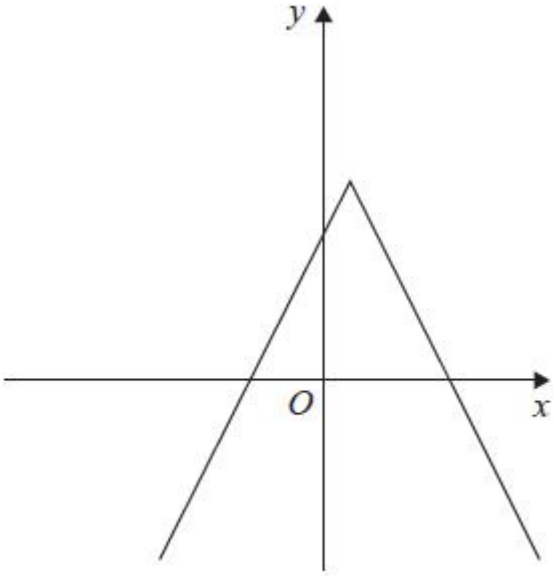


**Figure 2**

**In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

The graph shown in Figure 2 has equation

$$y = a - |2x - b|$$

where *a* and *b* are positive constants, *a* > *b*

(a) Find, giving your answer in terms of *a* and *b*,

(i) the coordinates of the maximum point of the graph,
(ii) the coordinates of the point of intersection of the graph with the *y*-axis,
(iii) the coordinates of the points of intersection of the graph with the *x*-axis.

**(5)**

(b) On Diagram 1, sketch the graph with equation

$$y = |x| - 1$$

**(2)**

Given that the graphs $y = |x| - 1$ and $y = a - |2x - b|$ intersect at *x* = –3 and *x* = 5

(c) find the value of *a* and the value of *b*

**(4)**

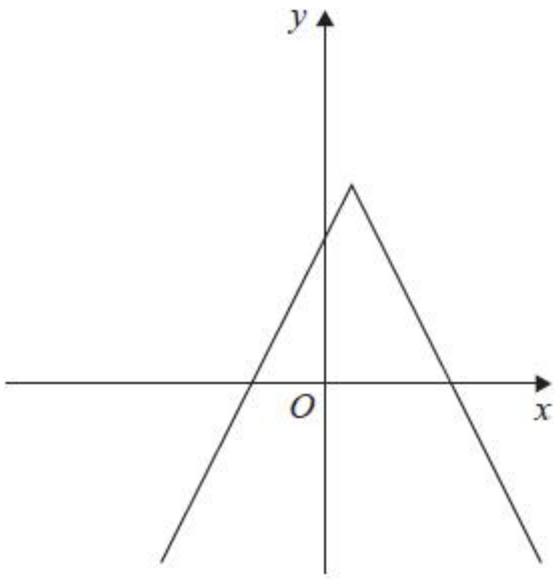


**Diagram 1**

**(Total for question = 11 marks)**

(**Q08 WMA13/01, Jan 2024**)

**Extra space for working:**

**Extra space for working:**

**Q5.**

The point $P(-4, -3)$ lies on the curve with equation $y = f(x)$, $x \in \mathbb{R}$

Find the point to which $P$ is mapped when the curve with equation $y = f(x)$ is transformed to the curve with equation

(a) $y = f(2x)$

**(1)**

(b) $y = 3f(x - 1)$

**(2)**

(c) $y = |f(x)|$

**(1)**

**(Total for question = 4 marks)**

(**Q01 WMA13/01, Jan 2024**)

**Extra space for working:**

**Q6.**

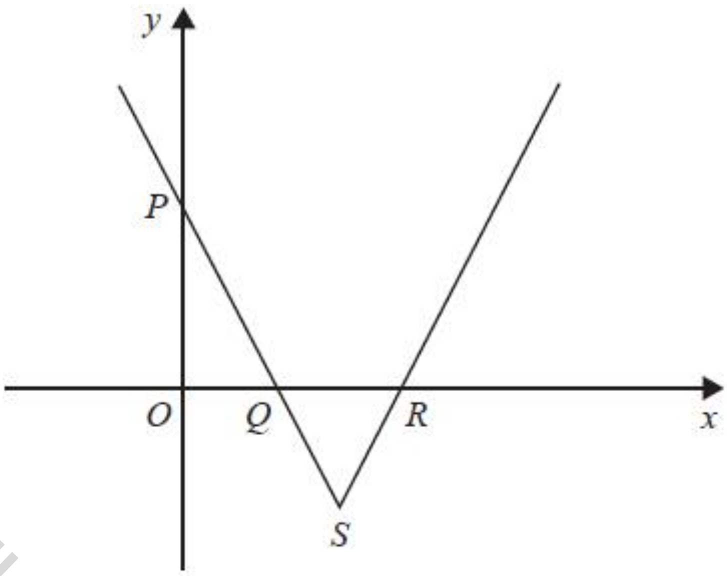


**Figure 2**

Figure 2 shows a sketch of the graph with equation

$$y = |3x - 5a| - 2a$$

where $a$ is a positive constant.

The graph

- cuts the $y$-axis at the point $P$
- cuts the $x$-axis at the points $Q$ and $R$
- has a minimum point at $S$

(a) Find, in simplest form in terms of $a$, the coordinates of

(i) point $P$
(ii) points $Q$ and $R$
(iii) point $S$

**(4)**

(b) Find, in simplest form in terms of $a$, the values of $x$ for which

$$|3x - 5a| - 2a = |x - 2a|$$

**(4)**

**(Total for question = 8 marks)**

(**Q06 WMA13/01, Jan 2023**)

**Extra space for working:**

**Q7.**

The function f is defined by

$$f(x) = \frac{5x - 3}{x - 4} \qquad x > 4$$

(a) Show, by using calculus, that f is a decreasing function.

**(3)**

(b) Find $f^{-1}$

**(3)**

(c) (i) Show that $ff(x) = \frac{ax + b}{x + c}$ where *a*, *b* and *c* are constants to be found.

(ii) Deduce the range of ff.

**(5)**

**(Total for question = 11 marks)**

(**Q06 WMA13/01, Jan 2022**)

**Extra space for working:**

**Q8.**

The function f is defined by

$$f(x) = 2x^2 - 5 \qquad x \geqslant 0 \quad x \in \mathbb{R}$$

(a) State the range of f

**(1)**

Below there is a diagram, labelled Diagram 1, which shows a sketch of the curve with equation $y = f(x)$.

(b) On Diagram 1, sketch the curve with equation $y = f^{-1}(x)$.

**(2)**

The curve with equation $y = f(x)$ meets the curve with equation $y = f^{-1}(x)$ at the point $P$

Using algebra and showing your working,

(c) find the exact $x$ coordinate of $P$

**(3)**

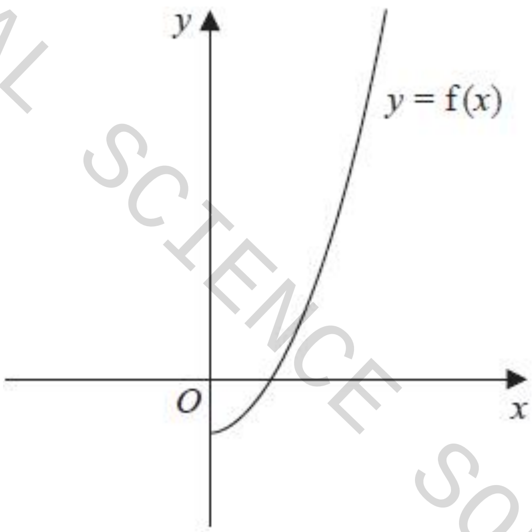


**Diagram 1**

**(Total for question = 6 marks)**

(**Q04 WMA13/01, June 2023**)

**Extra space for working:**

**Q9.**

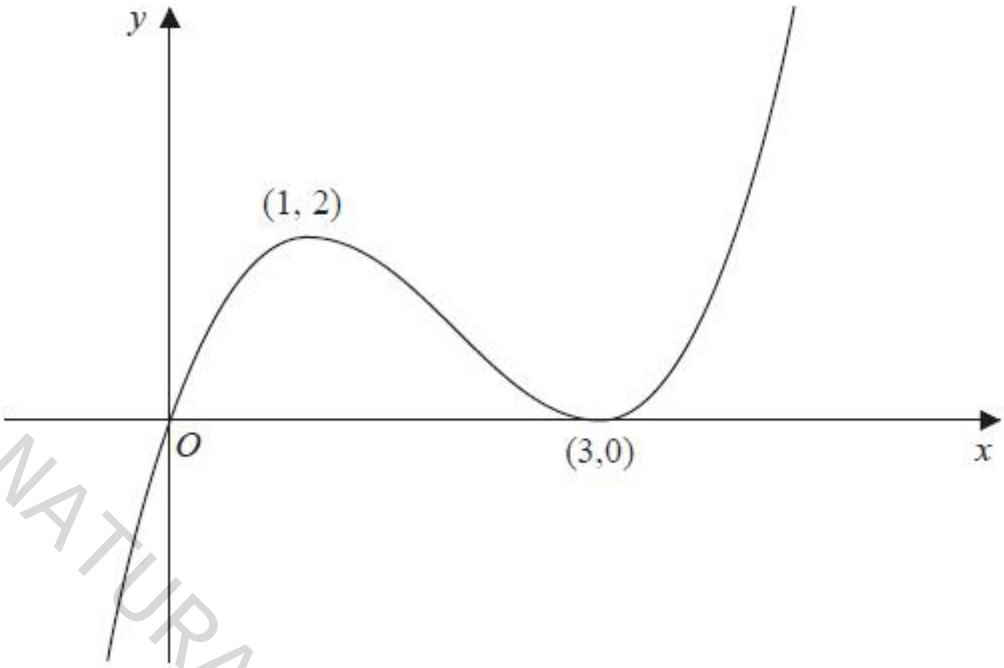


**Figure 1**



Figure 1 shows a sketch of the curve with equation $y = f(x)$, where $x \in \mathbb{R}$ and $f(x)$ is a polynomial.

The curve passes through the origin and touches the $x$-axis at the point (3, 0)

There is a maximum turning point at (1, 2) and a minimum turning point at (3, 0)

On separate diagrams, sketch the curve with equation

(i) $y = 3f(2x)$

**(3)**

(ii) $y = f(-x) - 1$

**(3)**

On each sketch, show clearly the coordinates of

- the point where the curve crosses the $y$-axis
- any maximum or minimum turning points

**(Total for question = 6 marks)**

(**Q02 WMA13/01, Jan 2021**)

**Extra space for working:**

**Q10.**

The functions f and g are defined by

$$f(x) = \frac{4x+6}{x-5} \qquad x \in \mathbb{R},\ x \neq 5$$

$$g(x) = 5 - 2x^2 \qquad x \in \mathbb{R},\ x \leqslant 0$$

(a) Solve the equation

$$fg(x) = 3$$

**(4)**

(b) Find $f^{-1}$

**(3)**

(c) Sketch and label, on the same axes, the curve with equation $y = g(x)$ and the curve with equation $y = g^{-1}(x)$ . Show on your sketch the coordinates of the points where each curve meets or cuts the coordinate axes.

**(3)**

**(Total for question = 10 marks)**

(**Q04 WMA13/01, June 2021**)

**Extra space for working:**

**Q11.**

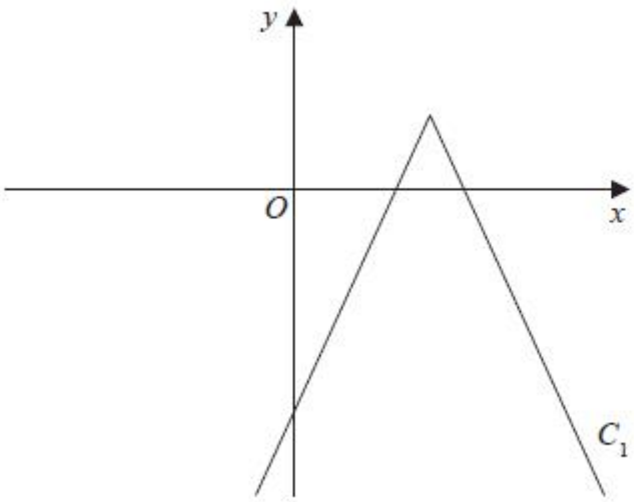


**Figure 3**

Figure 3 shows a sketch of the graph of $C_1$ with equation

$$y = 5 - |3x - 22|$$

(a) Write down the coordinates of

(i) the vertex of $C_1$
(ii) the intersection of $C_1$ with the $y$-axis.

**(2)**

(b) Find the x coordinates of the intersections of $C_1$ with the $x$-axis.

**(2)**

Diagram 1 is a copy of Figure 3.

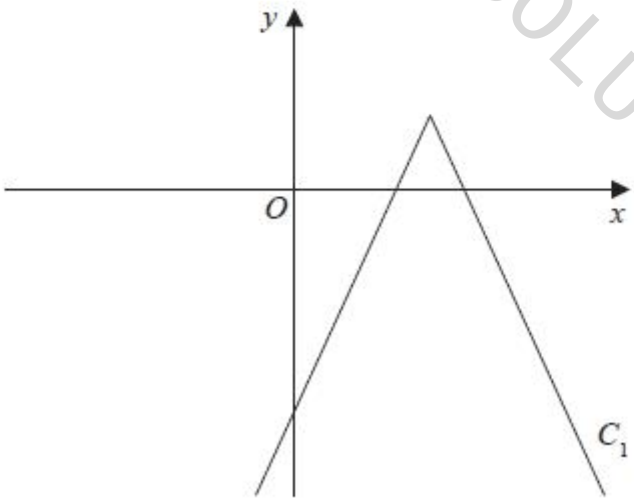


**Diagram 1**

(c) On Diagram 1, sketch the curve $C_2$ with equation

$$y = \frac{1}{9}x^2 - 9$$

Identify clearly the coordinates of any points of intersection of $C_2$ with the coordinate axes.

**(3)**

(d) Find the coordinates of the points of intersection of $C_1$ and $C_2$

(*Solutions relying entirely on calculator technology are not acceptable.*)

**(5)**

**(Total for question = 12 marks)**

(**Q07 WMA13/01, Oct 2022**)

**Extra space for working:**

**Q12.**

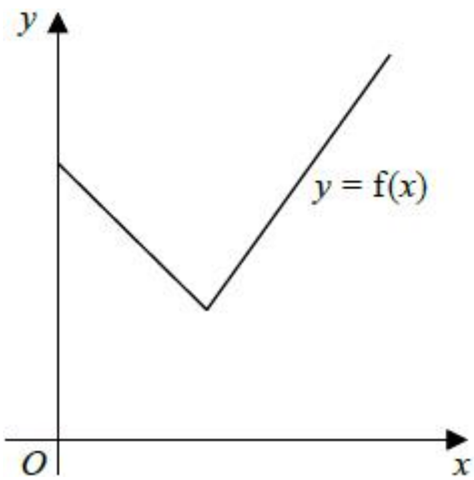


**Figure 1**

Figure 1 shows a sketch of part of the graph $y$ = f($x$) where

$$f(x) = 2|3 - x| + 5 \qquad x \geqslant 0$$

(a) Solve the equation

$$f(x) = \frac{1}{2}x + 30$$

**(3)**

Given that the equation f($x$) = $k$, where $k$ is a constant, has two distinct roots,

(b) state the set of possible values for $k$.

**(2)**

**(Total for question = 5 marks)**

(**Q03 WMA13/01, Specimen papers** )

**Extra space for working:**

**Q13.**

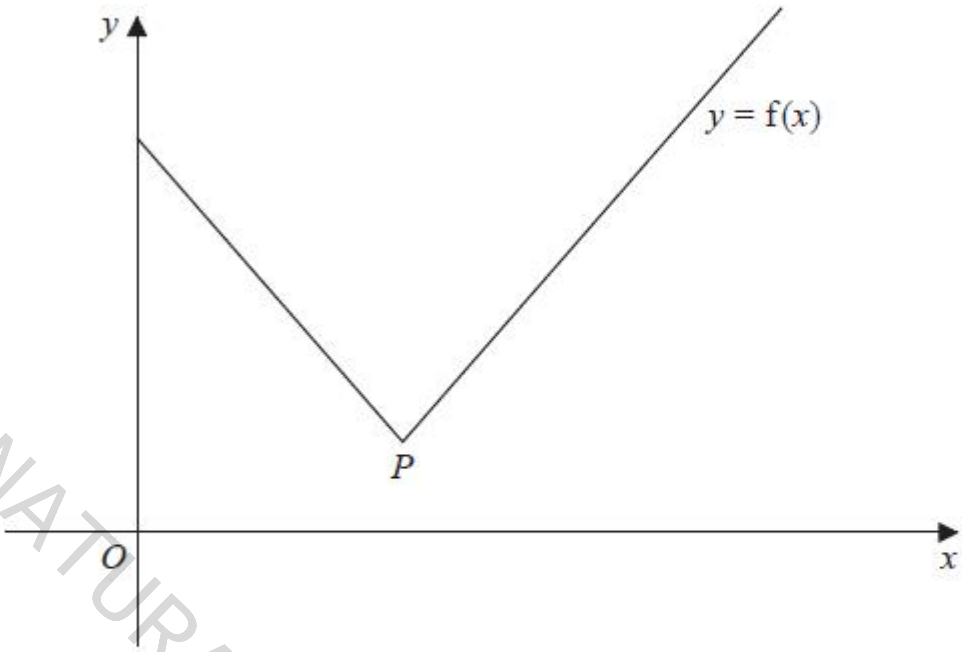


**Figure 2**

Figure 2 shows part of the graph with equation $y = \text{f}(x)$, where

$$\text{f}(x) = 2|2x - 5| + 3 \qquad x \geqslant 0$$

The vertex of the graph is at point $P$ as shown.

(a) State the coordinates of $P$.

**(2)**

(b) Solve the equation $\text{f}(x) = 3x - 2$

**(4)**

Given that the equation

$$\text{f}(x) = kx + 2$$

where $k$ is a constant, has exactly two roots,

(c) find the range of values of $k$.

**(3)**

**(Total for question = 9 marks)**

(**Q06 WMA13/01, Jan 2020**)

## Extra space for working:

**Q14.**

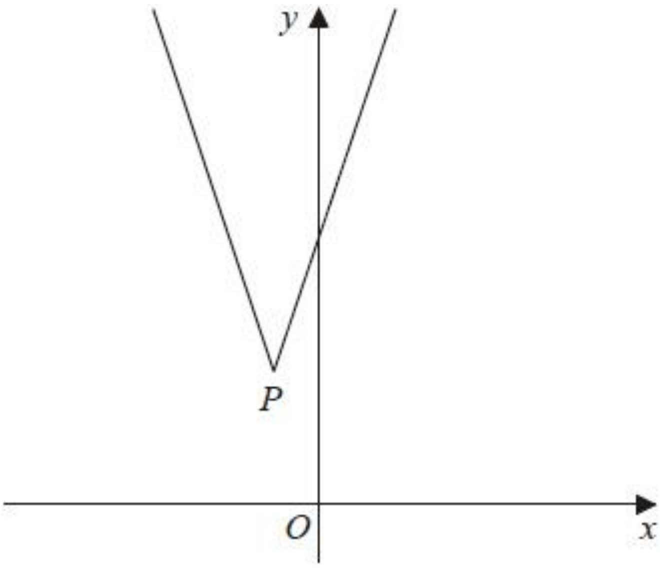


**Figure 2**

Figure 2 shows a sketch of the graph with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = |\,3x + a\,| + a$$

and where $a$ is a positive constant.

The graph has a vertex at the point $P$, as shown in Figure 2.

(a) Find, in terms of $a$, the coordinates of $P$.

**(2)**

(b) Sketch the graph with equation $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = |\,x + 5a\,|$$

On your sketch, show the coordinates, in terms of $a$, of each point where the graph cuts or meets the coordinate axes.

**(2)**

The graph with equation $y = \mathrm{g}(x)$ intersects the graph with equation $y = \mathrm{f}(x)$ at two points.

(c) Find, in terms of $a$, the coordinates of the two points.

**(5)**

**(Total for question = 9 marks)**

(**Q04 WMA13/01, Jan 2021**)

**Extra space for working:**

**Q15.**

Given that *k* is a positive constant,

(a) on separate diagrams, sketch the graph with equation

(i) $y = k - 2|x|$

(ii) $y = \left|2x - \dfrac{k}{3}\right|$

Show on each sketch the coordinates, in terms of *k*, of each point where the graph meets or cuts the axes.

**(4)**

(b) Hence find, in terms of *k*, the values of *x* for which

$$\left|2x - \frac{k}{3}\right| = k - 2|x|$$

giving your answers in simplest form.

**(4)**

**(Total for question = 8 marks)**

(**Q06 WMA13/01, June 2021**)

**Extra space for working:**

**Q16.**

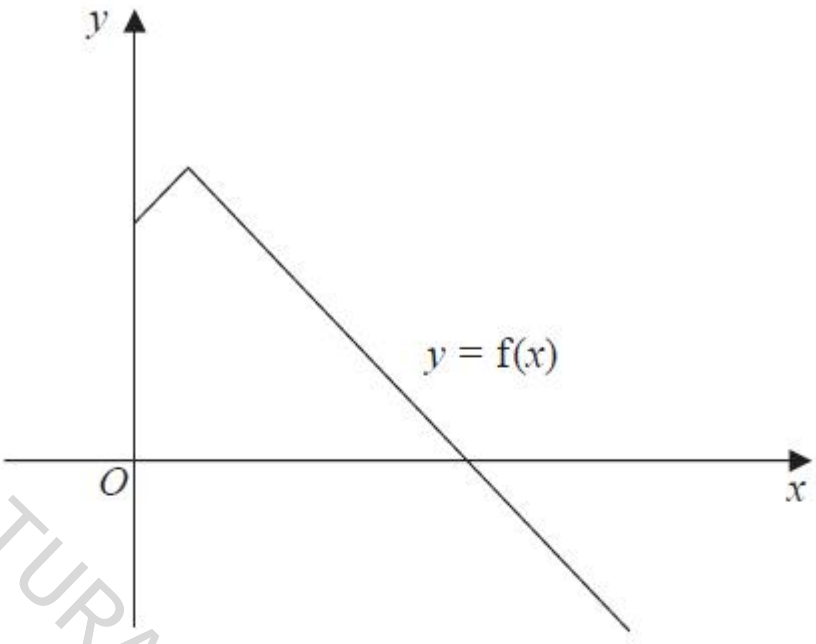


**Figure 2**

Figure 2 shows a sketch of part of the graph with equation y = f($x$) where

$$f(x) = 21 - 2|2 - x| \qquad x \geqslant 0$$

(a) Find ff(6)

**(2)**

(b) Solve the equation f($x$) = 5$x$

**(2)**

Given that the equation f($x$) = $k$ , where $k$ is a constant, has exactly two roots,

(c) state the set of possible values of $k$.

**(2)**

The graph with equation $y$ = f($x$) is transformed onto the graph with equation $y = a\,\mathrm{f}(x - b)$

The vertex of the graph with equation $y = a\,\mathrm{f}(x - b)$ is (6, 3).

Given that $a$ and $b$ are constants,

(d) find the value of $a$ and the value of $b$.

**(2)**

**(Total for question = 8 marks)**

(**Q04 WMA13/01, Oct 2020**)

**Extra space for working:**

Q17.

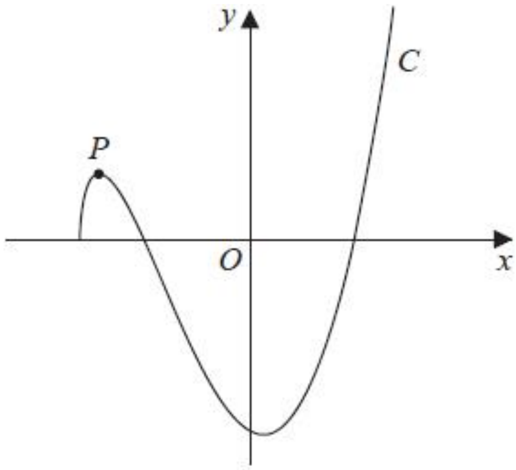


**Figure 3**

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

The function f is defined by

$$f(x) = 5(x^2 - 2)(4x + 9)^{\frac{1}{2}} \qquad x \geqslant -\frac{9}{4}$$

(a) Show that

$$f'(x) = \frac{k(5x^2 + 9x - 2)}{(4x + 9)^{\frac{1}{2}}}$$

where $k$ is an integer to be found.

**(4)**

(b) Hence, find the values of $x$ for which $f'(x) = 0$

**(1)**

Figure 3 shows a sketch of the curve $C$ with equation $y = f(x)$.

The curve has a local maximum at the point $P$

(c) Find the exact coordinates of $P$

**(2)**

The function g is defined by

$$g(x) = 2f(x) + 4 \qquad -\frac{9}{4} \leq x \leq 0$$

(d) Find the range of g

**(3)**

**(Total for question = 10 marks)**
(**Q06 WMA13/01, June 2022**)

**Extra space for working:**

**Q18.**

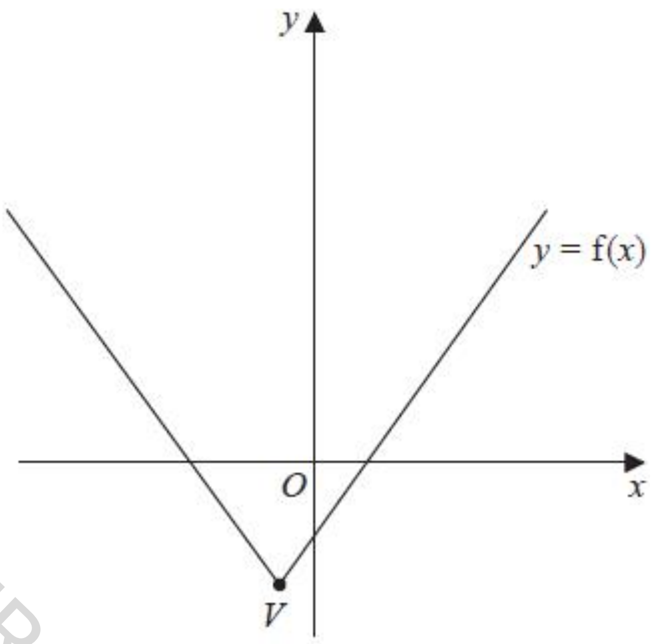


**Figure 2**

Figure 2 shows a sketch of part of the graph with equation $y = f(x)$, where

$$f(x) = \frac{1}{2}|2x + 7| - 10$$

(a) State the coordinates of the vertex, $V$, of the graph.

**(2)**

(b) Solve, using algebra,

$$\frac{1}{2}|2x + 7| - 10 \geqslant \frac{1}{3}x + 1$$

**(4)**

(c) Sketch the graph with equation

$$y = |f(x)|$$

stating the coordinates of the local maximum point and each local minimum point.

**(4)**

**(Total for question = 10 marks)**

(**Q07 WMA13/01, Jan 2022**)

**Extra space for working:**

**Q19.**

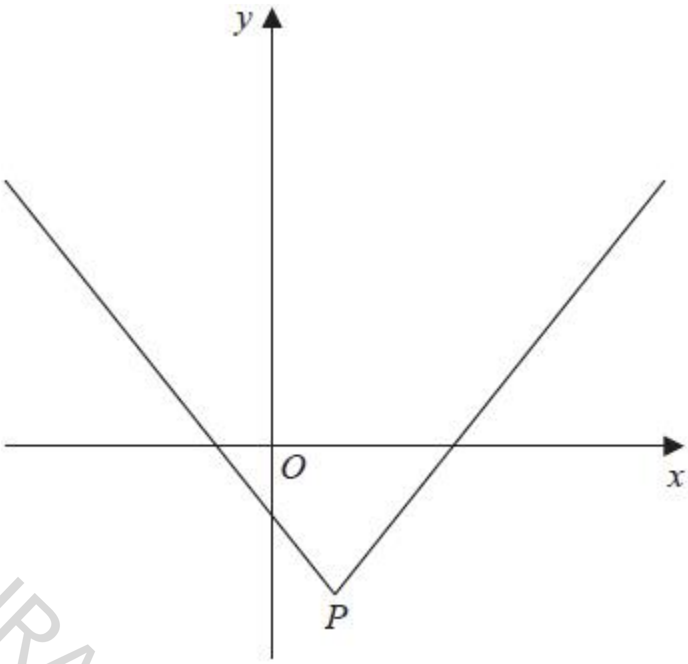


**Figure 2**

Figure 2 shows a sketch of the graph $y = f(x)$, where

$$f(x) = 3|x - 2| - 10$$

The vertex of the graph is at point $P$, shown in Figure 2.

(a) Find the coordinates of $P$

**(2)**

(b) Find ff(0)

**(2)**

(c) Solve the inequality

$$3|x - 2| - 10 < 5x + 10$$

**(2)**

(d) Solve the equation

$$f(|x|) = 0$$

**(3)**

**(Total for question = 9 marks)**

(**Q06 WMA13/01, June 2023**)

**Extra space for working:**

**Q20.**

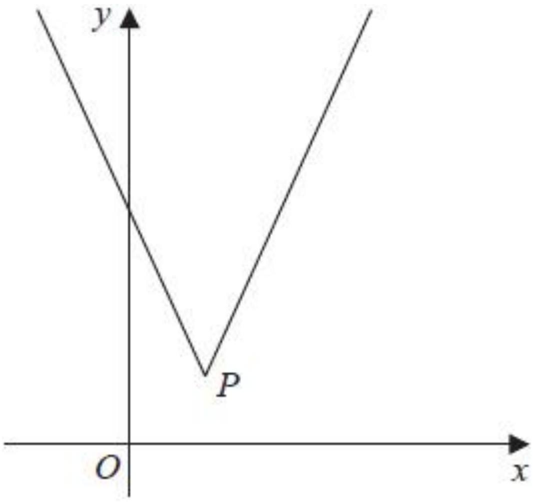


**Figure 1**

Figure 1 shows a sketch of part of the graph with equation $y = \text{f}(x)$, where

$$\text{f}(x) = |3x - 13| + 5 \qquad x \in \mathbb{R}$$

The vertex of the graph is at point *P*, as shown in Figure 1.

(a) State the coordinates of *P*.

**(2)**

(b) (i) State the range of f.

(ii) Find the value of ff(4)

**(2)**

(c) Solve, using algebra and showing your working,

$$16 - 2x > |3x - 13| + 5$$

**(4)**

The graph with equation $y = \text{f}(x)$ is transformed onto the graph with equation $y = a\text{f}(x + b)$

The vertex of the graph with equation $y = a\text{f}(x + b)$ is (4, 20)

Given that *a* and *b* are constants,

(d) find the value of *a* and the value of *b*.

**(2)**

**(Total for question = 10 marks)**

(**Q02 WMA13/01, Oct 2021**)

**Extra space for working:**

# Exponentials and Logarithms-(WMA13)

**Q1.**

In a controlled experiment, the number of microbes, $N$, present in a culture $T$ days after the start of the experiment were counted.

$N$ and $T$ are expected to satisfy a relationship of the form

$$N = aT^{b} \qquad \text{where } a \text{ and } b \text{ are constants}$$

(a) Show that this relationship can be expressed in the form

$$\log_{10}N = m\log_{10}T + c$$

giving $m$ and $c$ in terms of the constants $a$ and/or $b$.

**(2)**

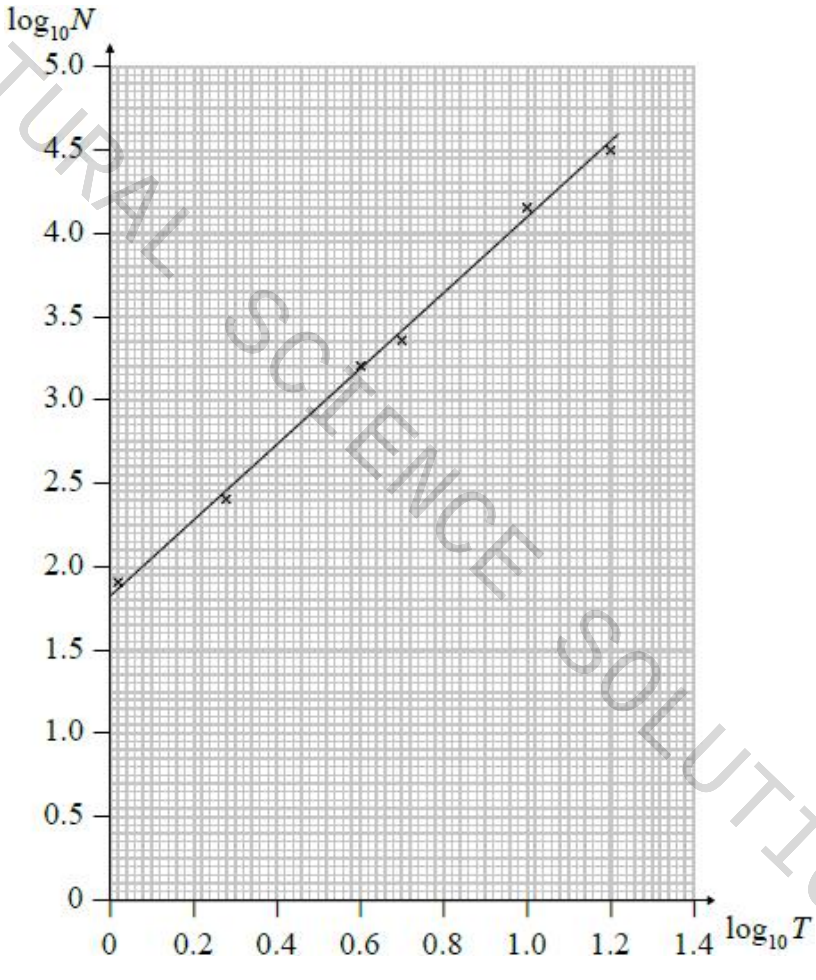




**Figure 2**

Figure 2 shows the line of best fit for values of $\log_{10}N$ plotted against values of $\log_{10}T$

(b) Use the information provided to estimate the number of microbes present in the culture 3 days after the start of the experiment.

**(4)**

(c) With reference to the model, interpret the value of the constant $a$.

**(1)**

**(Total for question = 7 marks)**

(**Q08 WMA13/01, Specimen papers** )

**Extra space for working:**

**Extra space for working:**

**Q2.**

The amount of an antibiotic in the bloodstream, from a given dose, is modelled by the formula

$$x = De^{-0.2t}$$

where $x$ is the amount of the antibiotic in the bloodstream in milligrams, $D$ is the dose given in milligrams and $t$ is the time in hours after the antibiotic has been given.

A first dose of 15 mg of the antibiotic is given.

(a) Use the model to find the amount of the antibiotic in the bloodstream 4 hours after the dose is given. Give your answer in mg to 3 decimal places.

**(2)**

A second dose of 15 mg is given 5 hours after the first dose has been given. Using the same model for the second dose,

(b) show that the total amount of the antibiotic in the bloodstream 2 hours after the second dose is given is 13.754 mg to 3 decimal places.

**(2)**

No more doses of the antibiotic are given. At time $T$ hours after the second dose is given, the total amount of the antibiotic in the bloodstream is 7.5 mg.

(c) Show that $T = a \ln\left(b + \frac{b}{e}\right)$, where $a$ and $b$ are integers to be determined.

**(4)**

**(Total for question = 8 marks)**

(**Q10 WMA13/01, Specimen papers** )

**Extra space for working:**

**Q3.**

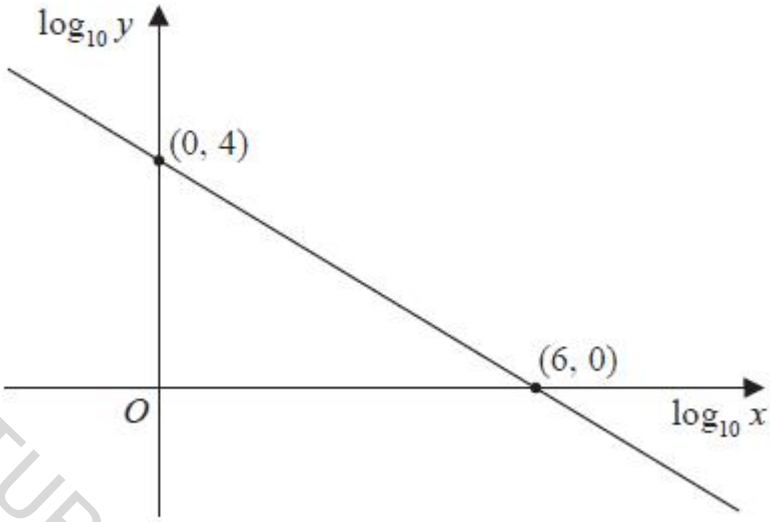


**Figure 1**

Figure 1 shows a linear relationship between $\log_{10}y$ and $\log_{10}x$

The line passes through the points (0, 4) and (6, 0) as shown.

(a) Find an equation linking $\log_{10}y$ with $\log_{10}x$

**(2)**

(b) Hence, or otherwise, express $y$ in the form $px^q$, where $p$ and $q$ are constants to be found.

**(3)**

**(Total for question = 5 marks)**

**(Q03 WMA13/01, Jan 2020)**

Accelerating the academic skills

**Extra space for working:**

**Q4.**

A scientist monitored the growth of bacteria on a dish over a 30-day period.

The area, $N$ mm$^2$, of the dish covered by bacteria, $t$ days after monitoring began, is modelled by the equation

$$\log_{10}N = 0.0646\,t + 1.478 \qquad 0 \le t \le 30$$

(a) Show that this equation may be written in the form

$$N = a\,b^t$$

where $a$ and $b$ are constants to be found. Give the value of $a$ to the nearest integer and give the value of $b$ to 3 significant figures.

**(4)**

(b) Use the model to find the area of the dish covered by bacteria 30 days after monitoring began. Give your answer, in mm$^2$, to 2 significant figures.

**(2)**

**(Total for question = 6 marks)**

(**Q02 WMA13/01, Oct 2020**)

**Extra space for working:**

**Q5.**

The percentage, $P$, of the population of a small country who have access to the internet, is modelled by the equation

$$P = ab^t$$

where $a$ and $b$ are constants and $t$ is the number of years after the start of 2005

Using the data for the years between the start of 2005 and the start of 2010, a graph is plotted of $\log_{10}P$ against $t$.

The points are found to lie approximately on a straight line with gradient 0.09 and intercept 0.68 on the $\log_{10}P$ axis.

(a) Find, according to the model, the value of $a$ and the value of $b$, giving your answers to 2 decimal places.

**(4)**

(b) In the context of the model, give a practical interpretation of the constant $a$.

**(1)**

(c) Use the model to estimate the percentage of the population who had access to the internet at the start of 2015

**(2)**

**(Total for question = 7 marks)**

(**Q08 WMA13/01, Jan 2021**)

**Extra space for working:**

**Q6.**

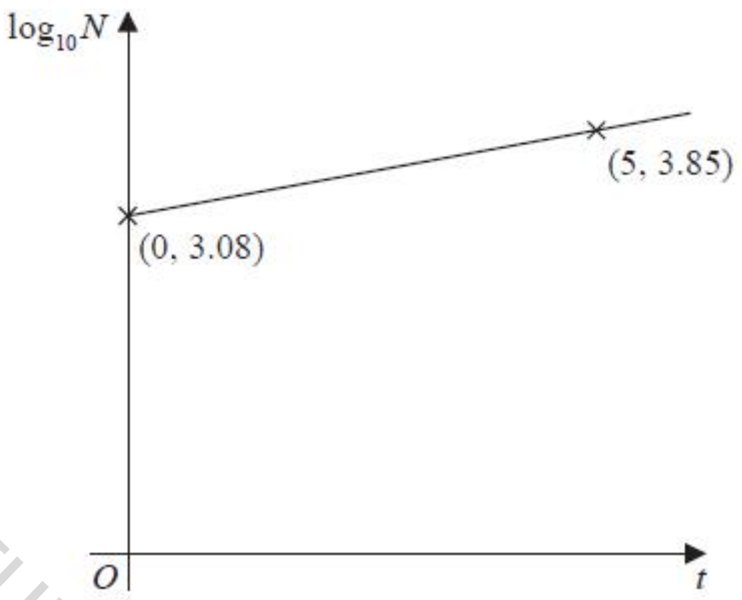


**Figure 1**

The number of subscribers to an online video streaming service, $N$, is modelled by the equation

$$N = ab^t$$

where $a$ and $b$ are constants and $t$ is the number of years since monitoring began.

The line in Figure 1 shows the linear relationship between $t$ and $\log_{10}N$

The line passes through the points (0, 3.08) and (5, 3.85)

Using this information,

(a) find an equation for this line.

**(2)**

(b) Find the value of $a$ and the value of $b$, giving your answers to 3 significant figures.

**(3)**

When $t = T$ the number of subscribers is 500 000

According to the model,

(c) find the value of $T$

**(2)**

**(Total for question = 7 marks)**

(**Q04 WMA13/01, June 2022**)

**Extra space for working:**

**Q7.**

The temperature, $\theta$ °C, inside an oven, $t$ minutes after the oven is switched on, is given by

$$\theta = A - 180e^{-kt}$$

where $A$ and $k$ are positive constants.

Given that the temperature inside the oven is initially 18 °C,

(a) find the value of $A$.

**(2)**

The temperature inside the oven, 5 minutes after the oven is switched on, is 90 °C.

(b) Show that $k = p \ln q$ where $p$ and $q$ are rational numbers to be found.

**(4)**

Hence find

(c) the temperature inside the oven 9 minutes after the oven is switched on, giving your answer to 3 significant figures,

**(2)**

(d) the rate of increase of the temperature inside the oven 9 minutes after the oven is switched on. Give your answer in °C $\text{min}^{-1}$ to 3 significant figures.

**(3)**

**(Total for question = 11 marks)**

(**Q05 WMA13/01, Jan 2021**)

**Extra space for working:**

**Extra space for working:**

**Q8.**

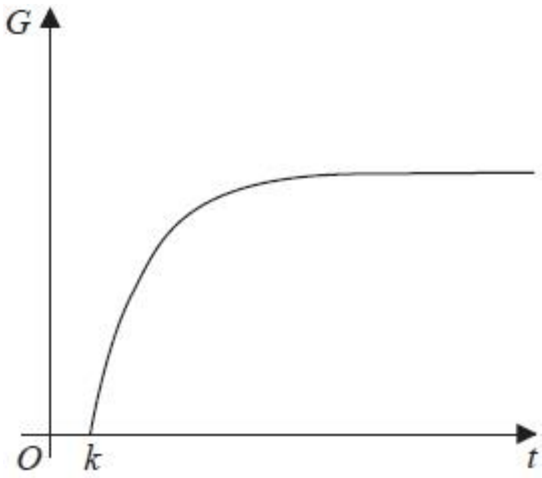


**Figure 2**

The total mass of gold, $G$ tonnes, extracted from a mine is modelled by the equation

$$G = 40 - 30e^{1-0.05t} \qquad t \geq k \qquad G \geq 0$$

where $t$ is the number of years after 1st January 1800.

Figure 2 shows a sketch of $G$ against $t$.

**Use the equation of the model to answer parts (a), (b) and (c).**

(a) (i) Find the value of $k$.

(ii) Hence find the year and month in which gold started being extracted from the mine.

**(3)**

(b) Find the total mass of gold extracted from the mine up to 1st January 1870.

**(2)**

There is a limit to the mass of gold that can be extracted from the mine.

(c) State the value of this limit.

**(1)**

**(Total for question = 6 marks)**

(**Q03 WMA13/01, Oct 2021**)

**Extra space for working:**

**Q9.**

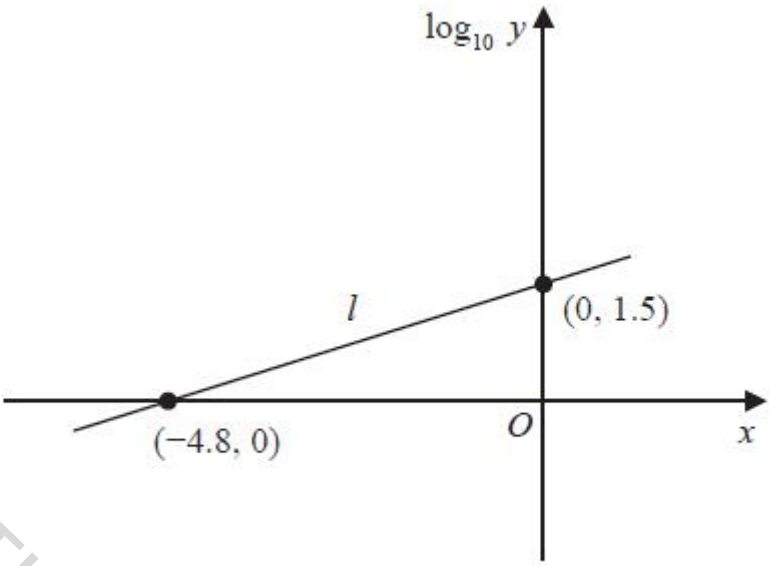


**Figure 1**

The line $l$ in Figure 1 shows a linear relationship between $\log_{10}y$ and $x$.

The line passes through the points (0, 1.5) and (−4.8, 0) as shown.

(a) Write down an equation for $l$.

**(2)**

(b) Hence, or otherwise, express $y$ in the form $kb^x$, giving the values of the constants $k$ and $b$ to 3 significant figures.

**(3)**

**(Total for question = 5 marks)**

(**Q03 WMA13/01, Jan 2023**)

**Extra space for working:**

**Q10.**

A dose of antibiotics is given to a patient.

The amount of the antibiotic, $x$ milligrams, in the patient's bloodstream $t$ hours after the dose was given, is found to satisfy the equation

$$\log_{10}x = 2.74 - 0.079t$$

(a) Show that this equation can be written in the form

$$x = pq^{-t}$$

where $p$ and $q$ are constants to be found. Give the value of $p$ to the nearest whole number and the value of $q$ to 2 significant figures.

**(4)**

(b) With reference to the equation in part (a), interpret the value of the constant $p$.

**(1)**

When a different dose of the antibiotic is given to another patient, the values of $x$ and $t$ satisfy the equation

$$x = 400 \times 1.4^{-t}$$

(c) Use calculus to find, to 2 significant figures, the value of $\frac{dx}{dt}$ when $t = 5$

**(3)**

**(Total for question = 8 marks)**

(**Q08 WMA13/01, Jan 2022**)

**Extra space for working:**

**Q11.**

**In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

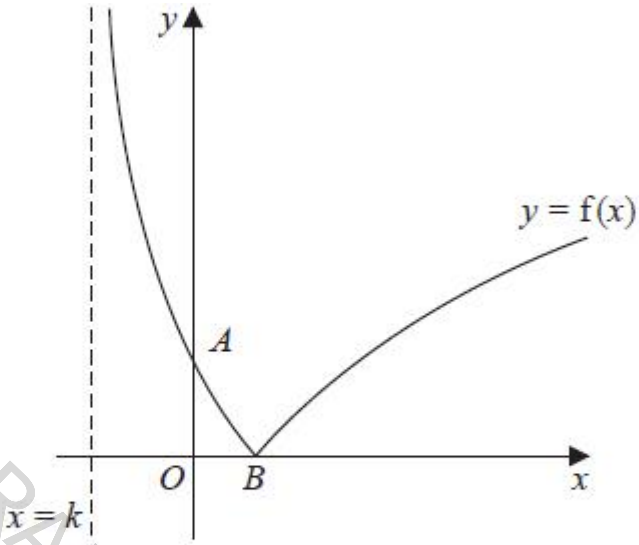


**Figure 2**

Figure 2 shows a sketch of the curve with equation

$$y = |2 - 4\ln(x + 1)| \qquad x > k$$

where *k* is a constant.

Given that the curve

- has an asymptote at $x = k$
- cuts the *y*-axis at point *A*
- meets the *x*-axis at point *B*

as shown in Figure 2,

(a) state the value of *k*

**(1)**

(b) (i) find the *y* coordinate of *A*

(ii) find the exact *x* coordinate of *B*

**(3)**

(c) Using algebra and showing your working, find the set of values of *x* such that

$$|2 - 4\ln(x + 1)| > 3$$

**(5)**

**(Total for question = 9 marks)**

**Extra space for working:**

**Extra space for working:**

**Q12.**

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

A population of fruit flies is being studied.

The number of fruit flies, $F$, in the population, $t$ days after the start of the study, is modelled by the equation

$$F = \frac{350e^{kt}}{9 + e^{kt}}$$

where $k$ is a constant.

**Use the equation of the model to answer parts (a), (b) and (c).**

(a) Find the number of fruit flies in the population at the start of the study.

**(1)**

Given that there are 200 fruit flies in the population 15 days after the start of the study,

(b) show that $k = \frac{1}{15}\ln 12$

**(3)**

Given also that, when $t = T$, the number of fruit flies in the population is increasing at a rate of 10 per day,

(c) find the possible values of $T$, giving your answers to one decimal place.

**(5)**

**(Total for question = 9 marks)**

(**Q10 WMA13/01, Jan 2023**)

**Extra space for working:**

**Q13.**

The functions f and g are defined by

$$f : x \mapsto e^x + 2 \qquad x \in \mathbb{R}$$

$$g : x \mapsto \ln x \qquad x > 0$$

(a) State the range of f.

(1)

(b) Find fg($x$), giving your answer in its simplest form.

(2)

(c) Find the exact value of $x$ for which f($2x + 3$) = 6

(4)

(d) Find $f^{-1}$ stating its domain.

(3)

(e) On the same axes sketch the curves with equation $y = f(x)$ and $y = f^{-1}(x)$, giving the coordinates of all the points where the curves cross the axes.

(4)

**(Total for question = 14 marks)**

(**Q06 WMA13/01, Specimen papers** )

**Extra space for working:**

**Q14.**

The function f and the function g are defined by

$$f(x) = \frac{12}{x+1} \qquad x > 0, x \in \mathbb{R}$$

$$g(x) = \frac{5}{2}\ln x \qquad x > 0, x \in \mathbb{R}$$

(a) Find, in simplest form, the value of $fg(e^2)$

**(2)**

(b) Find $f^{-1}$

**(3)**

(c) Hence, or otherwise, find all real solutions of the equation

$$f^{-1}(x) = f(x)$$

**(3)**

**(Total for question = 8 marks)**

(**Q02 WMA13/01, Jan 2020**)

**Extra space for working:**

**Q15.**

The amount of money raised for a charity is being monitored.

The total amount raised in the $t$ months after monitoring began, £$D$, is modelled by the equation

$$\log_{10} D = 1.04 + 0.38t$$

(a) Write this equation in the form

$$D = ab^t$$

where $a$ and $b$ are constants to be found. Give each value to 4 significant figures.

**(3)**

When $t = T$, the total amount of money raised is £45 000

According to the model,

(b) find the value of $T$, giving your answer to 3 significant figures.

**(2)**

The charity aims to raise a total of £350 000 within the first 12 months of monitoring.

According to the model,

(c) determine whether or not the charity will achieve its aim.

**(2)**

**(Total for question = 7 marks)**

(**Q03 WMA13/01, Jan 2024**)

**Extra space for working:**

**Q16.**

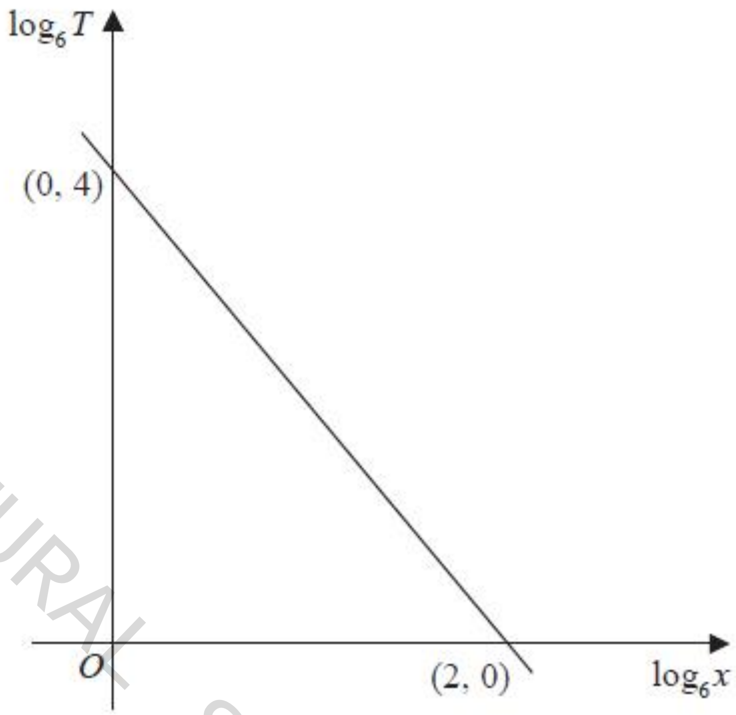


**Figure 1**

Figure 1 shows the linear relationship between $\log_6 T$ and $\log_6 x$

The line passes through the points (0, 4) and (2, 0) as shown.

(a) (i) Find an equation linking $\log_6 T$ and $\log_6 x$

(ii) Hence find the exact value of $T$ when $x = 216$

**(3)**

(b) Find an equation, not involving logs, linking $T$ with $x$

**(3)**

**(Total for question = 6 marks)**

(**Q02 WMA13/01, June 2023**)

**Extra space for working:**

**Extra space for working:**

**Q17.**

A population of a rare species of toad is being studied.

The number of toads, $N$, in the population, $t$ years after the start of the study, is modelled by the equation

$$N = \frac{900e^{0.12t}}{2e^{0.12t} + 1} \qquad t \geqslant 0, t \in \mathbb{R}$$

According to this model,

(a) calculate the number of toads in the population at the start of the study,

**(1)**

(b) find the value of $t$ when there are 420 toads in the population, giving your answer to 2 decimal places.

**(4)**

(c) Explain why, according to this model, the number of toads in the population can never reach 500

**(1)**

**(Total for question = 6 marks)**

(**Q01 WMA13/01, Jan 2020**)

**Extra space for working:**

**Q18.**

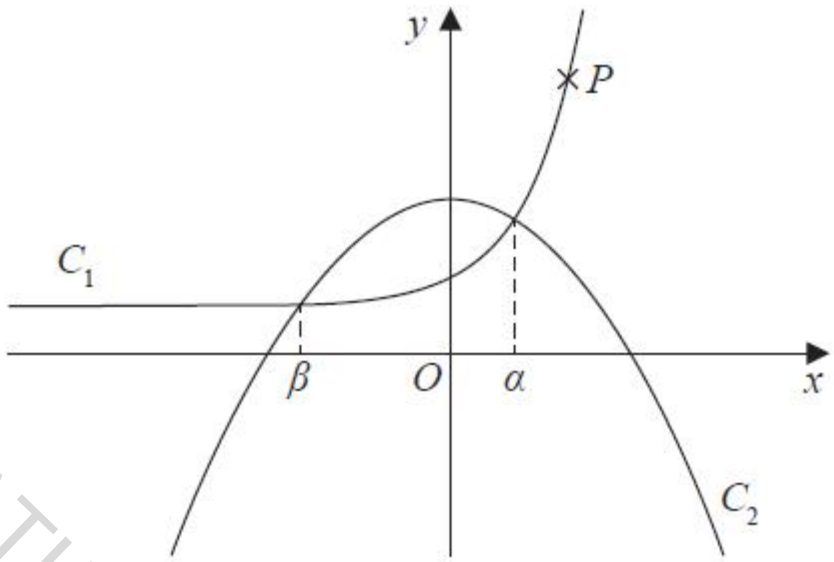


**Figure 3**



Figure 3 shows a sketch of curve $C_1$ with equation $y = 5e^{x-1} + 3$

and of curve $C_2$ with equation $y = 10 - x^2$

The point $P$ lies on $C_1$ and has $y$ coordinate 18

(a) Find the $x$ coordinate of $P$, writing your answer in the form $\ln k$, where $k$ is a constant to be found.

(3)

The curve $C_1$ meets the curve $C_2$ at $x = \alpha$ and at $x = \beta$, as shown in Figure 3.

(b) Using a suitable interval and a suitable function that should be stated, show that to 3 decimal places $\alpha = 1.134$

(3)

The iterative equation

$$x_{n+1} = -\sqrt{7 - 5e^{x_n - 1}}$$

is used to find an approximation to $\beta$.

Using this iterative formula with $x_1 = -3$

(c) find the value of $x_2$ and the value of $\beta$, giving each answer to 6 decimal places.

(3)

**(Total for question = 9 marks)**

(**Q06 WMA13/01, Oct 2020**)

**Extra space for working:**

# Trigonometry-(WMA13)

**Q1.**

**In this question you should show detailed reasoning.**

**Solutions relying entirely on calculator technology are not acceptable.**

(a) Show that the equation

$$2 \sin (\theta - 30°) = 5 \cos \theta$$

can be written in the form

$$\tan \theta = 2\sqrt{3}$$

**(4)**

(b) Hence, or otherwise, solve for $0 \leq x \leq 360°$

$$2 \sin (x - 10°) = 5 \cos (x + 20°)$$

giving your answers to one decimal place.

**(3)**

**(Total for question = 7 marks)**

(**Q04 WMA13/01, Oct 2021**)

**Extra space for working:**

**Q2.**

(a) Prove that

$$2\ \text{cosec}^2 2\theta\ (1 - \cos 2\theta) \equiv 1 + \tan^2\theta$$

**(4)**

(b) Hence solve for $0 < x < 360°$ , where $x \neq (90n)°$ , $n \in \mathbb{N}$ , the equation

$$2\ \text{cosec}^2 2x\ (1 - \cos 2x) = 4 + 3 \sec x$$

giving your answers to one decimal place.

(*Solutions relying entirely on calculator technology are not acceptable.*)

**(4)**

**(Total for question = 8 marks)**

(**Q08 WMA13/01, Oct 2023**)

**Extra space for working:**

**Q3.**

Solve, for $0 \leq x < 360°$, the equation

$$2 \cos 2x = 7 \cos x$$

giving your solution to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

**(Total for question = 5 marks)**

(**Q01 WMA13/01, Oct 2020**)

**Q4.**

(a) Express $\cos x + 4\sin x$ in the form $R\cos(x - \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of $R$ and give the value of $\alpha$, in radians, to 3 decimal places.

**(3)**

A scientist is studying the behaviour of seabirds in a colony.

She models the height above sea level, $H$ metres, of one of the birds in the colony by the equation

$$H = \frac{24}{3 + \cos\left(\frac{1}{2}t\right) + 4\sin\left(\frac{1}{2}t\right)} \qquad 0 \leqslant t \leqslant 6.5$$

where $t$ seconds is the time after it leaves the nest.

Find, according to the model,

(b) the minimum height of the seabird above sea level, giving your answer to the nearest cm,

**(2)**

(c) the value of $t$, to 2 decimal places, when $H = 10$

**(4)**

**(Total for question = 9 marks)**

(**Q07 WMA13/01, Oct 2020**)

**Extra space for working:**

**Q5.**

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

(a) Prove that

$$\frac{\sin 2x}{\cos x} + \frac{\cos 2x}{\sin x} \equiv \operatorname{cosec} x \qquad x \neq \frac{n\pi}{2} \quad n \in \mathbb{Z}$$

**(3)**

(b) Hence solve, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$$7 + \frac{\sin 4\theta}{\cos 2\theta} + \frac{\cos 4\theta}{\sin 2\theta} = 3\cot^2 2\theta$$

giving your answers in radians to 3 significant figures where appropriate.

**(6)**

**(Total for question = 9 marks)**

(**Q07 WMA13/01, Jan 2021**)

**Extra space for working:**

**Q6.**

(a) Show that

$$\frac{1-\cos 2x}{2\sin 2x} \equiv k\tan x \qquad x \neq (90n)^\circ \qquad n \in \mathbb{Z}$$

where *k* is a constant to be found.

**(3)**

(b) Hence solve, for $0 < \theta < 90°$

$$\frac{9(1-\cos 2\theta)}{2\sin 2\theta} = 2\sec^2\theta$$

giving your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

**(6)**

**(Total for question = 9 marks)**

(**Q02 WMA13/01, June 2021**)

**Extra space for working:**

**Q7.**

(a) Express 12 sin$x$ − 5 cos$x$ in the form $R$ sin($x$ − $\alpha$), where $R$ and $\alpha$ are constants, $R$ > 0 and $0 < \alpha < \frac{\pi}{2}$.
Give the exact value of $R$ and give the value of $\alpha$ in radians, to 3 decimal places.

**(3)**

The function g is defined by

$$g(\theta) = 10 + 12\sin\left(2\theta - \frac{\pi}{6}\right) - 5\cos\left(2\theta - \frac{\pi}{6}\right) \qquad \theta > 0$$

Find

(b) (i) the minimum value of g($\theta$)

(ii) the smallest value of $\theta$ at which the minimum value occurs.

**(3)**

The function h is defined by

$$h(\beta) = 10 - (12\sin\beta - 5\cos\beta)^2$$

(c) Find the range of h.

**(2)**

**(Total for question = 8 marks)**

(**Q09 WMA13/01, June 2021**)

**Q8.**

(a) Show that

$$\sin 3x \equiv 3\sin x - 4\sin^3 x$$

**(4)**

(b) Hence find, using algebraic integration,

$$\int_0^{\frac{\pi}{3}} \sin^3 x \, dx$$

**(4)**

**(Total for question = 8 marks)**

(**Q05 WMA13/01, Oct 2020**)

**Q9.**

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

(a) Show that

$$\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x} \equiv \operatorname{cosec} x \qquad x \neq \frac{n\pi}{2} \qquad n \in \mathbb{Z}$$

**(3)**

(b) Hence solve, for $0 < \theta < \frac{\pi}{2}$

$$\left(\frac{\cos 2\theta}{\sin \theta} + \frac{\sin 2\theta}{\cos \theta}\right)^2 = 6\cot\theta - 4$$

giving your answers to 3 significant figures as appropriate.

**(5)**

(c) Using the result from part (a), or otherwise, find the exact value of

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \left(\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x}\right)\cot x \, dx$$

**(2)**

**(Total for question = 10 marks)**

(**Q09 WMA13/01, June 2023**)

**Extra space for working:**

Accelerating the academic skills

**Q10.**

(a) Use the substitution $t = \tan x$ to show that the equation

$$12 \tan 2x + 5 \cot x \sec^2 x = 0$$

can be written in the form

$$5t^4 - 24t^2 - 5 = 0$$

**(4)**

(b) Hence solve, for $0 \leq x < 360°$, the equation

$$12 \tan 2x + 5 \cot x \sec^2 x = 0$$

Show each stage of your working and give your answers to one decimal place.

**(4)**

**(Total for question = 8 marks)**

(**Q05 WMA13/01, Jan 2020**)

**Extra space for working:**

**Q11.**

$$f(\theta) = 5\cos\theta - 4\sin\theta \qquad \theta \in \mathbb{R}$$

(a) Express f($\theta$) in the form $R$ cos ($\theta + \alpha$), where $R$ and $\alpha$ are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the exact value of $R$ and give the value of $\alpha$, in radians, to 3 decimal places.

**(3)**

The curve with equation $y = \cos\theta$ is transformed onto the curve with equation $y = f(\theta)$ by a sequence of two transformations.

Given that the first transformation is a stretch and the second a translation,

(b) (i) describe fully the transformation that is a stretch,

(ii) describe fully the transformation that is a translation.

**(2)**

Given

$$g(\theta) = \frac{90}{4 + (f(\theta))^2} \qquad \theta \in \mathbb{R}$$

(c) find the range of g.

**(2)**

**(Total for question = 7 marks)**

(**Q09 WMA13/01, Jan 2020**)

**Extra space for working:**

**Q12.**

**In this question you must show detailed reasoning.**

**Solutions relying entirely on calculator technology are not acceptable.**

(i) Solve, for $0 < x \leq \pi$, the equation

$$2\sec^2 x - 3\tan x = 2$$

giving the answers, as appropriate, to 3 significant figures.

**(4)**

(ii) Prove that

$$\frac{\sin 3\theta}{\sin \theta} - \frac{\cos 3\theta}{\cos \theta} \equiv 2$$

**(4)**

**(Total for question = 8 marks)**

(**Q09 WMA13/01, Jan 2022**)

**Q13.**

(a) Show that the equation

$$8 \cos \theta = 3 \operatorname{cosec} \theta$$

can be written in the form

$$\sin 2\theta = k$$

where $k$ is a constant to be found.

**(3)**

(b) Hence find the smallest positive solution of the equation

$$8 \cos \theta = 3 \operatorname{cosec} \theta$$

giving your answer, in degrees, to one decimal place.

**(2)**

**(Total for question = 5 marks)**

(**Q02 WMA13/01, Jan 2022**)

**Q14.**

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

Given that $\cos 2\theta - \sin 3\theta \neq 0$

(a) prove that

$$\frac{\cos^2\theta}{\cos 2\theta - \sin 3\theta} \equiv \frac{1+\sin\theta}{1-2\sin\theta-4\sin^2\theta}$$

**(4)**

(b) Hence solve, for $0 < \theta \leqslant 360°$

$$\frac{\cos^2\theta}{\cos 2\theta - \sin 3\theta} = 2\operatorname{cosec}\theta$$

Give your answers to one decimal place.

**(5)**

**(Total for question = 9 marks)**

(**Q09 WMA13/01, Oct 2022**)

**Extra space for working:**

**Q15.**

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

(a) Show that the equation

$$2\sin\theta\,(3\cot^2 2\theta - 7) = 13\sec\theta$$

can be written as

$$3\operatorname{cosec}^2 2\theta - 13\operatorname{cosec} 2\theta - 10 = 0$$

**(4)**

(b) Hence solve, for $0 < \theta < \frac{\pi}{2}$, the equation

$$2\sin\theta\,(3\cot^2 2\theta - 7) = 13\sec\theta$$

giving your answers to 3 significant figures.

**(4)**

**(Total for question = 8 marks)**

(**Q07 WMA13/01, June 2022**)

**Extra space for working:**

**Q16.**

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

(a) Show that the equation

$$\frac{3\sin\theta\cos\theta}{\cos\theta + \sin\theta} = (2 + \sec 2\theta)(\cos\theta - \sin\theta)$$

can be written in the form

$$3\sin 2\theta - 4\cos 2\theta = 2$$

**(3)**

(b) Hence solve for $\pi < x < \frac{3\pi}{2}$

$$\frac{3\sin x\cos x}{\cos x + \sin x} = (2 + \sec 2x)(\cos x - \sin x)$$

giving the answer to 3 significant figures.

**(5)**

**(Total for question = 8 marks)**

(**Q09 WMA13/01, Jan 2024**)

**Q17.**

(a) Using the identity for $\cos(A + B)$ , prove that

$$\cos 2A \equiv 2\cos^2 A - 1$$

**(2)**

(b) Hence, using algebraic integration, find the exact value of

$$\int_{\frac{\pi}{12}}^{\frac{\pi}{8}} (5 - 4\cos^2 3x)\,dx$$

**(4)**

**(Total for question = 6 marks)**

(**Q03 WMA13/01, Oct 2023**)

**Q18.**

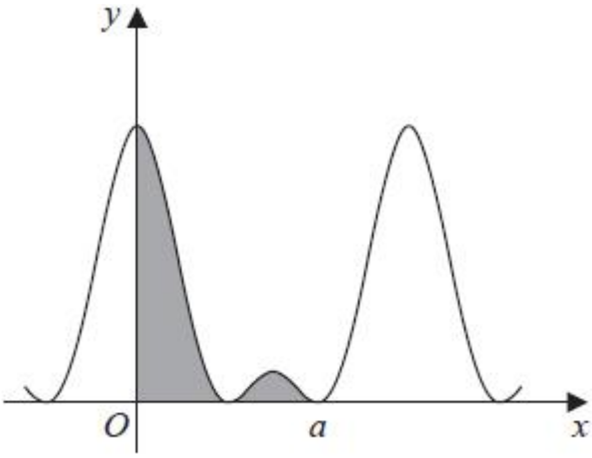


**Figure 4**

Figure 4 shows a sketch of part of the curve with equation

$$y = (1 + 2\cos 2x)^2$$

(a) Show that

$$(1 + 2\cos 2x)^2 \equiv p + q\cos 2x + r\cos 4x$$

where $p$, $q$ and $r$ are constants to be found.

**(2)**

The curve touches the positive $x$-axis for the second time when $x = a$, as shown in Figure 4.

The regions bounded by the curve, the $y$-axis and the $x$-axis up to $x = a$ are shown shaded in Figure 4.

(b) Find, using algebraic integration and making your method clear, the exact total area of the shaded regions. Write your answer in simplest form.

**(5)**

**(Total for question = 7 marks)**

(**Q10 WMA13/01, Oct 2021**)

**Extra space for working:**

**Q19.**

$$f(x) = \cos x + 2\sin x$$

(a) Express f(*x*) in the form $R\cos(x - \alpha)$, where *R* and *α* are constants,

$R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of *R* and give the value of *α*, in radians, to 3 decimal places.

**(3)**

$$g(x) = 3 - 7f(2x)$$

(b) Using the answer to part (a),

(i) write down the exact maximum value of g(*x*),

(ii) find the smallest positive value of *x* for which this maximum value occurs, giving your answer to 2 decimal places.

**(3)**

**(Total for question = 6 marks)**

(**Q02 WMA13/01, Jan 2023**)

**Extra space for working:**

**Q20.**

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

(i) Solve, for $0 < x < \pi$

$$(x-2)\left(\sqrt{3}\sec x + 2\right) = 0$$

**(3)**

(ii) Solve, for $0 < \theta < 360°$

$$10 \sin\theta = 3 \cos 2\theta$$

**(4)**

**(Total for question = 7 marks)**

(**Q05 WMA13/01, June 2023**)

**Q21.**

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

(a) Express 8 sin $x$ − 15 cos $x$ in the form $R$ sin ($x$ − α) , where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of $R$, and give the value of $\alpha$, in radians, to 4 significant figures.

**(3)**

$$f(x) = \frac{15}{41 + 16\sin x - 30\cos x} \qquad x > 0$$

(b) Find

(i) the minimum value of f($x$)

(ii) the smallest value of $x$ at which this minimum value occurs.

**(4)**

(c) State the $y$ coordinate of the minimum points on the curve with equation

$$y = 2f(x) - 5 \qquad x > 0$$

**(1)**

(d) State the smallest value of $x$ at which a maximum point occurs for the curve with equation

$$y = -f(2x) \qquad x > 0$$

**(1)**

**(Total for question = 9 marks)**

(**Q08 WMA13/01, Oct 2022**)

Accelerating the academic skills

**Extra space for working:**

**Q22.**

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

(a) Prove that

$$\cot^2 x - \tan^2 x \equiv 4\cot 2x \operatorname{cosec} 2x \qquad x \neq \frac{n\pi}{2} \quad n \in \mathbb{Z}$$

**(4)**

(b) Hence solve, for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$$4\cot 2\theta \operatorname{cosec} 2\theta = 2\tan^2 \theta$$

giving your answers to 2 decimal places.

**(5)**

**(Total for question = 9 marks)**

(**Q05 WMA13/01, Jan 2023**)

**Extra space for working:**

# Differentiation-(WMA13)

**Q1.**

The function f is defined by

$$f(x) = \frac{5x}{x^2 + 7x + 12} + \frac{5x}{x + 4} \qquad x > 0$$

(a) Show that f($x$) = $\frac{5x}{x + 3}$

(3)

(b) Find $f^{-1}$

(3)

(c) (i) Find, in simplest form, f′($x$).

(ii) Hence, state whether f is an increasing or a decreasing function, giving a reason for your answer.

(3)

**(Total for question = 9 marks)**

(**Q01 WMA13/01, Oct 2021**)

**Extra space for working:**

## Q2.

Given that

$$x = 6\sin^2 2y \qquad 0 < y < \frac{\pi}{4}$$

show that

$$\frac{dy}{dx} = \frac{1}{A\sqrt{(Bx - x^2)}}$$

where *A* and *B* are integers to be found.

**(Total for question = 5 marks)**

(**Q07 WMA13/01, June 2021**)

**Q3.**

The point *P* lies on the curve with equation

$$x = (4y - \sin 2y)^2$$

Given that *P* has (*x*, *y*) coordinates $\left(p, \frac{\pi}{2}\right)$, where *p* is a constant,

(a) find the exact value of *p*

(1)

The tangent to the curve at *P* cuts the *y*-axis at the point *A*.

(b) Use calculus to find the coordinates of *A*.

(6)

**(Total for question = 7 marks)**

(**Q07 WMA13/01, Specimen papers** )

Accelerating the academic skills

**Extra space for working:**

**Q4.**

(i)

$$f(x) = \frac{(2x+5)^2}{x-3} \qquad x \neq 3$$

(a) Find f′(*x*) in the form $\frac{P(x)}{Q(x)}$ where *P*(*x*) and *Q*(*x*) are fully factorised quadratic expressions.
(b) Hence find the range of values of *x* for which f(*x*) is increasing.

**(6)**

(ii)

$$g(x) = x\sqrt{\sin 4x} \qquad 0 \leqslant x < \frac{\pi}{4}$$

The curve with equation *y* = g(*x*) has a maximum at the point *M*.
Show that the *x* coordinate of *M* satisfies the equation

$$\tan 4x + kx = 0$$

where *k* is a constant to be found.

**(5)**

**(Total for question = 11 marks)**

(**Q04 WMA13/01, Jan 2020**)

Accelerating the academic skills

**Extra space for working:**

**Q5.**

(i) The curve $C$ has equation $y = \mathrm{g}(x)$ where

$$\mathrm{g}(x) = \mathrm{e}^{3x} \sec 2x \qquad -\frac{\pi}{4} < x < \frac{\pi}{4}$$

(a) Find g'($x$)

**(2)**

(b) Hence find the $x$ coordinate of the stationary point of $C$.

**(3)**

(ii) A different curve has equation

$$x = \ln(\sin y) \qquad 0 < y < \frac{\pi}{2}$$

Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{e}^x}{\mathrm{f}(x)}$$

where f($x$) is a function of $\mathrm{e}^x$ that should be found.

**(4)**

**(Total for question = 9 marks)**

(**Q08 WMA13/01, Oct 2020**)

**Extra space for working:**

**Q6.**

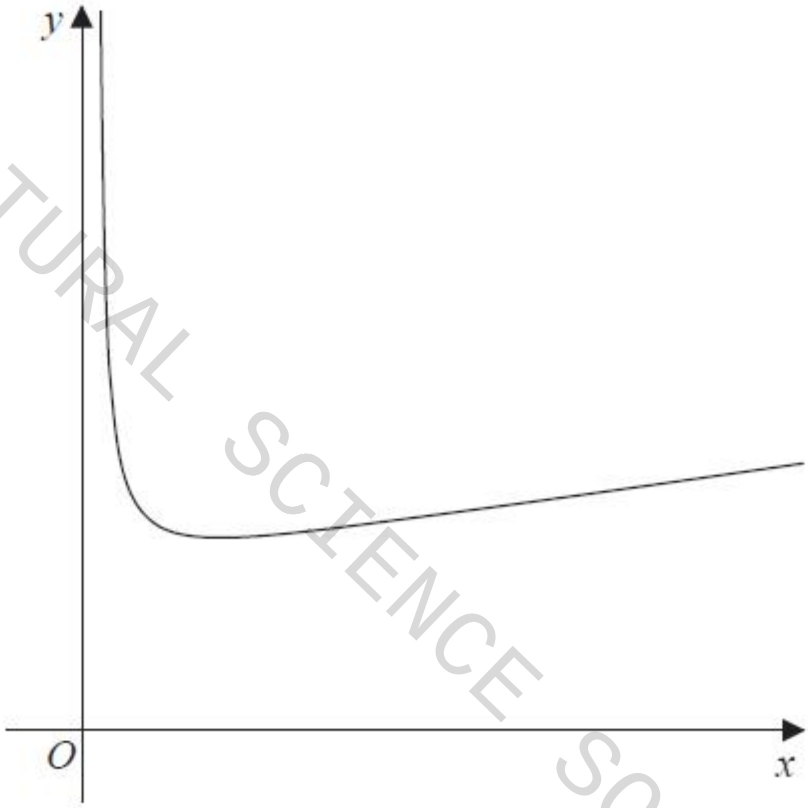


**Figure 1**



Figure 1 shows a sketch of a curve with equation $y = f(x)$ where

$$f(x) = \frac{2x+3}{\sqrt{4x-1}} \qquad x > \frac{1}{4}$$

(a) Find, in simplest form, $f'(x)$.

**(4)**

(b) Hence find the range of f.

**(3)**

**(Total for question = 7 marks)**

(**Q03 WMA13/01, Oct 2020**)

**Extra space for working:**

**Q7.**

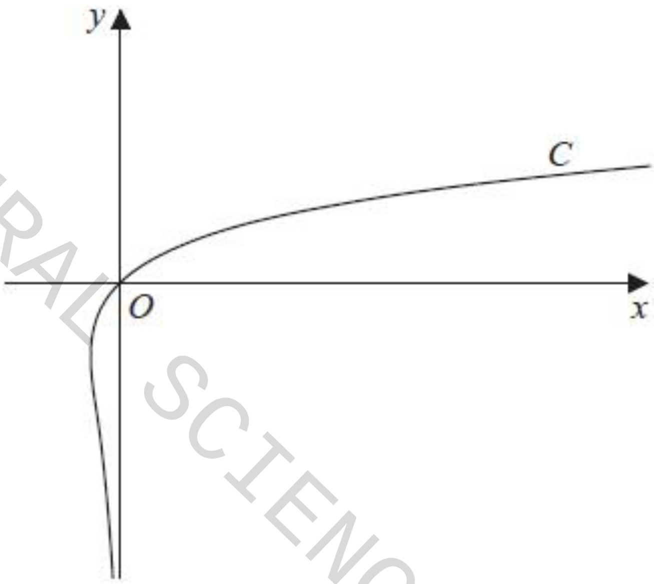


**Figure 3**

Figure 3 shows a sketch of the curve *C* with equation

$$x = ye^{2y} \qquad y \in \mathbb{R}$$

(a) Show that

$$\frac{dy}{dx} = \frac{y}{x(1+2y)}$$

**(4)**

Given that the straight line with equation *x* = *k*, where *k* is a constant, cuts *C* at exactly two points,

(b) find the range of possible values for *k*.

**(3)**

**(Total for question = 7 marks)**

(**Q10 WMA13/01, Jan 2022**)

**Extra space for working:**

**Q8.**

Find, using calculus, the $x$ coordinate of the stationary point on the curve with equation

$$y = (2x + 5)e^{3x}$$

**(Total for question = 4 marks)**

(**Q01 WMA13/01, Jan 2022**)

**Q9.**

A curve $C$ has equation $y = f(x)$, where

$$f(x) = \arcsin\left(\frac{1}{2}x\right) \qquad -2 \leqslant x \leqslant 2 \qquad -\frac{\pi}{2} \leqslant y \leqslant \frac{\pi}{2}$$

(a) Sketch $C$.

**(1)**

(b) Given $x = 2 \sin y$, show that

$$\frac{dy}{dx} = \frac{1}{\sqrt{A - x^2}}$$

where $A$ is a constant to be found.

**(3)**

The point $P$ lies on $C$ and has $y$ coordinate $\frac{\pi}{4}$

(c) Find the equation of the tangent to $C$ at $P$. Write your answer in the form $y = mx + c$, where $m$ and $c$ are constants to be found.

**(3)**

**(Total for question = 7 marks)**

(**Q08 WMA13/01, Oct 2021**)

**Extra space for working:**

**Q10.**

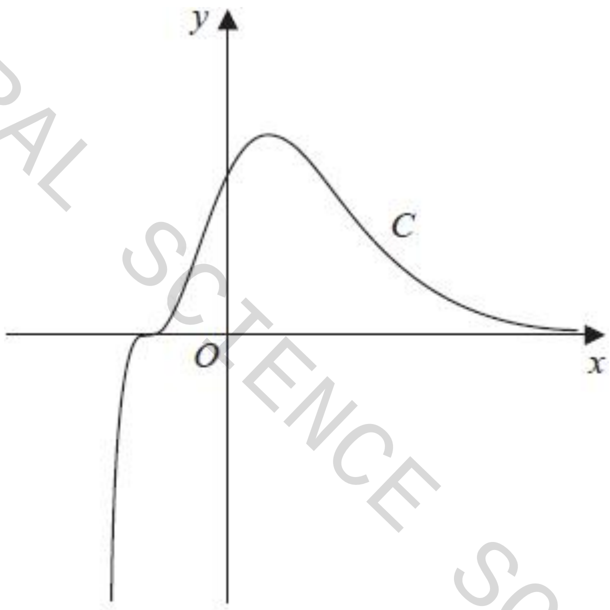


**Figure 3**



Figure 3 shows a sketch of the curve $C$ with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (2x + 1)^3 \mathrm{e}^{-4x}$$

(a) Show that

$$\mathrm{f}'(x) = A(2x + 1)^2 (1 - 4x)\, \mathrm{e}^{-4x}$$

where $A$ is a constant to be found.

**(4)**

(b) Hence find the exact coordinates of the two stationary points on $C$.

**(3)**

The function g is defined by

$$\mathrm{g}(x) = 8\mathrm{f}(x - 2)$$

(c) Find the coordinates of the maximum stationary point on the curve with equation $y = \mathrm{g}(x)$ .

(2)

**(Total for question = 9 marks)**

(**Q08 WMA13/01, June 2023**)

**Extra space for working:**

**Q11.**

The curve *C* has equation

$$y = (3x - 2)^6$$

(a) Find $\frac{dy}{dx}$

**(2)**

Given that the point $P\left(\frac{1}{3}, 1\right)$ lies on *C*,

(b) find the equation of the normal to *C* at *P*. Write your answer in the form $ax + by + c = 0$ where *a*, *b* and *c* are integers to be found.

**(4)**

**(Total for question = 6 marks)**

(**Q01 WMA13/01, June 2022**)

**Q12.**

$$y = \log_{10}(2x + 1)$$

(a) Express $x$ in terms of $y$.

**(2)**

(b) Hence, giving your answer in terms of $x$, find $\frac{dy}{dx}$

**(3)**

**(Total for question = 5 marks)**

(**Q04 WMA13/01, Oct 2022**)

**Q13.**

The curve $C$ has equation

$$x = 3\tan\left(y - \frac{\pi}{6}\right) \qquad x \in \mathbb{R} \quad -\frac{\pi}{3} < y < \frac{2\pi}{3}$$

(a) Show that

$$\frac{dy}{dx} = \frac{a}{x^2 + b}$$

where $a$ and $b$ are integers to be found.

**(4)**

The point $P$ with $y$ coordinate $\frac{\pi}{3}$ lies on $C$

Given that the tangent to $C$ at $P$ crosses the $x$-axis at the point $Q$.

(b) find, in simplest form, the exact $x$ coordinate of $Q$.

**(5)**

**(Total for question = 9 marks)**

(**Q07 WMA13/01, Jan 2023**)

**Extra space for working:**

**Q14.**

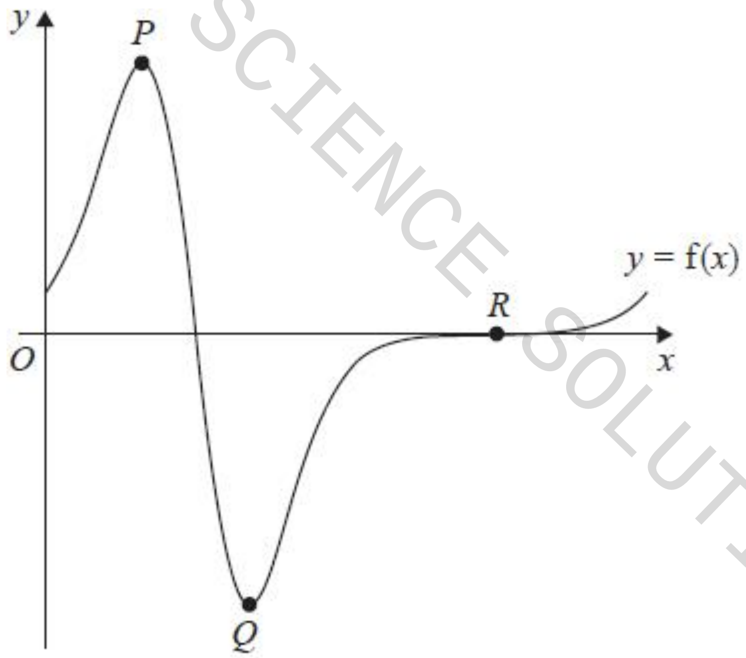


**Figure 1**

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

Figure 1 shows a sketch of the curve with equation $y = f(x)$, where

$$f(x) = 2e^{3\sin x}\cos x \qquad 0 \leq x \leq 2\pi$$

The curve intersects the $x$-axis at point $R$, as shown in Figure 1.

(a) State the coordinates of *R*

**(1)**

The curve has two turning points, at point *P* and point *Q*, also shown in Figure 1.

(b) Show that, at points *P* and *Q*,

$$a\sin^2 x + b\sin x + c = 0$$

where *a*, *b* and *c* are integers to be found.

**(4)**

(c) Hence find the *x* coordinate of point *Q*, giving your answer to 3 decimal places.

**(2)**

**(Total for question = 7 marks)**

(**Q06 WMA13/01, Jan 2024**)



**Extra space for working:**

**Q15.**

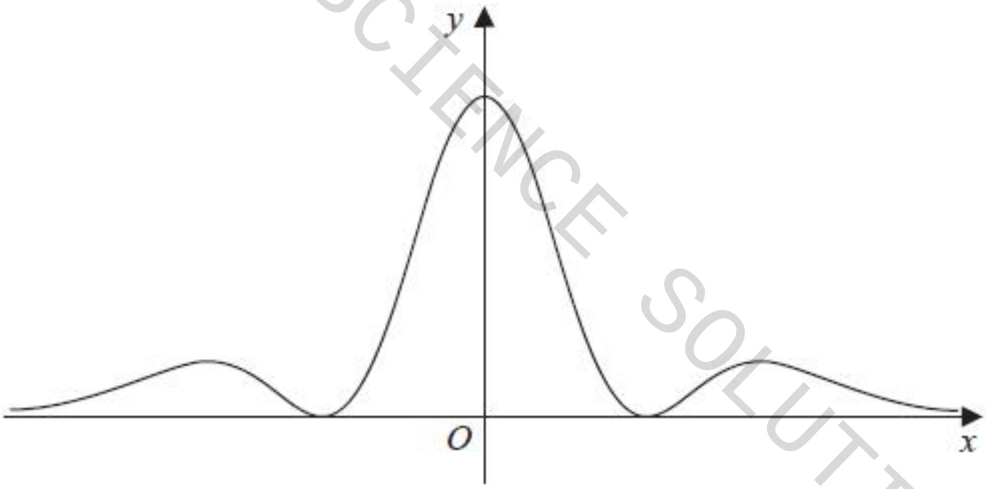


**Figure 1**

Figure 1 shows a sketch of the curve $C$ with equation $y = f(x)$ where

$$f(x) = e^{-x^2}(2x^2 - 3)^2$$

(a) Find the range of f

**(2)**

(b) Show that

$$f'(x) = 2x(2x^2 - 3)e^{-x^2}(A - Bx^2)$$

where $A$ and $B$ are constants to be found.

**(4)**

Given that the line $y = k$, where $k$ is a constant, $k > 0$, intersects the curve at exactly two distinct points,

(c) find the exact range of values of $k$

**(4)**

**(Total for question = 10 marks)**

(**Q07 WMA13/01, Oct 2023**)



**Extra space for working:**

**Q16.**

The curve $C$ has equation

$$y = \frac{\ln(x^2 + k)}{x^2 + k} \qquad x \in \mathbb{R}$$

where $k$ is a positive constant.

(a) Show that

$$\frac{dy}{dx} = \frac{Ax(B - \ln(x^2 + k))}{(x^2 + k)^2}$$

where $A$ and $B$ are constants to be found.

**(3)**

Given that $C$ has exactly three turning points,

(b) find the $x$ coordinate of each of these points. Give your answer in terms of $k$ where appropriate.

**(3)**

(c) find the upper limit to the value for $k$.

**(1)**

**(Total for question = 7 marks)**

(**Q05 WMA13/01, Oct 2023**)

**Extra space for working:**

**Q17.**

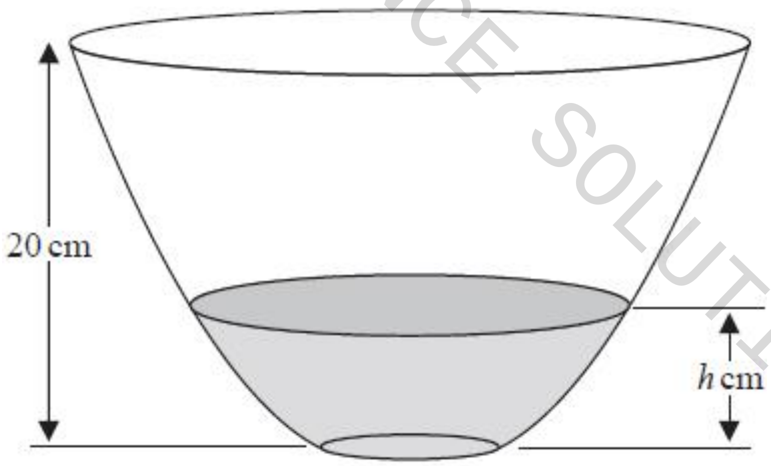


**Figure 2**

A bowl with circular cross section and height 20 cm is shown in Figure 2.

The bowl is initially empty and water starts flowing into the bowl.

When the depth of water is $h$ cm, the volume of water in the bowl, $V$ cm$^3$, is modelled by the equation

$V = \frac{1}{3}h^2(h + 4) \qquad 0 \leq h \leq 20$

Given that the water flows into the bowl at a constant rate of 160 $cm^3 s^{-1}$, find, according to the model,

(a) the time taken to fill the bowl,

**(2)**

(b) the rate of change of the depth of the water, in $cm s^{-1}$, when $h = 5$

**(5)**

**(Total for question = 7 marks)**

(**Q03 WMA14/01, June 2021**)



**Extra space for working:**

**Q18.**

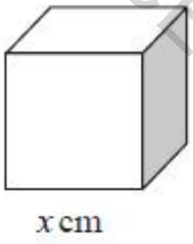


**Figure 1**

Figure 1 shows a cube which is increasing in size.

At time *t* seconds,

- the length of each edge of the cube is $x$ cm
- the surface area of the cube is $S$ $cm^2$
- the volume of the cube is $V$ $cm^3$

Given that the surface area of the cube is increasing at a constant rate of 4 $cm^2$ $s^{-1}$

(a) show that $\frac{dx}{dt} = \frac{k}{x}$ where $k$ is a constant to be found,

**(4)**

(b) show that $\frac{dV}{dt} = V^p$ where $p$ is a constant to be found.

**(3)**

**(Total for question = 7 marks)**

(**Q02 WMA14/01, Oct 2023**)



**Extra space for working:**

**Q19.**

Given that

$$y = \frac{5x^2 - 10x + 9}{(x-1)^2} \qquad x \neq 1$$

show that $\frac{dy}{dx} = \frac{k}{(x-1)^3}$, where *k* is a constant to be found.

**(Total for question = 6 marks)**

(**Q05 WMA13/01, Specimen papers** )

**Q20.**

The function f is defined by

$$f(x) = \frac{2x^2 - 32}{3x^2 + 7x - 20} + \frac{8}{3x - 5} \qquad x \in \mathbb{R} \quad x > 2$$

(a) Show that $f(x) = \frac{2x}{3x-5}$

**(3)**

(b) Show, using calculus, that f is a decreasing function.

You must make your reasoning clear.

**(3)**

The function g is defined by

$$g(x) = 3 + 2 \ln x \qquad x \geq 1$$

(c) Find $g^{-1}$

**(3)**

(d) Find the exact value of *a* for which

$$gf(a) = 5$$

**(4)**

**(Total for question = 13 marks)**

(**Q04 WMA13/01, Jan 2024**)



**Extra space for working:**

**Q21.**

(a) Given that

$$\frac{x^4 - x^3 - 10x^2 + 3x - 9}{x^2 - x - 12} \equiv x^2 + P + \frac{Q}{x - 4} \qquad x > -3$$

find the value of the constant $P$ and show that $Q = 5$

**(4)**

The curve $C$ has equation $y = \text{g}(x)$, where

$$\text{g}(x) = \frac{x^4 - x^3 - 10x^2 + 3x - 9}{x^2 - x - 12} \qquad -3 < x < 3.5 \qquad x \in \mathbb{R}$$

(b) Find the equation of the tangent to $C$ at the point where $x = 2$
Give your answer in the form $y = mx + c$ , where $m$ and $c$ are constants to be found.

(5)

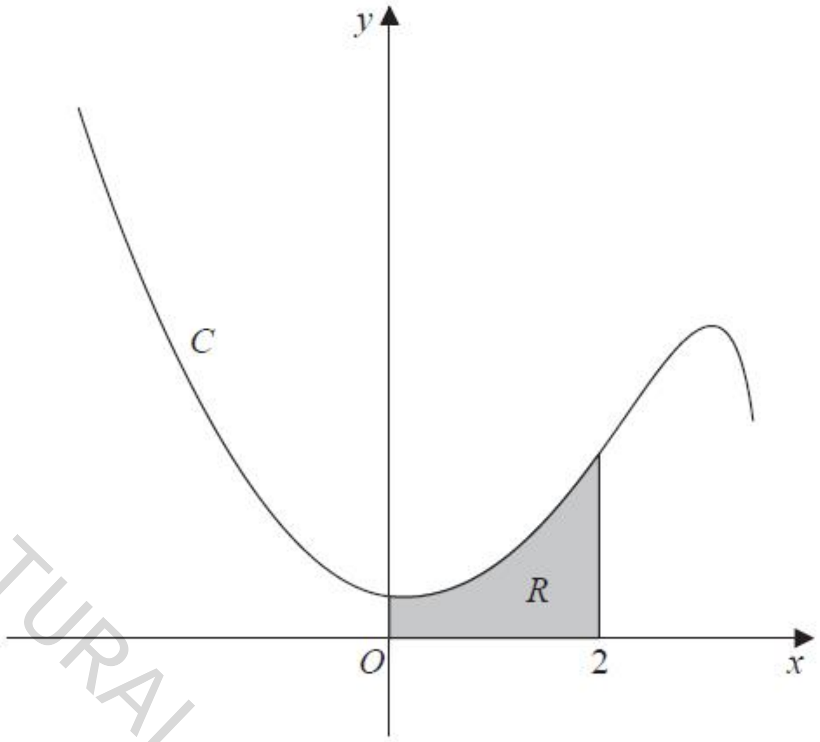


**Figure 4**

Figure 4 shows a sketch of the curve *C*.
The region *R*, shown shaded in Figure 4, is bounded by *C*, the *y*-axis, the *x*-axis and the line with equation $x = 2$

(c) Find the exact area of *R*, writing your answer in the form $a + b\ln 2$ , where *a* and *b* are constants to be found.

(5)
**(Total for question = 14 marks)**

(**Q09 WMA13/01, Oct 2020**)

**Extra space for working:**

**Q22.**

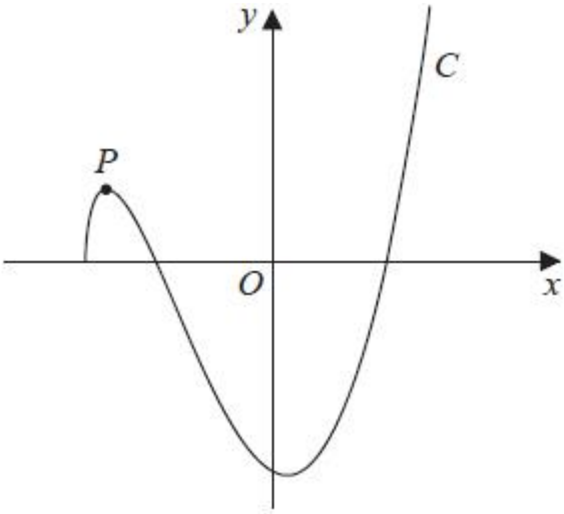


**Figure 3**

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

The function f is defined by

$$f(x) = 5(x^2 - 2)(4x + 9)^{\frac{1}{2}} \qquad x \geqslant -\frac{9}{4}$$

(a) Show that

$$f'(x) = \frac{k(5x^2 + 9x - 2)}{(4x + 9)^{\frac{1}{2}}}$$

where $k$ is an integer to be found.

(4)

(b) Hence, find the values of $x$ for which $f'(x) = 0$

(1)

Figure 3 shows a sketch of the curve $C$ with equation $y = f(x)$.

The curve has a local maximum at the point $P$

(c) Find the exact coordinates of $P$

(2)

The function g is defined by

$g(x) = 2f(x) + 4 \qquad -\frac{9}{4} \leq x \leq 0$

(d) Find the range of g

(3)

**(Total for question = 10 marks)**

(**Q06 WMA13/01, June 2022**)

**Extra space for working:**

**Q23.**

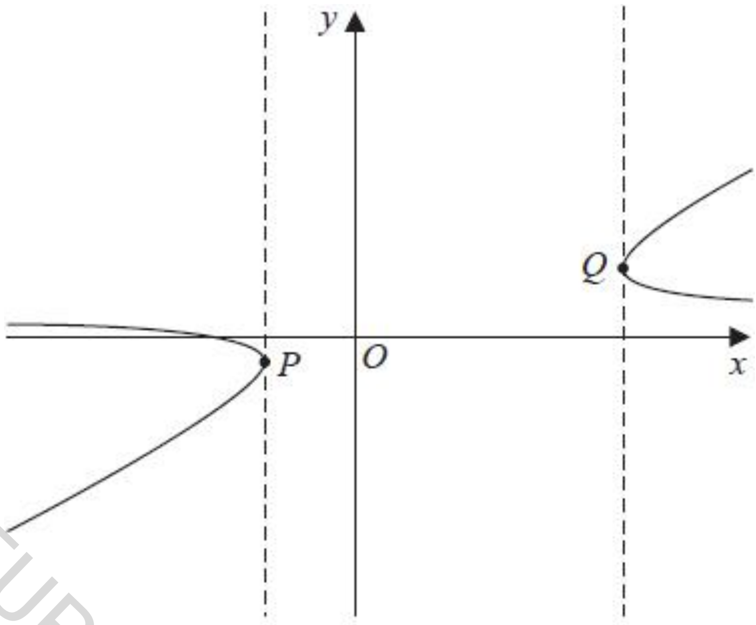


**Figure 4**

Figure 4 shows a sketch of the curve with equation

$$x = \frac{2y^2 + 6}{3y - 3}$$

(a) Find $\frac{dx}{dy}$ giving your answer as a fully simplified fraction.

**(4)**

The tangents at points *P* and *Q* on the curve are parallel to the *y*-axis, as shown in Figure 4.

(b) Use the answer to part (a) to find the equations of these two tangents.

**(4)**

**(Total for question = 8 marks)**

(**Q10 WMA13/01, June 2023**)

**Extra space for working:**

# Integration-(WMA13)

**Q1.**

(i) Find

$$\int_5^{13} \frac{1}{(2x-1)}\,dx$$

writing your answer in its simplest form.

**(4)**

(ii) Use integration to find the exact value of

$$\int_0^{\frac{\pi}{2}} \sin 2x + \sec\frac{1}{3}x\tan\frac{1}{3}x\,dx$$

**(3)**

**(Total for question = 7 marks)**

(**Q04 WMA13/01, Specimen papers** )

Extra space for working:

**Q2.**

(i) Find

$$\int \frac{12}{(2x-1)^2}\,dx$$

giving your answer in simplest form.

**(2)**

(ii) (a) Write $\frac{4x+3}{x+2}$ in the form

$A + \frac{B}{x+2}$ where $A$ and $B$ are constants to be found

(b) Hence find, using algebraic integration, the exact value of

$$\int_{-8}^{-5} \frac{4x+3}{x+2}\,dx$$

giving your answer in simplest form.

**(6)**

**(Total for question = 8 marks)**

(**Q03 WMA13/01, June 2021**)

**<u>Extra space for working:</u>**

**Extra space for working:**

**Q3.**

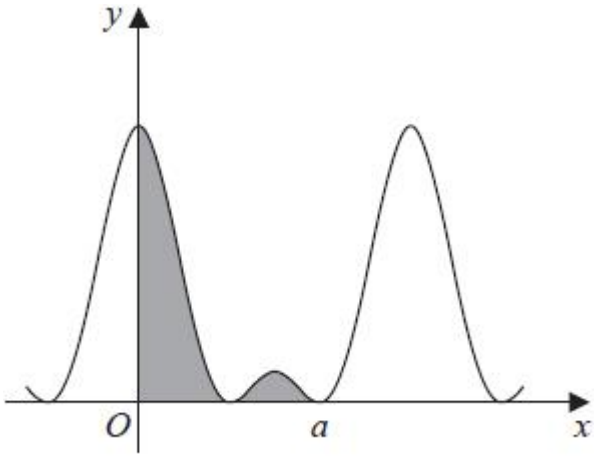


**Figure 4**

Figure 4 shows a sketch of part of the curve with equation

$$y = (1 + 2\cos 2x)^2$$

(a) Show that

$$(1 + 2\cos 2x)^2 \equiv p + q\cos 2x + r\cos 4x$$

where $p$, $q$ and $r$ are constants to be found.

**(2)**

The curve touches the positive $x$-axis for the second time when $x = a$, as shown in Figure 4.

The regions bounded by the curve, the $y$-axis and the $x$-axis up to $x = a$ are shown shaded in Figure 4.

(b) Find, using algebraic integration and making your method clear, the exact total area of the shaded regions. Write your answer in simplest form.

**(5)**

**(Total for question = 7 marks)**

(**Q10 WMA13/01, Oct 2021**)

Extra space for working:

**Q4.**

Find

$$\int \frac{x^2 - 5}{2x^3} \mathrm{d}x \qquad x > 0$$

giving your answer in simplest form.

**(3)**

**(Total for question = 3 marks)**

(**Q01 WMA13/01, Jan 2021**)

**Q5.**

Find

(i) $\displaystyle\int \frac{3x-2}{3x^2-4x+5}\,dx$

**(2)**

(ii) $\displaystyle\int \frac{e^{2x}}{(e^{2x}-1)^3}\,dx \qquad x \neq 0$

**(2)**

**(Total for question = 4 marks)**

(**Q09 WMA13/01, Jan 2021**)

**Extra space for working:**

**Q6.**

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

Given that $k$ is a positive constant,

(a) find

$$\int \frac{9x}{3x^2 + k}\,dx$$

**(2)**

Given also that

$$\int_2^5 \frac{9x}{3x^2 + k}\,dx = \ln 8$$

(b) find the value of $k$

**(4)**

**(Total for question = 6 marks)**

(**Q03 WMA13/01, June 2022**)

**Extra space for working:**

**Q7.**

(i) Find, using algebraic integration, the exact value of

$$\int_{3}^{42} \frac{2}{3x-1}\,dx$$

giving your answer in simplest form.

**(4)**

(ii)

$$h(x) = \frac{2x^3 - 7x^2 + 8x + 1}{(x-1)^2} \qquad x > 1$$

Given $h(x) = Ax + B + \dfrac{C}{(x-1)^2}$ where *A*, *B* and *C* are constants to be found, find

$$\int h(x)\,dx$$

**(6)**

**(Total for question = 10 marks)**
(**Q08 WMA13/01, Jan 2020**)

**Extra space for working:**

**Q8.**

(a) Given that

$$\frac{x^4 - x^3 - 10x^2 + 3x - 9}{x^2 - x - 12} \equiv x^2 + P + \frac{Q}{x - 4} \qquad x > -3$$

find the value of the constant $P$ and show that $Q = 5$

**(4)**

The curve $C$ has equation $y = \text{g}(x)$, where

$$\text{g}(x) = \frac{x^4 - x^3 - 10x^2 + 3x - 9}{x^2 - x - 12} \qquad -3 < x < 3.5 \qquad x \in \mathbb{R}$$

(b) Find the equation of the tangent to $C$ at the point where $x = 2$
Give your answer in the form $y = mx + c$, where $m$ and $c$ are constants to be found.

**(5)**

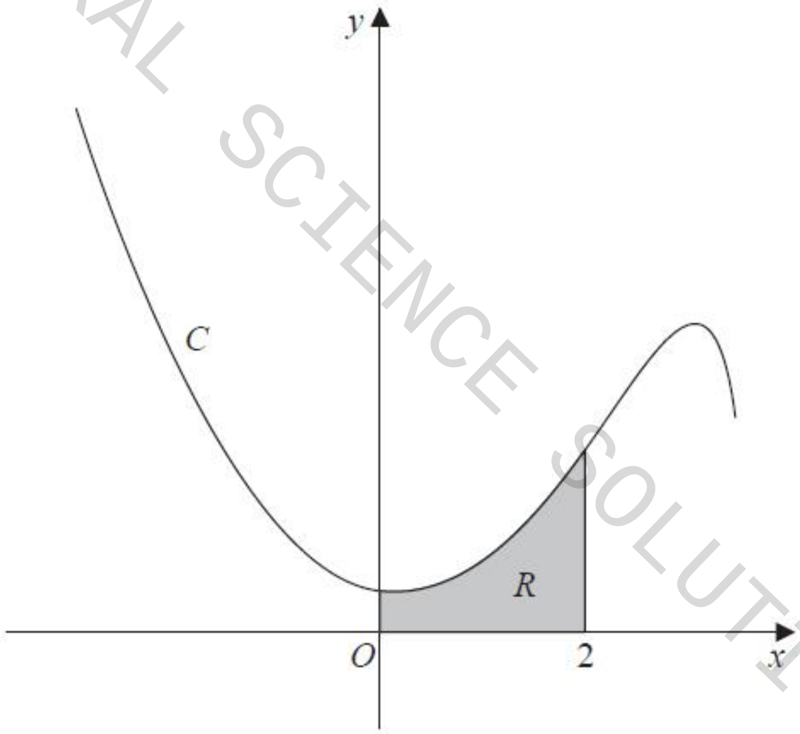


**Figure 4**

Figure 4 shows a sketch of the curve $C$.
The region $R$, shown shaded in Figure 4, is bounded by $C$, the $y$-axis, the $x$-axis and the line with equation $x = 2$

(c) Find the exact area of $R$, writing your answer in the form $a + b\ln 2$, where $a$ and $b$ are constants to be found.

**(5)**

**(Total for question = 14 marks)**

(**Q09 WMA13/01, Oct 2020**)

**Extra space for working:**

**Q9.**

(a) Show that

$$\sin 3x \equiv 3 \sin x - 4 \sin^3 x$$

**(4)**

(b) Hence find, using algebraic integration,

$$\int_0^{\frac{\pi}{3}} \sin^3 x \, dx$$

**(4)**



**(Total for question = 8 marks)**

(**Q05 WMA13/01, Oct 2020**)

**Q10.**

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

(a) Show that

$$\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x} \equiv \operatorname{cosec} x \qquad x \neq \frac{n\pi}{2} \qquad n \in \mathbb{Z}$$

**(3)**

(b) Hence solve, for $0 < \theta < \frac{\pi}{2}$ $\left(\frac{\cos 2\theta}{\sin \theta} + \frac{\sin 2\theta}{\cos \theta}\right)^2 = 6 \cot \theta - 4$ giving your answers to 3 significant figures as appropriate.

**(5)**

(c) Using the result from part (a), or otherwise, find the exact value of

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \left(\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x}\right) \cot x \, dx$$

**(2)**

**(Total for question = 10 marks)**

(**Q09 WMA13/01, June 2023**)

**Extra space for working:**

# Numerical Method-(WMA13)

**Q1.**

A curve has equation $y = f(x)$ where

$$f(x) = x^2 - 5x + e^x \qquad x \in \mathbb{R}$$

(a) Show that the equation $f(x) = 0$ has a root, $\alpha$, in the interval [1, 2]

**(2)**

The iterative formula

$$x_{n+1} = \sqrt{5x_n - e^{x_n}}$$

with $x^1 = 1$ is used to find an approximate value for the root $\alpha$.

(b) (i) Find the value of $x_2$ to 4 decimal places.

(ii) Find, by repeated iteration, the value of $\alpha$, giving your answer to 4 decimal places.

**(3)**

**(Total for question = 5 marks)**

(**Q01 WMA13/01, Oct 2023**)

**Extra space for working:**

**Q2.**

$$f(x) = x^3 + 3x^2 + 4x - 12$$

(a) Show that the equation f($x$) = 0 can be written as

$$x = \sqrt{\left(\frac{4(3-x)}{(3+x)}\right)} \qquad x \neq -3$$

**(3)**

The equation $x^3 + 3x^2 + 4x - 12 = 0$ has a single root which is between 1 and 2

(b) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{4(3-x_n)}{(3+x_n)}\right)} \qquad n \geqslant 0$$

with $x_0 = 1$ to find, to 2 decimal places, the value of $x_1$, $x_2$ and $x_3$

**(3)**

The root of f($x$) = 0 is $\alpha$.

(c) By choosing a suitable interval, prove that $\alpha = 1.272$ to 3 decimal places.

**(2)**

**(Total for question = 8 marks)**

(**Q02 WMA13/01, Specimen papers** )

**Extra space for working:**

**Q3.**

$$g(x) = x^6 + 2x - 1000$$

(a) Show that $g(x) = 0$ has a root *ɑ* in the interval [3, 4]

**(2)**

Using the iteration formula

$$x_{n+1} = \sqrt[6]{1000 - 2x_n} \qquad \text{with } x_1 = 3$$

(b) (i) find, to 4 decimal places, the value of $x^2$

(ii) find, by repeated iteration, the value of *ɑ*.
Give your answer to 4 decimal places.

**(3)**

**(Total for question = 5 marks)**

(**Q01 WMA13/01, June 2023**)

**Extra space for working:**

**Q4.**

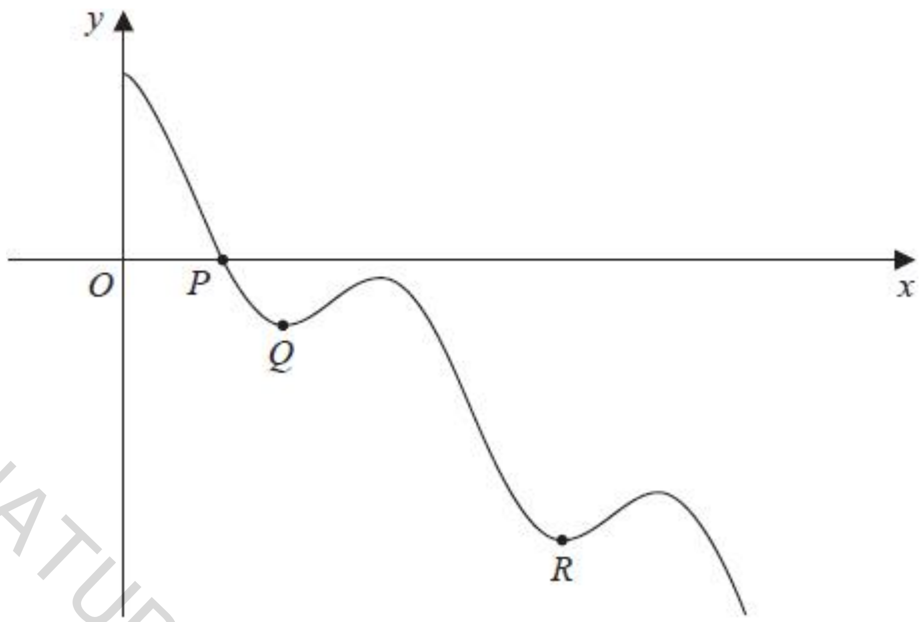


**Figure 3**



Figure 3 shows a sketch of part of the curve with equation

$$y = 2\cos 3x - 3x + 4 \qquad x > 0$$

where $x$ is measured in radians.

The curve crosses the $x$-axis at the point $P$, as shown in Figure 3.

Given that the $x$ coordinate of $P$ is $\alpha$,

(a) show that $\alpha$ lies between 0.8 and 0.9

**(2)**

The iteration formula

$$x_{n+1} = \frac{1}{3}\arccos(1.5x_n - 2)$$

can be used to find an approximate value for $\alpha$.

(b) Using this iteration formula with $x_1 = 0.8$ find, to 4 decimal places, the value of

(i) $x_2$
(ii) $x_5$

**(3)**

The point $Q$ and the point $R$ are local minimum points on the curve, as shown in Figure 3.

Given that the $x$ coordinates of $Q$ and $R$ are $\beta$ and $\lambda$ respectively, and that they are the two smallest values of $x$ at which local minima occur,

(c) find, using calculus, the exact value of $\beta$ and the exact value of $\lambda$.

**(6)**

**(Total for question = 11 marks)**

(**Q07 WMA13/01, Jan 2020**)

**Q5.**

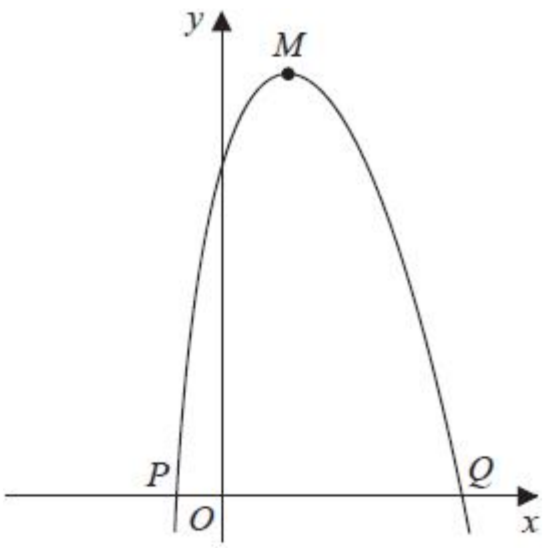


**Figure 1**

Figure 1 shows a sketch of part of the curve with equation

$$y = 6\ln(2x+3) - \frac{1}{2}x^2 + 4 \qquad x > -\frac{3}{2}$$

The curve cuts the negative *x*-axis at the point *P*, as shown in Figure 1.

(a) Show that the *x* coordinate of *P* lies in the interval [–1.25, –1.2]

**(2)**

The curve cuts the positive *x*-axis at the point *Q*, also shown in Figure 1.

Using the iterative formula

$$x_{n+1} = \sqrt{12\ln(2x_n + 3) + 8} \quad \text{with} \quad x_1 = 6$$

(b) (i) find, to 4 decimal places, the value of $x^2$

(ii) find, by continued iteration, the *x* coordinate of *Q*. Give your answer to 4 decimal places.

**(3)**

The curve has a maximum turning point at *M*, as shown in Figure 1.

(c) Using calculus and showing each stage of your working, find the *x* coordinate of *M*.

**(4)**

**(Total for question = 9 marks)**

(**Q05 WMA13/01, Jan 2022**)

**Extra space for working:**

**Q6.**

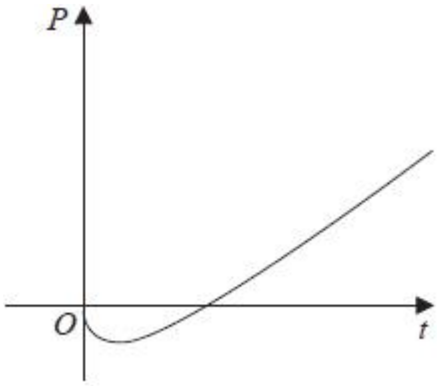


**Figure 2**

The profit made by a company, £ $P$ million, $t$ years after the company started trading, is modelled by the equation

$$P = \frac{4t-1}{10} + \frac{3}{4}\ln\left[\frac{t+1}{(2t+1)^2}\right]$$

The graph of $P$ against $t$ is shown in Figure 2.

According to the model,

(a) show that exactly one year after it started trading, the company had made a loss of approximately £ 830 000

**(2)**

A manager of the company wants to know the value of $t$ for which $P = 0$

(b) Show that this value of $t$ occurs in the interval [6, 7]

**(2)**

(c) Show that the equation $P = 0$ can be expressed in the form

$$t = \frac{1}{4} + \frac{15}{8}\ln\left[\frac{(2t+1)^2}{t+1}\right]$$

**(2)**

(d) Using the iteration formula

$$t_{n+1} = \frac{1}{4} + \frac{15}{8}\ln\left[\frac{(2t_n+1)^2}{t_n+1}\right] \quad \text{with} \quad t_1 = 6$$

find the value of $t_2$ and the value of $t_6$ , giving your answers to 3 decimal places.

**(3)**

(e) Hence find, according to the model, how many months it takes in total, from when the company started trading, for it to make a profit.

**(2)**

**(Total for question = 11 marks)**

(**Q05 WMA13/01, Oct 2022**)

**Extra space for working:**

**Q7.**

A curve has equation y = f($x$) where

$$f(x) = x^4 - 5x^2 + 4x - 7 \qquad x \in \mathbb{R}$$

(a) Show that the equation f($x$) = 0 has a root, $\alpha$, in the interval [2, 3]

**(2)**

(b) Show that the equation f($x$) = 0 can be written as

$$x = \sqrt[3]{\frac{5x^2 - 4x + 7}{x}}$$

**(1)**

The iterative formula

$$x_{n+1} = \sqrt[3]{\frac{5x_n^{\ 2} - 4x_n + 7}{x_n}}$$

is used to find $\alpha$

(c) Starting with $x_1 = 2$ and using the iterative formula,

(i) find, to 4 decimal places, the value of $x_2$
(ii) find, to 4 decimal places, the value of $\alpha$

**(3)**

**(Total for question = 6 marks)**

(**Q02 WMA13/01, Jan 2024**)

**Extra space for working:**

**Q8.**

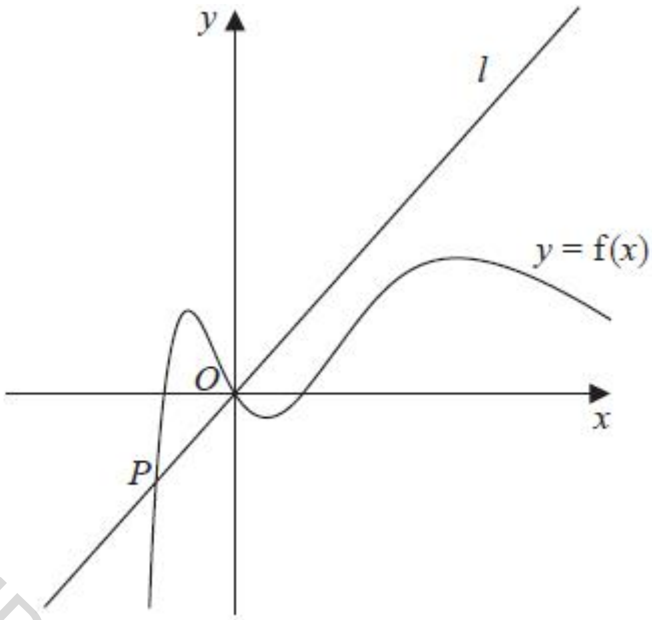


**Figure 3**

Figure 3 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = x(x^2 - 4)e^{-\frac{1}{2}x}$$

(a) Find $f'(x)$.

**(2)**

The line *l* is the normal to the curve at *O* and meets the curve again at the point *P*.

The point *P* lies in the 3rd quadrant, as shown in Figure 3.

(b) Show that the *x* coordinate of *P* is a solution of the equation

$$x = -\frac{1}{2}\sqrt{16 + e^{\frac{1}{2}x}}$$

**(4)**

(c) Using the iterative formula

$$x_{n+1} = -\frac{1}{2}\sqrt{16 + e^{\frac{1}{2}x_n}} \quad \text{with } x_1 = -2$$

find, to 4 decimal places,
(i) the value of $x_2$
(ii) the *x* coordinate of *P*.

**(3)**

**(Total for question = 9 marks)**

(**Q09 WMA13/01, Oct 2021**)

**Extra space for working:**

**Q9.**

The curve $C$ has equation

$$y = x^2 \cos\left(\frac{1}{2}x\right) \qquad 0 < x \leq \pi$$

The curve has a stationary point at the point $P$.

(a) Show, using calculus, that the $x$ coordinate of $P$ is a solution of the equation

$$x = 2\arctan\left(\frac{4}{x}\right)$$

**(4)**

Using the iteration formula

$$x_{n+1} = 2\arctan\left(\frac{4}{x_n}\right) \qquad x_1 = 2$$

(b) find the value of $x_2$ and the value of $x_6$, giving your answers to 3 decimal places.

**(3)**

**(Total for question = 7 marks)**

(**Q01 WMA13/01, June 2021**)

**Extra space for working:**

**Q10.**

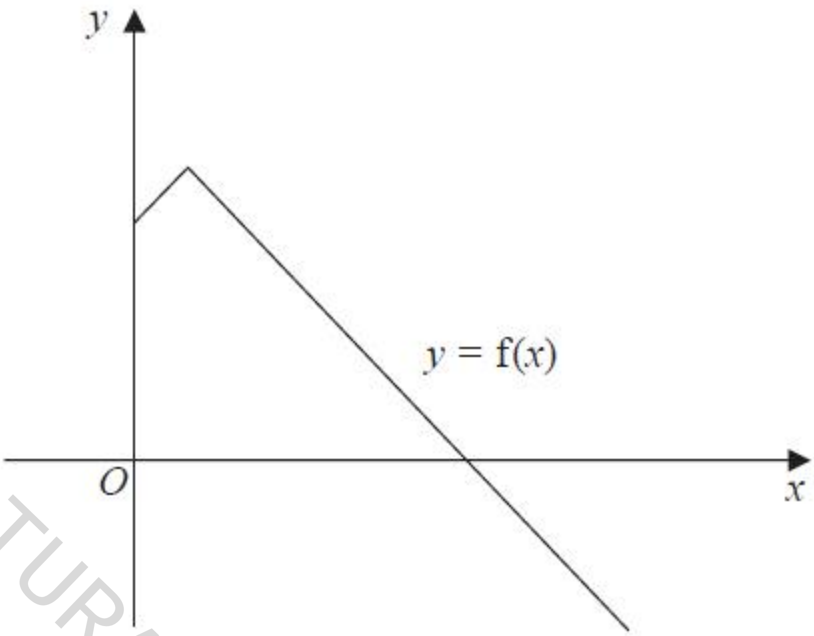


**Figure 2**

Figure 2 shows a sketch of part of the graph with equation y = f($x$) where

$$f(x) = 21 - 2|2 - x| \qquad x \geqslant 0$$

(a) Find ff(6)

**(2)**

(b) Solve the equation f($x$) = 5$x$

**(2)**

Given that the equation f($x$) = $k$ , where $k$ is a constant, has exactly two roots,

(c) state the set of possible values of $k$.

**(2)**

The graph with equation $y$ = f($x$) is transformed onto the graph with equation $y = a\,f(x - b)$

The vertex of the graph with equation $y = a\,f(x - b)$ is (6, 3).

Given that $a$ and $b$ are constants,

(d) find the value of $a$ and the value of $b$.

**(2)**

**(Total for question = 8 marks)**

(**Q04 WMA13/01, Oct 2020**)

**Extra space for working:**

**Q11.**

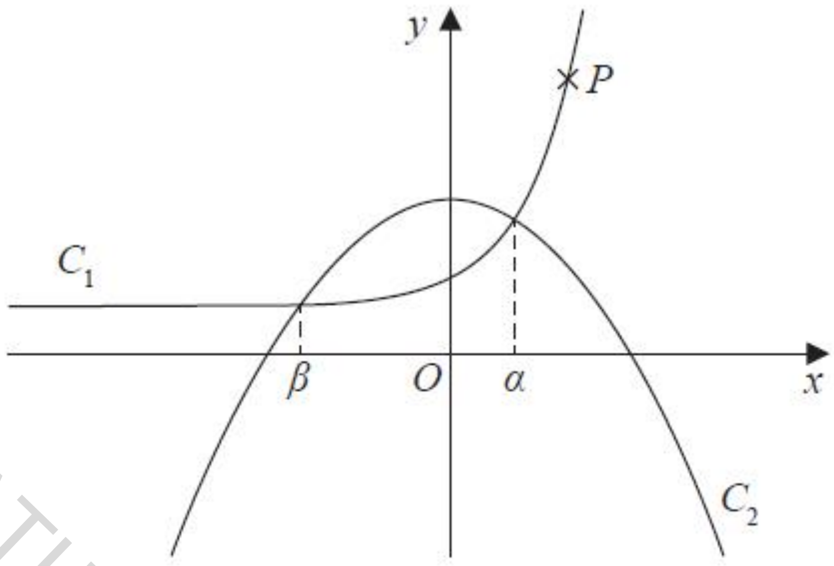


**Figure 3**

Figure 3 shows a sketch of curve $C_1$ with equation $y = 5e^{x-1} + 3$

and of curve $C_2$ with equation $y = 10 - x^2$

The point $P$ lies on $C_1$ and has $y$ coordinate 18

(a) Find the $x$ coordinate of $P$, writing your answer in the form $\ln k$, where $k$ is a constant to be found.

**(3)**

The curve $C_1$ meets the curve $C_2$ at $x = \alpha$ and at $x = \beta$, as shown in Figure 3.

(b) Using a suitable interval and a suitable function that should be stated, show that to 3 decimal places $\alpha = 1.134$

**(3)**

The iterative equation

$$x_{n+1} = -\sqrt{7 - 5e^{x_n - 1}}$$

is used to find an approximation to $\beta$.

Using this iterative formula with $x_1 = -3$

(c) find the value of $x_2$ and the value of $\beta$, giving each answer to 6 decimal places.

**(3)**

**(Total for question = 9 marks)**

(**Q06 WMA13/01, Oct 2020**)

**Extra space for working:**

**Q12.**

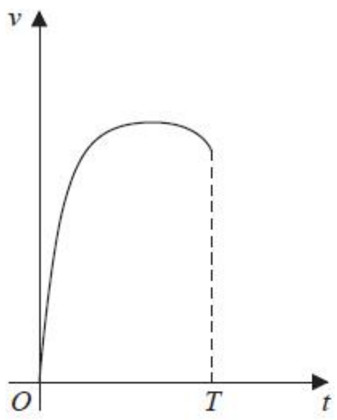


**Figure 4**

Figure 4 is a graph showing the velocity of a sprinter during a 100 m race.

The sprinter's velocity during the race, $v$ m s$^{-1}$, is modelled by the equation

$$v = 12 - e^{t-10} - 12e^{-0.75t} \qquad t \geq 0$$

where $t$ seconds is the time after the sprinter begins to run.

According to the model,

(a) find, using calculus, the sprinter's maximum velocity during the race.

**(5)**

Given that the sprinter runs 100 m in $T$ seconds, such that

$$\int_0^T v \, dt = 100$$

(b) show that $T$ is a solution of the equation

$$T = \frac{1}{12}(116 - 16e^{-0.75T} + e^{T-10} - e^{-10})$$

**(4)**

The iteration formula

$$T_{n+1} = \frac{1}{12}(116 - 16e^{-0.75T_n} + e^{T_n-10} - e^{-10})$$

is used to find an approximate value for $T$

Using this iteration formula with $T_1 = 10$

(c) find, to 4 decimal places,

(i) the value of $T_2$

(ii) the time taken by the sprinter to run the race, according to the model.

**(3)**

**(Total for question = 12 marks)**

(**Q08 WMA13/01, June 2022**)

**Extra space for working:**

**Q13.**

$$f(x) = x\cos\left(\frac{x}{3}\right) \qquad x > 0$$

(a) Find fʹ($x$)

**(2)**

(b) Show that the equation fʹ($x$) = 0 can be written as

$$x = k\arctan\left(\frac{k}{x}\right)$$

where $k$ is an integer to be found.

**(2)**

(c) Starting with $x_1 = 2.5$ use the iteration formula

$$x_{n+1} = k\arctan\left(\frac{k}{x_n}\right)$$

with the value of $k$ found in part (b), to calculate the values of $x_2$ and $x_6$ giving your answers to 3 decimal places.

**(2)**

(d) Using a suitable interval and a suitable function that should be stated, show that a root of f'($x$) = 0 is 2.581 correct to 3 decimal places.

**(2)**

**(Total for question = 8 marks)**

(**Q06 WMA13/01, Jan 2021**)

**Extra space for working:**