Accelerating the academic skills

> *"Knowing the path is good but not enough, walking the path with determination leads to destiny"*

AS CAMBRIDGE Paper 1/9709 CLASSIFIED QUESTIONS

2019 - 2022

Topic 1: Quadratics

1. M20/12

Express $2x^2 + 12x + 11$ in the form $2(x+a)^2 + b$, where a and b are constants. [2]

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2. S20/13

The functions f and g are defined by

$$f(x) = x^2 - 4x + 3 \quad \text{for } x > c, \text{ where } c \text{ is a constant,}$$

$$g(x) = \frac{1}{x+1} \quad \text{for } x > -1.$$

(a) Express $f(x)$ in the form $(x-a)^2 + b$. [2]

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3. W20/13 (only a part)

(a) Express $x^2 + 6x + 5$ in the form $(x+a)^2 + b$, where a and b are constants. [2]

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4. M21/12

By using a suitable substitution, solve the equation

$$(2x-3)^2 - \frac{4}{(2x-3)^2} - 3 = 0. \qquad [4]$$

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5. S21/12

(a) Express $16x^2 - 24x + 10$ in the form $(4x + a)^2 + b$. [2]

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(b) It is given that the equation $16x^2 - 24x + 10 = k$, where k is a constant, has exactly one root.

Find the value of this root. [2]

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6. W21/11

A curve has equation $y = kx^2 + 2x - k$ and a line has equation $y = kx - 2$, where k is a constant.

Find the set of values of k for which the curve and line do not intersect. [5]

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7. W21/11 (only part a)

(a) Express $-3x^2 + 12x + 2$ in the form $-3(x - a)^2 + b$, where a and b are constants. [2]

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TOPIC 2: Function

1 **M20/12**

(a) Express $2x^2 + 12x + 11$ in the form $2(x + a)^2 + b$, where a and b are constants. [2]

The function f is defined by $f(x) = 2x^2 + 12x + 11$ for $x \leqslant -4$.

(b) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [3]

The function g is defined by $g(x) = 2x - 3$ for $x \leqslant k$.

(c) For the case where $k = -1$, solve the equation $fg(x) = 193$. [2]

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(d) State the largest value of k possible for the composition fg to be defined. [1]

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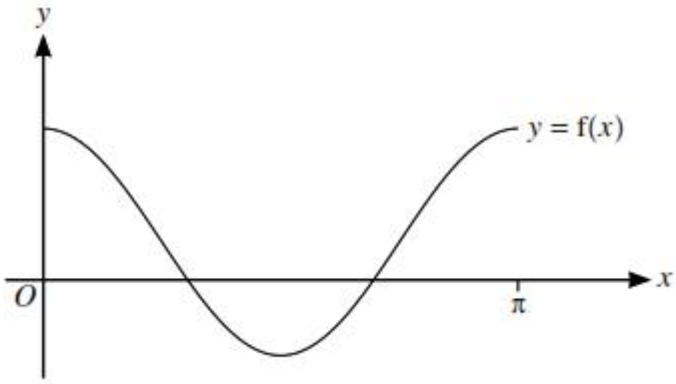
2 **S20/11**



The diagram shows the graph of $y = f(x)$, where $f(x) = \frac{3}{2}\cos 2x + \frac{1}{2}$ for $0 \leqslant x \leqslant \pi$.

(a) State the range of f. [2]

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3 **S20/11**

Functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto \frac{1}{2}x - a,$$
$$g : x \mapsto 3x + b,$$

where a and b are constants.

(a) Given that $gg(2) = 10$ and $f^{-1}(2) = 14$, find the values of a and b. [4]

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(b) Using these values of a and b, find an expression for $gf(x)$ in the form $cx + d$, where c and d are constants. [2]

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4 **S20/12**

The function f is defined for $x \in \mathbb{R}$ by

$$f : x \mapsto a - 2x,$$

where a is a constant.

(a) Express $ff(x)$ and $f^{-1}(x)$ in terms of a and x. [4]

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(b) Given that $ff(x) = f^{-1}(x)$, find x in terms of a. [2]

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5 **S20/12**

Functions f and g are such that

$$f(x) = 2 - 3\sin 2x \quad \text{for } 0 \leqslant x \leqslant \pi,$$
$$g(x) = -2f(x) \quad \text{for } 0 \leqslant x \leqslant \pi.$$

(a) State the ranges of f and g. [3]

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6 **S20/13**

The functions f and g are defined by

$$f(x) = x^2 - 4x + 3 \quad \text{for } x > c, \text{ where } c \text{ is a constant,}$$
$$g(x) = \frac{1}{x+1} \quad \text{for } x > -1.$$

(a) Express f(x) in the form $(x - a)^2 + b$. [2]

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It is given that f is a one-one function.

(b) State the smallest possible value of c. [1]

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It is now given that $c = 5$.

(c) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [3]

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(d) Find an expression for $gf(x)$ and state the range of gf. [3]

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7 **W20/11**

The functions f and g are defined by

$$f(x) = x^2 + 3 \quad \text{for } x > 0,$$
$$g(x) = 2x + 1 \quad \text{for } x > -\tfrac{1}{2}.$$

(a) Find an expression for fg(x). [1]

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(b) Find an expression for $(fg)^{-1}(x)$ and state the domain of $(fg)^{-1}$. [4]

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(c) Solve the equation $fg(x) - 3 = gf(x)$. [4]

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8 **W20/12**

Functions f and g are defined by

$$f(x) = 4x - 2, \quad \text{for } x \in \mathbb{R},$$

$$g(x) = \frac{4}{x+1}, \quad \text{for } x \in \mathbb{R},\ x \neq -1.$$

(a) Find the value of $fg(7)$. [1]

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(b) Find the values of x for which $f^{-1}(x) = g^{-1}(x)$. [5]

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9 **W20/13**

The function f is defined by $f(x) = \dfrac{2x}{3x-1}$ for $x > \frac{1}{3}$.

(a) Find an expression for $f^{-1}(x)$. [3]

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(b) Show that $\frac{2}{3} + \frac{2}{3(3x-1)}$ can be expressed as $\frac{2x}{3x-1}$. [2]

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(c) State the range of f. [1]

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10 M21/12

Functions f and g are defined as follows:

$$f : x \mapsto x^2 + 2x + 3 \text{ for } x \leqslant -1,$$
$$g : x \mapsto 2x + 1 \text{ for } x \geqslant -1.$$

(a) Express $f(x)$ in the form $(x+a)^2 + b$ and state the range of f. [3]

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(b) Find an expression for $f^{-1}(x)$. [2]

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(c) Solve the equation $gf(x) = 13$. [3]

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11 **S21/11**

Functions f and g are defined as follows:

$$f(x) = (x-2)^2 - 4 \text{ for } x \geqslant 2,$$
$$g(x) = ax + 2 \text{ for } x \in \mathbb{R},$$

where a is a constant.

(a) State the range of f. [1]

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(b) Find $f^{-1}(x)$. [2]

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(c) Given that $a = -\frac{5}{3}$, solve the equation $f(x) = g(x)$. [3]

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(d) Given instead that $gg f^{-1}(12) = 62$, find the possible values of a. [5]

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12 **S21/12**

The function f is defined by $f(x) = 2x^2 + 3$ for $x \geqslant 0$.

(a) Find and simplify an expression for $ff(x)$. [2]

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(b) Solve the equation $ff(x) = 34x^2 + 19$. [4]

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13 **S21/13**

Functions f and g are defined as follows:

$$f : x \mapsto x^2 - 1 \text{ for } x < 0,$$

$$g : x \mapsto \frac{1}{2x+1} \text{ for } x < -\tfrac{1}{2}.$$

(a) Solve the equation $fg(x) = 3$. [4]

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(b) Find an expression for $(fg)^{-1}(x)$. [3]

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14 W21/11

(a) Express $-3x^2 + 12x + 2$ in the form $-3(x - a)^2 + b$, where a and b are constants. [2]

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The one-one function f is defined by $f : x \mapsto -3x^2 + 12x + 2$ for $x \leqslant k$.

(b) State the largest possible value of the constant k. [1]

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It is now given that $k = -1$.

(c) State the range of f. [1]

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(d) Find an expression for $f^{-1}(x)$. [3]

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The result of translating the graph of $y = f(x)$ by $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$ is the graph of $y = g(x)$.

(e) Express $g(x)$ in the form $px^2 + qx + r$, where p, q and r are constants. [3]

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TOPIC 3: Transformation

1. M20/12

The graph of $y = \mathrm{f}(x)$ is transformed to the graph of $y = 1 + \mathrm{f}(\frac{1}{2}x)$.

Describe fully the two single transformations which have been combined to give the resulting transformation. [4]

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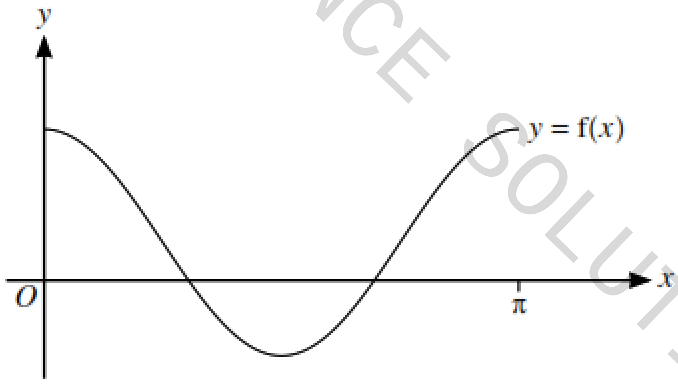
2. S20/11



The diagram shows the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \frac{3}{2}\cos 2x + \frac{1}{2}$ for $0 \leqslant x \leqslant \pi$.

(a) State the range of f. [2]

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A function g is such that $g(x) = f(x) + k$, where k is a positive constant. The x-axis is a tangent to the curve $y = g(x)$.

(b) State the value of k and hence describe fully the transformation that maps the curve $y = f(x)$ on to $y = g(x)$. [2]

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(c) State the equation of the curve which is the reflection of $y = f(x)$ in the x-axis. Give your answer in the form $y = a\cos 2x + b$, where a and b are constants. [1]

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3. S20/12

Functions f and g are such that

$$f(x) = 2 - 3\sin 2x \quad \text{for } 0 \leqslant x \leqslant \pi,$$
$$g(x) = -2f(x) \quad \text{for } 0 \leqslant x \leqslant \pi.$$

(a) State the ranges of f and g. [3]

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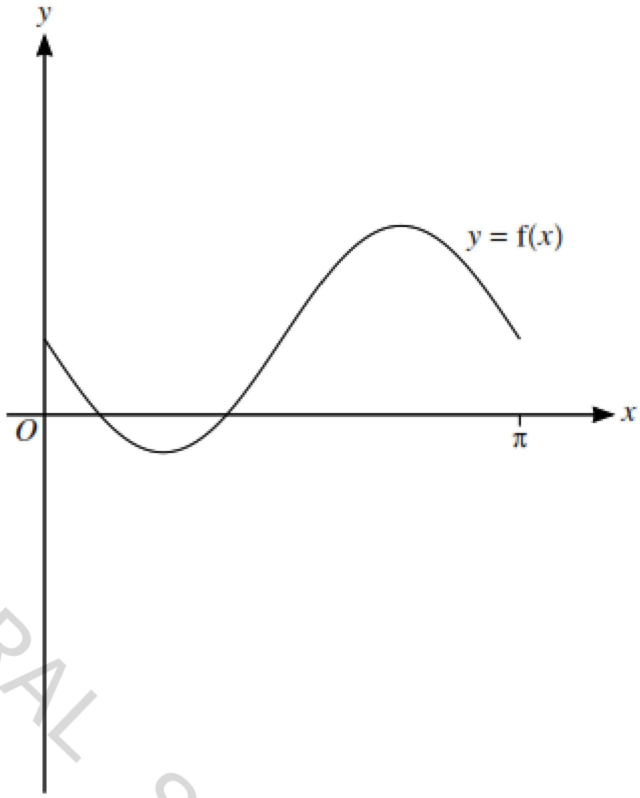
The diagram below shows the graph of $y = \mathrm{f}(x)$.



(b) Sketch, on this diagram, the graph of $y = \mathrm{g}(x)$. [2]

The function h is such that

$$\mathrm{h}(x) = \mathrm{g}(x + \pi) \quad \text{for } -\pi \leqslant x \leqslant 0.$$

(c) Describe fully a sequence of transformations that maps the curve $y = \mathrm{f}(x)$ on to $y = \mathrm{h}(x)$. [3]

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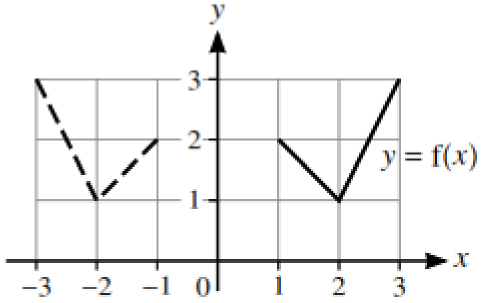
4. S20/13

In each of parts **(a)**, **(b)** and **(c)**, the graph shown with solid lines has equation $y = f(x)$. The graph shown with broken lines is a transformation of $y = f(x)$.

(a)



State, in terms of f, the equation of the graph shown with broken lines. [1]

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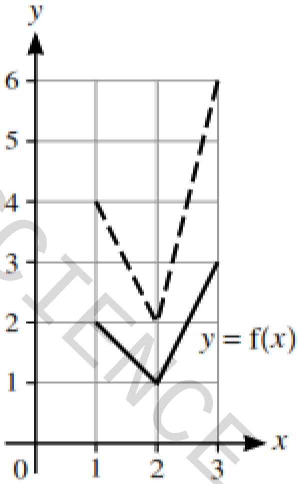
(b)



State, in terms of f, the equation of the graph shown with broken lines. [1]

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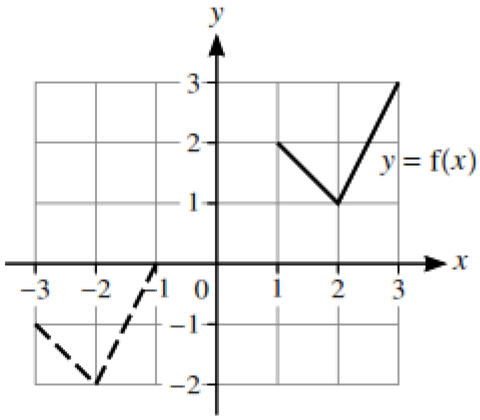
(c)



State, in terms of f, the equation of the graph shown with broken lines. [2]

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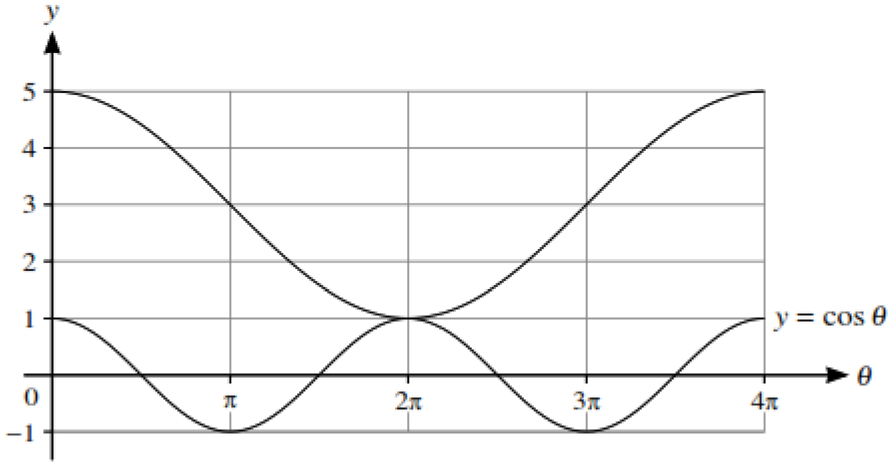
5. W20/11



In the diagram, the lower curve has equation $y = \cos\theta$. The upper curve shows the result of applying a combination of transformations to $y = \cos\theta$.

Find, in terms of a cosine function, the equation of the upper curve. [3]

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6. W20/12 (only d and e part)

Functions f, g and h are defined for $x \in \mathbb{R}$ by

$$f(x) = 3\cos 2x + 2,$$
$$g(x) = f(2x) + 4,$$
$$h(x) = 2f\left(x + \tfrac{1}{2}\pi\right).$$

(d) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = g(x)$. [2]

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(e) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = h(x)$. [2]

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7. W20/13

(a) Express $x^2 + 6x + 5$ in the form $(x + a)^2 + b$, where a and b are constants. [2]

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(b) The curve with equation $y = x^2$ is transformed to the curve with equation $y = x^2 + 6x + 5$.

Describe fully the transformation(s) involved. [2]

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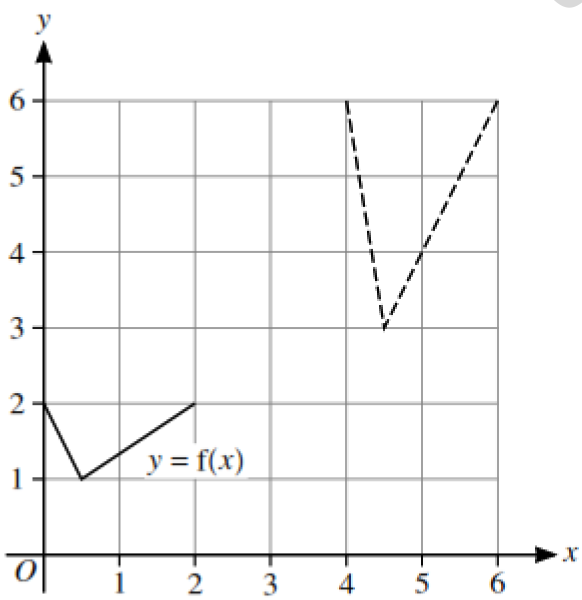
8. M21/12



In the diagram, the graph of $y = \mathrm{f}(x)$ is shown with solid lines. The graph shown with broken lines is a transformation of $y = \mathrm{f}(x)$.

(a) Describe fully the two single transformations of $y = \mathrm{f}(x)$ that have been combined to give the resulting transformation. [4]

(b) State in terms of y, f and x, the equation of the graph shown with broken lines. [2]

9. S21/12

(a) The graph of $y = \mathrm{f}(x)$ is transformed to the graph of $y = 2\mathrm{f}(x - 1)$.

Describe fully the two single transformations which have been combined to give the resulting transformation. [3]

(b) The curve $y = \sin 2x - 5x$ is reflected in the y-axis and then stretched by scale factor $\frac{1}{3}$ in the x-direction.

Write down the equation of the transformed curve. [2]

10. S21/13

Functions f and g are both defined for $x \in \mathbb{R}$ and are given by

$$f(x) = x^2 - 2x + 5,$$
$$g(x) = x^2 + 4x + 13.$$

(a) By first expressing each of f(x) and g(x) in completed square form, express g(x) in the form f($x + p$) + q, where p and q are constants. [4]

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(b) Describe fully the transformation which transforms the graph of $y = f(x)$ to the graph of $y = g(x)$. [2]

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TOPIC 4: Coordinate Geometry

1. M20/12

A diameter of a circle C_1 has end-points at $(-3, -5)$ and $(7, 3)$.

(a) Find an equation of the circle C_1. [3]

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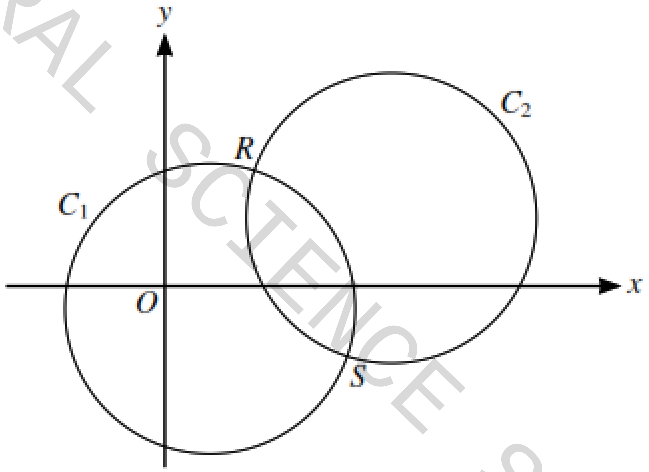

The circle C_1 is translated by $\begin{pmatrix} 8 \\ 4 \end{pmatrix}$ to give circle C_2, as shown in the diagram.

(b) Find an equation of the circle C_2. [2]

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The two circles intersect at points R and S.

(c) Show that the equation of the line RS is $y = -2x + 13$. [4]

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(d) Hence show that the x-coordinates of R and S satisfy the equation $5x^2 - 60x + 159 = 0$. [2]

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2. S20/11

The equation of a line is $y = mx + c$, where m and c are constants, and the equation of a curve is $xy = 16$.

(a) Given that the line is a tangent to the curve, express m in terms of c. [3]

(b) Given instead that $m = -4$, find the set of values of c for which the line intersects the curve at two distinct points. [3]

3. S20/11

The coordinates of the points A and B are $(-1, -2)$ and $(7, 4)$ respectively.

(a) Find the equation of the circle, C, for which AB is a diameter. [4]

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(b) Find the equation of the tangent, T, to circle C at the point B. [4]

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(c) Find the equation of the circle which is the reflection of circle C in the line T. [3]

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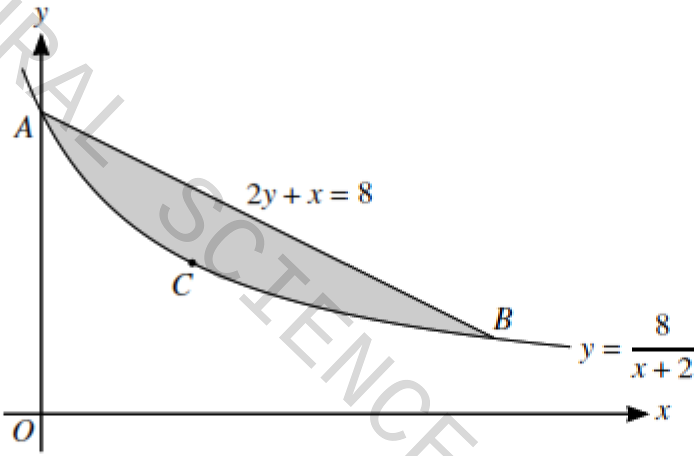
4. S/20/11



The diagram shows part of the curve $y = \frac{8}{x+2}$ and the line $2y + x = 8$, intersecting at points A and B. The point C lies on the curve and the tangent to the curve at C is parallel to AB.

(a) Find, by calculation, the coordinates of A, B and C. [6]

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5. S20/12

The equation of a curve is $y = 2x^2 + kx + k - 1$, where k is a constant.

(a) Given that the line $y = 2x + 3$ is a tangent to the curve, find the value of k. [3]

It is now given that $k = 2$.

(b) Express the equation of the curve in the form $y = 2(x + a)^2 + b$, where a and b are constants, and hence state the coordinates of the vertex of the curve. [3]

6. S20/12

The equation of a circle with centre C is $x^2 + y^2 - 8x + 4y - 5 = 0$.

(a) Find the radius of the circle and the coordinates of C. [3]

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The point $P(1, 2)$ lies on the circle.

(b) Show that the equation of the tangent to the circle at P is $4y = 3x + 5$. [3]

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The point Q also lies on the circle and PQ is parallel to the x-axis.

(c) Write down the coordinates of Q. [2]

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The tangents to the circle at P and Q meet at T.

(d) Find the coordinates of T. [3]

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7. S20/13

Find the set of values of m for which the line with equation $y = mx + 1$ and the curve with equation $y = 3x^2 + 2x + 4$ intersect at two distinct points. [4]

8. S20/13

(a) The coordinates of two points A and B are $(-7, 3)$ and $(5, 11)$ respectively.

Show that the equation of the perpendicular bisector of AB is $3x + 2y = 11$. [4]

(b) A circle passes through A and B and its centre lies on the line $12x - 5y = 70$.

Find an equation of the circle. [5]

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9. W/20/11

Find the set of values of m for which the line with equation $y = mx - 3$ and the curve with equation $y = 2x^2 + 5$ do not meet. [3]

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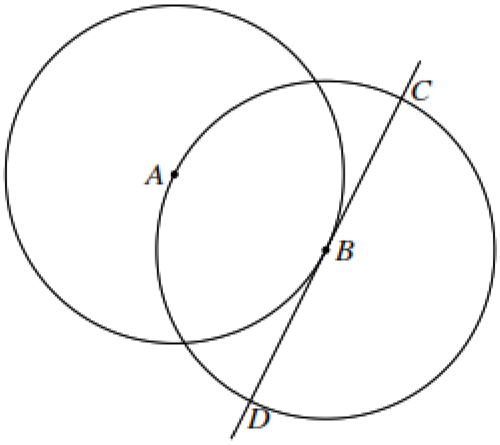
10. W20/11



The diagram shows a circle with centre A passing through the point B. A second circle has centre B and passes through A. The tangent at B to the first circle intersects the second circle at C and D.

The coordinates of A are $(-1, 4)$ and the coordinates of B are $(3, 2)$.

(a) Find the equation of the tangent CBD. [2]

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(b) Find an equation of the circle with centre B. [3]

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(c) Find, by calculation, the x-coordinates of C and D. [3]

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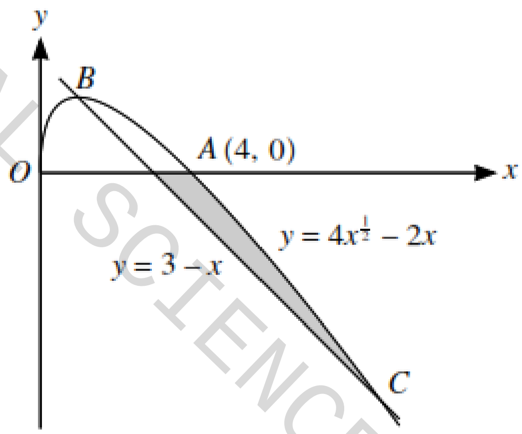
11. W20/11



The diagram shows a curve with equation $y = 4x^{\frac{1}{2}} - 2x$ for $x \geqslant 0$, and a straight line with equation $y = 3 - x$. The curve crosses the x-axis at $A\,(4, 0)$ and crosses the straight line at B and C.

(a) Find, by calculation, the x-coordinates of B and C. [4]

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12. W20/12

The equation of a curve is $y = 2x^2 + m(2x + 1)$, where m is a constant, and the equation of a line is $y = 6x + 4$.

Show that, for all values of m, the line intersects the curve at two distinct points. [5]

13. W20/12

A circle has centre at the point $B\,(5,\ 1)$. The point $A\,(-1,\ -2)$ lies on the circle.

(a) Find the equation of the circle. [3]

Point C is such that AC is a diameter of the circle. Point D has coordinates (5, 16).

(b) Show that DC is a tangent to the circle. [4]

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The other tangent from D to the circle touches the circle at E.

(c) Find the coordinates of E. [2]

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14. W20/13

A curve has equation $y = 3x^2 - 4x + 4$ and a straight line has equation $y = mx + m - 1$, where m is a constant.

Find the set of values of m for which the curve and the line have two distinct points of intersection.

[5]

15. W20/13

A circle with centre C has equation $(x - 8)^2 + (y - 4)^2 = 100$.

(a) Show that the point $T\,(-6, 6)$ is outside the circle. [3]

Two tangents from T to the circle are drawn.

(b) Show that the angle between one of the tangents and CT is exactly 45°. [2]

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The two tangents touch the circle at A and B.

(c) Find the equation of the line AB, giving your answer in the form $y = mx + c$. [4]

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(d) Find the x-coordinates of A and B. [3]

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16. M21/12

A line has equation $y = 3x + k$ and a curve has equation $y = x^2 + kx + 6$, where k is a constant.

Find the set of values of k for which the line and curve have two distinct points of intersection. [5]

17. M21/12

The points $A\,(7, 1)$, $B\,(7, 9)$ and $C\,(1, 9)$ are on the circumference of a circle.

(a) Find an equation of the circle. [5]

(b) Find an equation of the tangent to the circle at B. [2]

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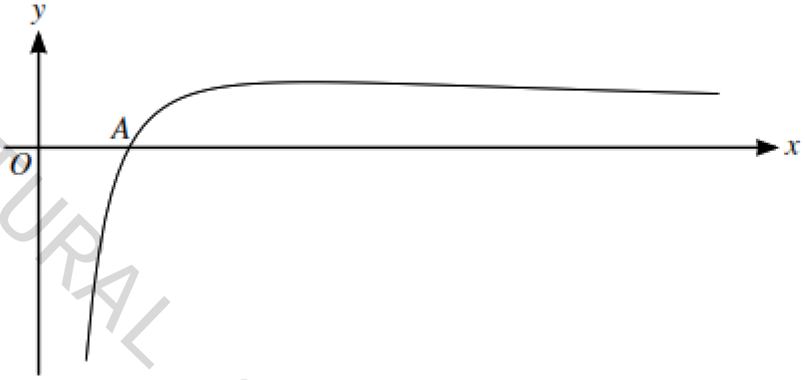
18. M21/12 (Only a, and b part)



The diagram shows the curve with equation $y = 9(x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}})$. The curve crosses the x-axis at the point A.

(a) Find the x-coordinate of A. [2]

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(b) Find the equation of the tangent to the curve at A. [4]

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19. S21/11

The equation of a curve is $y = (2k - 3)x^2 - kx - (k - 2)$, where k is a constant. The line $y = 3x - 4$ is a tangent to the curve.

Find the value of k. [5]

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20. S21/11

The equation of a circle is $x^2 + y^2 - 4x + 6y - 77 = 0$.

(a) Find the x-coordinates of the points A and B where the circle intersects the x-axis. [2]

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(b) Find the point of intersection of the tangents to the circle at A and B. [6]

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21. S21/12

The equation of a curve is $y = (x - 3)\sqrt{x + 1} + 3$. The following points lie on the curve. Non-exact values are rounded to 4 decimal places.

$A\,(2, k)$ $B\,(2.9, 2.8025)$ $C\,(2.99, 2.9800)$ $D\,(2.999, 2.9980)$ $E\,(3, 3)$

(a) Find k, giving your answer correct to 4 decimal places. [1]

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(b) Find the gradient of AE, giving your answer correct to 4 decimal places. [1]

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The gradients of BE, CE and DE, rounded to 4 decimal places, are 1.9748, 1.9975 and 1.9997 respectively.

(c) State, giving a reason for your answer, what the values of the four gradients suggest about the gradient of the curve at the point E. [2]

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22. S21/12

Points A and B have coordinates $(8, 3)$ and (p, q) respectively. The equation of the perpendicular bisector of AB is $y = -2x + 4$.

Find the values of p and q. [4]

23. S21/12

The point A has coordinates $(1, 5)$ and the line l has gradient $-\frac{2}{3}$ and passes through A. A circle has centre $(5, 11)$ and radius $\sqrt{52}$.

(a) Show that l is the tangent to the circle at A. [2]

(b) Find the equation of the other circle of radius $\sqrt{52}$ for which l is also the tangent at A. [3]

24. S21/13

A line with equation $y = mx - 6$ is a tangent to the curve with equation $y = x^2 - 4x + 3$.

Find the possible values of the constant m, and the corresponding coordinates of the points at which the line touches the curve. [6]

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25. S21/13

Points $A\,(-2, 3)$, $B\,(3, 0)$ and $C\,(6, 5)$ lie on the circumference of a circle with centre D.

(a) Show that angle $ABC = 90°$. [2]

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(b) Hence state the coordinates of D. [1]

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(c) Find an equation of the circle. [2]

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The point E lies on the circumference of the circle such that BE is a diameter.

(d) Find an equation of the tangent to the circle at E. [5]

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26. W21/11

A circle with centre (5, 2) passes through the point (7, 5).

(a) Find an equation of the circle. [2]

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The line $y = 5x - 10$ intersects the circle at A and B.

(b) Find the exact length of the chord AB. [7]

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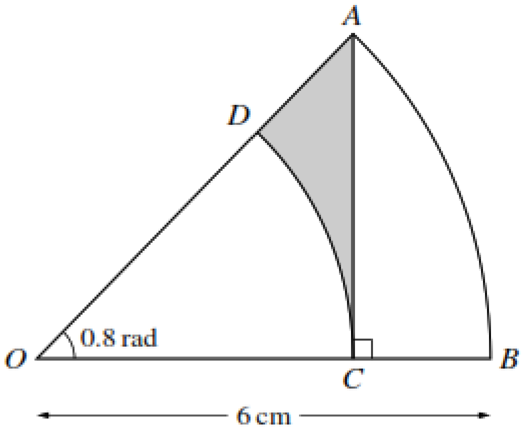
TOPIC 5: Circular measure

1. M20/12



The diagram shows a sector AOB which is part of a circle with centre O and radius 6 cm and with angle $AOB = 0.8$ radians. The point C on OB is such that AC is perpendicular to OB. The arc CD is part of a circle with centre O, where D lies on OA.

Find the area of the shaded region. [6]

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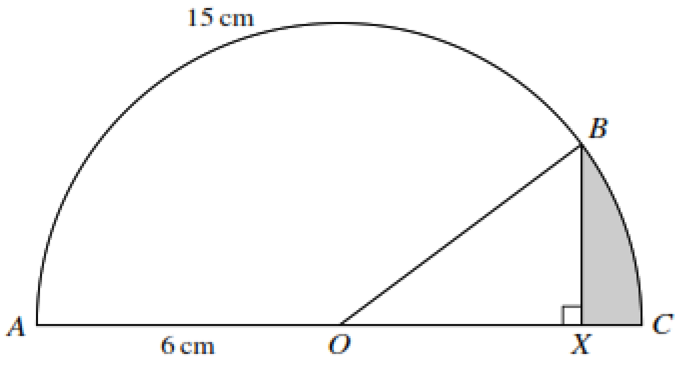
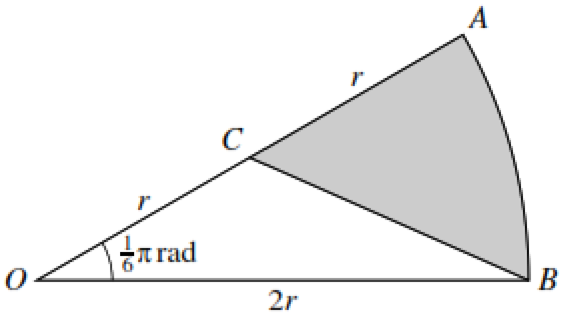
2. **S20/11**



In the diagram, ABC is a semicircle with diameter AC, centre O and radius 6 cm. The length of the arc AB is 15 cm. The point X lies on AC and BX is perpendicular to AX.

Find the perimeter of the shaded region BXC. [6]

3. S20/12



In the diagram, OAB is a sector of a circle with centre O and radius $2r$, and angle $AOB = \frac{1}{6}\pi$ radians. The point C is the midpoint of OA.

(a) Show that the exact length of BC is $r\sqrt{5 - 2\sqrt{3}}$. [2]

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(b) Find the exact perimeter of the shaded region. [2]

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(c) Find the exact area of the shaded region. [3]

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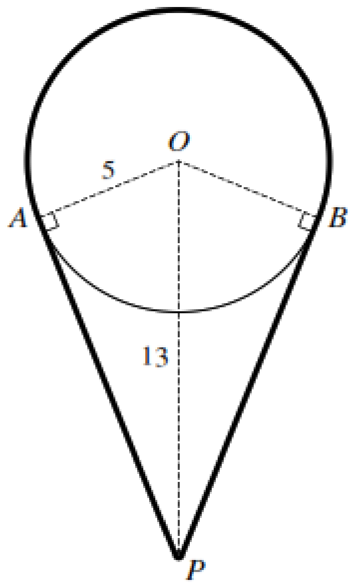
4. **S20/13**



The diagram shows a cord going around a pulley and a pin. The pulley is modelled as a circle with centre O and radius 5 cm. The thickness of the cord and the size of the pin P can be neglected. The pin is situated 13 cm vertically below O. Points A and B are on the circumference of the circle such that AP and BP are tangents to the circle. The cord passes over the major arc AB of the circle and under the pin such that the cord is taut.

Calculate the length of the cord. [6]

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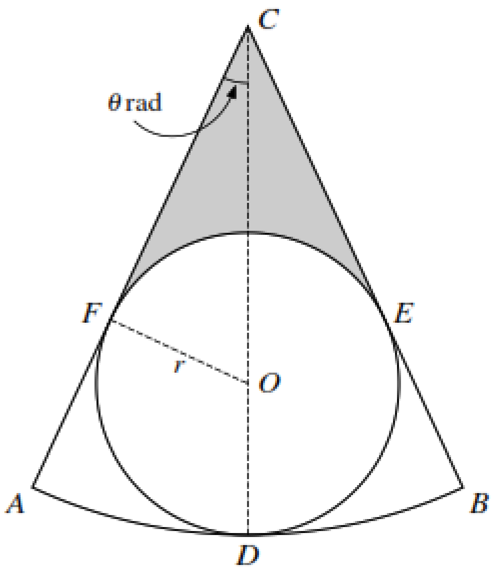
5. W20/11



The diagram shows a sector CAB which is part of a circle with centre C. A circle with centre O and radius r lies within the sector and touches it at D, E and F, where COD is a straight line and angle ACD is θ radians.

(a) Find CD in terms of r and $\sin\theta$. [3]

It is now given that $r = 4$ and $\theta = \frac{1}{6}\pi$.

(b) Find the perimeter of sector *CAB* in terms of π. [3]

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(c) Find the area of the shaded region in terms of π and $\sqrt{3}$. [4]

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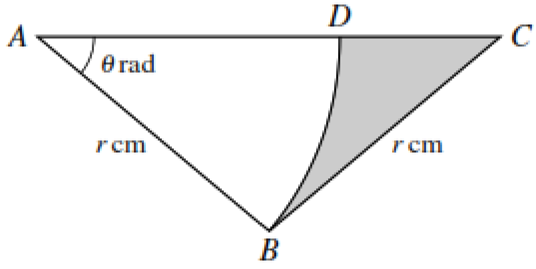
6. W20/12



In the diagram, ABC is an isosceles triangle with $AB = BC = r$ cm and angle $BAC = \theta$ radians. The point D lies on AC and ABD is a sector of a circle with centre A.

(a) Express the area of the shaded region in terms of r and θ. [3]

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(b) In the case where $r = 10$ and $\theta = 0.6$, find the perimeter of the shaded region. [4]

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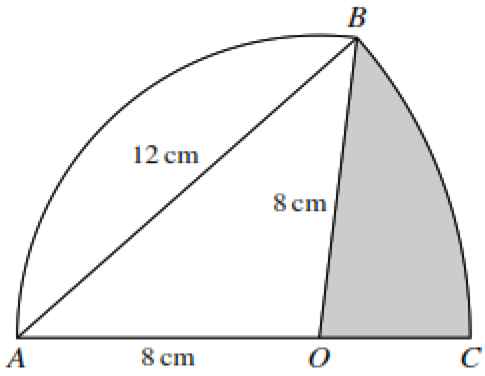
7. W20/13



In the diagram, arc AB is part of a circle with centre O and radius 8 cm. Arc BC is part of a circle with centre A and radius 12 cm, where AOC is a straight line.

(a) Find angle BAO in radians. [2]

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(b) Find the area of the shaded region. [4]

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(c) Find the perimeter of the shaded region. [3]

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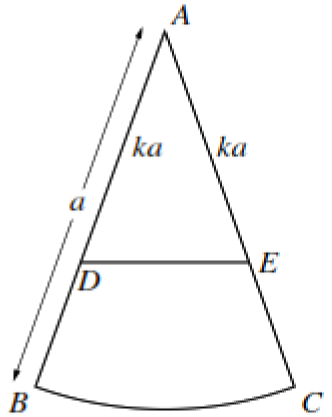
8. M21/12



The diagram shows a sector ABC which is part of a circle of radius a. The points D and E lie on AB and AC respectively and are such that $AD = AE = ka$, where $k < 1$. The line DE divides the sector into two regions which are equal in area.

(a) For the case where angle $BAC = \frac{1}{6}\pi$ radians, find k correct to 4 significant figures. [5]

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(b) For the general case in which angle $BAC = \theta$ radians, where $0 < \theta < \frac{1}{2}\pi$, it is given that $\frac{\theta}{\sin\theta} > 1$.

Find the set of possible values of k. [3]

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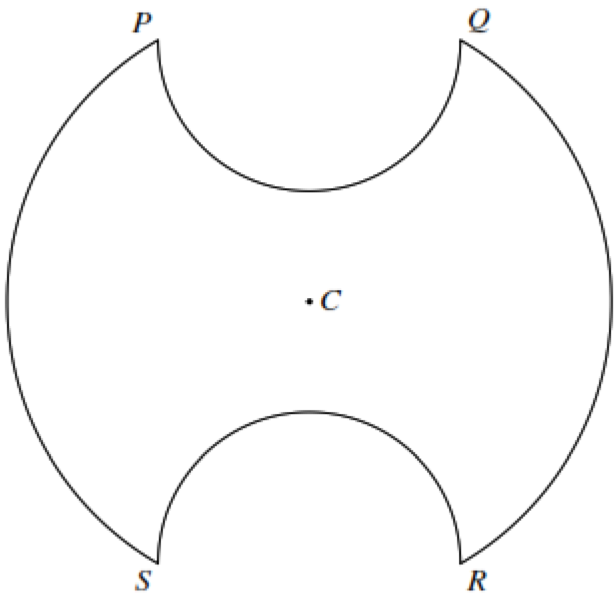
9. **S21/11**



The diagram shows a symmetrical metal plate. The plate is made by removing two identical pieces from a circular disc with centre C. The boundary of the plate consists of two arcs PS and QR of the original circle and two semicircles with PQ and RS as diameters. The radius of the circle with centre C is 4 cm, and $PQ = RS = 4$ cm also.

(a) Show that angle $PCS = \frac{2}{3}\pi$ radians. [2]

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(b) Find the exact perimeter of the plate. [3]

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(c) Show that the area of the plate is $(\frac{20}{3}\pi + 8\sqrt{3})$ cm^2. [5]

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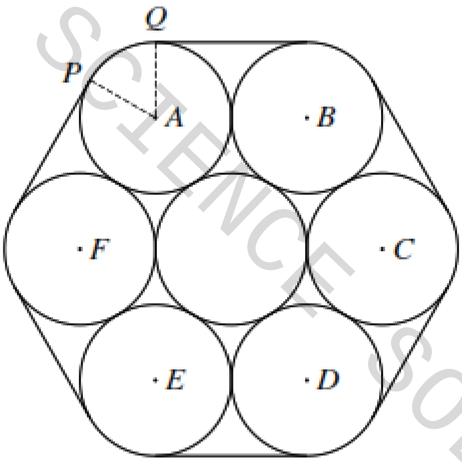
10. S21/12



The diagram shows a cross-section of seven cylindrical pipes, each of radius 20 cm, held together by a thin rope which is wrapped tightly around the pipes. The centres of the six outer pipes are A, B, C, D, E and F. Points P and Q are situated where straight sections of the rope meet the pipe with centre A.

(a) Show that angle $PAQ = \frac{1}{3}\pi$ radians. [2]

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(b) Find the length of the rope. [4]

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(c) Find the area of the hexagon *ABCDEF*, giving your answer in terms of $\sqrt{3}$. [2]

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(d) Find the area of the complete region enclosed by the rope. [3]

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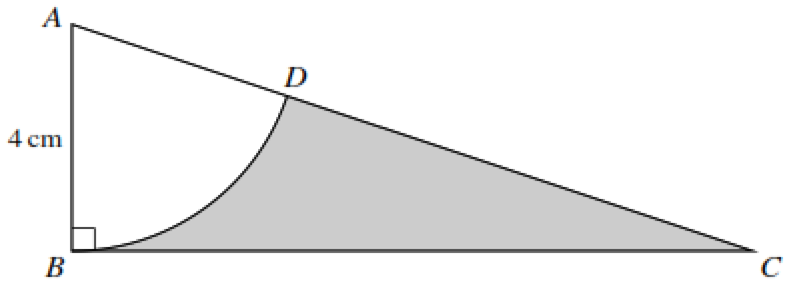
11. S21/13



The diagram shows a triangle ABC, in which angle $ABC = 90°$ and $AB = 4$ cm. The sector ABD is part of a circle with centre A. The area of the sector is $10\,\text{cm}^2$.

(a) Find angle BAD in radians. [2]

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(b) Find the perimeter of the shaded region. [4]

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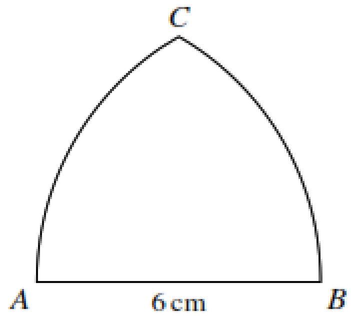
12. W21/11



The diagram shows a metal plate ABC in which the sides are the straight line AB and the arcs AC and BC. The line AB has length 6 cm. The arc AC is part of a circle with centre B and radius 6 cm, and the arc BC is part of a circle with centre A and radius 6 cm.

(a) Find the perimeter of the plate, giving your answer in terms of π. [3]

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(b) Find the area of the plate, giving your answer in terms of π and $\sqrt{3}$. [4]

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TOPIC 6: Trigonometry

1. **M20/12**

Solve the equation

$$\frac{\tan\theta + 3\sin\theta + 2}{\tan\theta - 3\sin\theta + 1} = 2$$

for $0° \leqslant \theta \leqslant 90°$. [5]

2. M20/12

(a) Solve the equation $3\tan^2 x - 5\tan x - 2 = 0$ for $0° \leqslant x \leqslant 180°$. [4]

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(b) Find the set of values of k for which the equation $3\tan^2 x - 5\tan x + k = 0$ has no solutions. [2]

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(c) For the equation $3\tan^2 x - 5\tan x + k = 0$, state the value of k for which there are three solutions in the interval $0° \leqslant x \leqslant 180°$, and find these solutions. [3]

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3. S20/11

(a) Prove the identity $\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta} \equiv \frac{2}{\cos\theta}$. [3]

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(b) Hence solve the equation $\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta} = \frac{3}{\sin\theta}$, for $0 \leqslant \theta \leqslant 2\pi$. [3]

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4. S20/12

(a) Express the equation $3\cos\theta = 8\tan\theta$ as a quadratic equation in $\sin\theta$. [3]

(b) Hence find the acute angle, in degrees, for which $3\cos\theta = 8\tan\theta$. [2]

5. S20/13

(a) Show that $\frac{\tan\theta}{1+\cos\theta}+\frac{\tan\theta}{1-\cos\theta}\equiv\frac{2}{\sin\theta\cos\theta}$. [4]

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(b) Hence solve the equation $\frac{\tan\theta}{1+\cos\theta}+\frac{\tan\theta}{1-\cos\theta}=\frac{6}{\tan\theta}$ for $0° < \theta < 180°$. [4]

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6. W20/11

(a) Show that $\frac{\sin\theta}{1-\sin\theta} - \frac{\sin\theta}{1+\sin\theta} \equiv 2\tan^2\theta$. [3]

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(b) Hence solve the equation $\frac{\sin\theta}{1-\sin\theta} - \frac{\sin\theta}{1+\sin\theta} = 8$, for $0° < \theta < 180°$. [3]

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7. W20/12

(a) Prove the identity $\left(\frac{1}{\cos x} - \tan x\right)\left(\frac{1}{\sin x} + 1\right) \equiv \frac{1}{\tan x}$. [4]

(b) Hence solve the equation $\left(\frac{1}{\cos x} - \tan x\right)\left(\frac{1}{\sin x} + 1\right) = 2\tan^2 x$ for $0° \leqslant x \leqslant 180°$. [2]

8. W20/12

A curve has equation $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$.

(a) State the greatest and least values of y. [2]

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(b) Sketch the graph of $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$. [2]

(c) By considering the straight line $y = kx$, where k is a constant, state the number of solutions of the equation $3\cos 2x + 2 = kx$ for $0 \leqslant x \leqslant \pi$ in each of the following cases.

(i) $k = -3$ [1]

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(ii) $k = 1$ [1]

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(iii) $k = 3$ [1]

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9. W20/13

Solve the equation $3\tan^2\theta + 1 = \dfrac{2}{\tan^2\theta}$ for $0° < \theta < 180°$. [5]

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10. M21/12

Solve the equation $\dfrac{\tan\theta + 2\sin\theta}{\tan\theta - 2\sin\theta} = 3$ for $0° < \theta < 180°$. [4]

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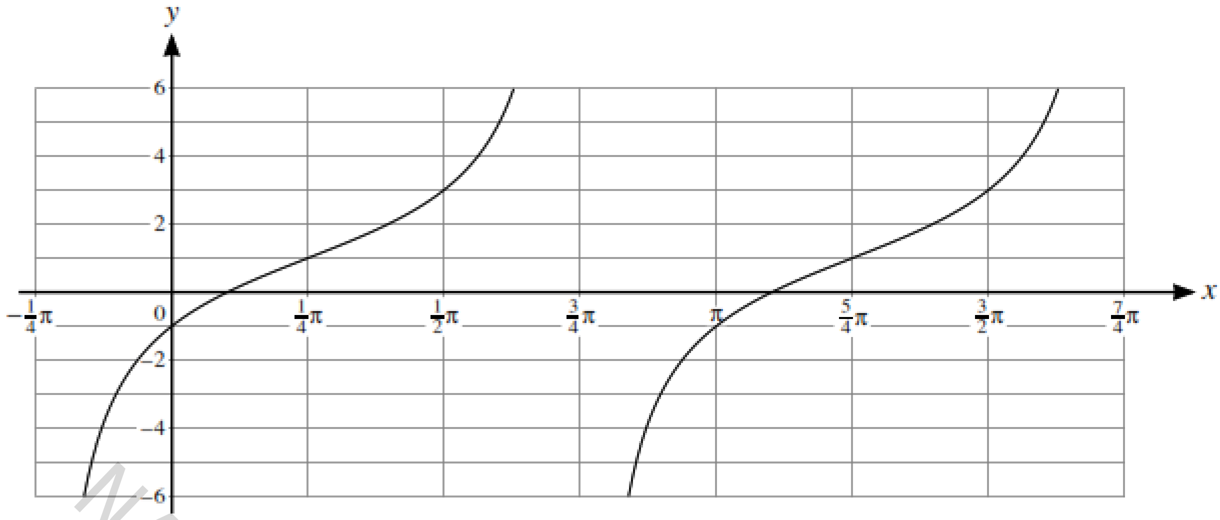
11. S21/11



The diagram shows part of the graph of $y = a\tan(x - b) + c$.

Given that $0 < b < \pi$, state the values of the constants a, b and c. [3]

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12. S21/11

(a) Prove the identity $\frac{1 - 2\sin^2\theta}{1 - \sin^2\theta} \equiv 1 - \tan^2\theta$. [2]

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(b) Hence solve the equation $\dfrac{1-2\sin^2\theta}{1-\sin^2\theta} = 2\tan^4\theta$ for $0° \leqslant \theta \leqslant 180°$. [3]

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13. S21/12

(a) Prove the identity $\dfrac{1+\sin x}{1-\sin x} - \dfrac{1-\sin x}{1+\sin x} \equiv \dfrac{4\tan x}{\cos x}$. [4]

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(b) Hence solve the equation $\frac{1+\sin x}{1-\sin x} - \frac{1-\sin x}{1+\sin x} = 8\tan x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. [3]

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14. S21/13

(a) Show that the equation

$$\frac{\tan x + \sin x}{\tan x - \sin x} = k,$$

where k is a constant, may be expressed as

$$\frac{1+\cos x}{1-\cos x} = k.$$ [2]

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(b) Hence express $\cos x$ in terms of k. [2]

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(c) Hence solve the equation $\dfrac{\tan x + \sin x}{\tan x - \sin x} = 4$ for $-\pi < x < \pi$. [2]

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15. W21/11

Solve, by factorising, the equation

$$6\cos\theta\tan\theta - 3\cos\theta + 4\tan\theta - 2 = 0,$$

for $0° \leqslant \theta \leqslant 180°$. [4]

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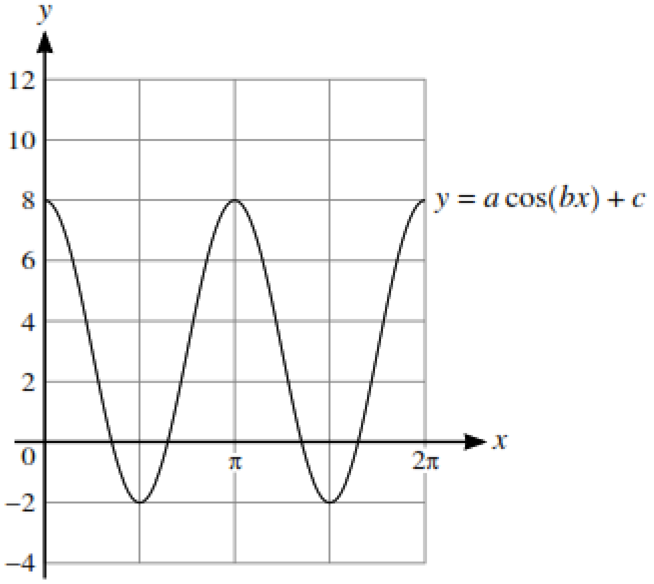
16. W21/11



The diagram shows part of the graph of $y = a\cos(bx) + c$.

(a) Find the values of the positive integers a, b and c. [3]

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(b) For these values of a, b and c, use the given diagram to determine the number of solutions in the interval $0 \leqslant x \leqslant 2\pi$ for each of the following equations.

(i) $a\cos(bx) + c = \frac{6}{\pi}x$ [1]

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(ii) $a\cos(bx) + c = 6 - \frac{6}{\pi}x$ [1]

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TOPIC 7: Binomial

1. m/20/12

The coefficient of $\frac{1}{x}$ in the expansion of $\left(2x + \frac{a}{x^2}\right)^5$ is 720.

(a) Find the possible values of the constant a. [3]

(b) Hence find the coefficient of $\frac{1}{x^7}$ in the expansion. [2]

2. S/20/12

(a) Find the coefficient of x^2 in the expansion of $\left(x - \frac{2}{x}\right)^6$. [2]

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(b) Find the coefficient of x^2 in the expansion of $(2 + 3x^2)\left(x - \frac{2}{x}\right)^6$. [3]

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3. S/20/13

(a) Expand $(1 + a)^5$ in ascending powers of a up to and including the term in a^3. [1]

(b) Hence expand $[1 + (x + x^2)]^5$ in ascending powers of x up to and including the term in x^3, simplifying your answer. [3]

4. S/20/11

The coefficient of $\frac{1}{x}$ in the expansion of $\left(kx+\frac{1}{x}\right)^5+\left(1-\frac{2}{x}\right)^8$ is 74.

Find the value of the positive constant k. [5]

5. w/20/11

In the expansion of $\left(2x^2 + \frac{a}{x}\right)^6$, the coefficients of x^6 and x^3 are equal.

(a) Find the value of the non-zero constant a. [4]

(b) Find the coefficient of x^6 in the expansion of $(1 - x^3)\left(2x^2 + \frac{a}{x}\right)^6$. [1]

6. w/20/12

The coefficient of x^3 in the expansion of $(1 + kx)(1 - 2x)^5$ is 20.

Find the value of the constant k. [4]

7. w/20/13

In the expansion of $(a + bx)^7$, where a and b are non-zero constants, the coefficients of x, x^2 and x^4 are the first, second and third terms respectively of a geometric progression.

Find the value of $\frac{a}{b}$. [5]

8. m/21/12

(a) Find the first three terms in the expansion, in ascending powers of x, of $(1+x)^5$. [1]

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(b) Find the first three terms in the expansion, in ascending powers of x, of $(1-2x)^6$. [2]

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(c) Hence find the coefficient of x^2 in the expansion of $(1+x)^5(1-2x)^6$. [2]

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9. S/21/11

(a) Find the first three terms in the expansion of $(3 - 2x)^5$ in ascending powers of x. [3]

(b) Hence find the coefficient of x^2 in the expansion of $(4 + x)^2(3 - 2x)^5$. [3]

10. s/21/12

The coefficient of x in the expansion of $\left(4x+\frac{10}{x}\right)^3$ is p. The coefficient of $\frac{1}{x}$ in the expansion of $\left(2x+\frac{k}{x^2}\right)^5$ is q.

Given that $p = 6q$, find the possible values of k. [5]

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11. s/21/13

(a) Write down the first four terms of the expansion, in ascending powers of x, of $(a-x)^6$. [2]

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(b) Given that the coefficient of x^2 in the expansion of $\left(1+\frac{2}{ax}\right)(a-x)^6$ is -20, find in exact form the possible values of the constant a. [5]

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12. w/21/11

(a) Expand $\left(1 - \frac{1}{2x}\right)^2$. [1]

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(b) Find the first four terms in the expansion, in ascending powers of x, of $(1 + 2x)^6$. [2]

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(c) Hence find the coefficient of x in the expansion of $\left(1 - \frac{1}{2x}\right)^2 (1 + 2x)^6$. [2]

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13. w/21/12

(a) It is given that in the expansion of $(4+2x)(2-ax)^5$, the coefficient of x^2 is -15.

Find the possible values of a. [4]

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(b) It is given instead that in the expansion of $(4+2x)(2-ax)^5$, the coefficient of x^2 is k. It is also given that there is only one value of a which leads to this value of k.

Find the values of k and a. [4]

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14. w/21/13

(a) Find the first three terms, in ascending powers of x, in the expansion of $(1 + ax)^6$. [1]

(b) Given that the coefficient of x^2 in the expansion of $(1 - 3x)(1 + ax)^6$ is -3, find the possible values of the constant a. [4]

15. m/22/12

Find the term independent of x in each of the following expansions.

(a) $\left(3x+\frac{2}{x^2}\right)^6$ [3]

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(b) $\left(3x+\frac{2}{x^2}\right)^6(1-x^3)$ [3]

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16. s/22/11

The coefficient of x^4 in the expansion of $\left(2x^2 + \frac{k^2}{x}\right)^5$ is a. The coefficient of x^2 in the expansion of $(2kx - 1)^4$ is b.

(a) Find a and b in terms of the constant k. [3]

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(b) Given that $a + b = 216$, find the possible values of k. [3]

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17. s/22/12

The coefficient of x^4 in the expansion of $(3+x)^5$ is equal to the coefficient of x^2 in the expansion of $\left(2x+\frac{a}{x}\right)^6$.

Find the value of the positive constant a. [4]

18. s/22/13

The coefficient of x^3 in the expansion of $\left(p+\frac{1}{p}x\right)^4$ is 144.

Find the possible values of the constant p. [4]

19. w/22/11

The coefficient of x^2 in the expansion of $\left(1+\frac{2}{p}x\right)^5+(1+px)^6$ is 70.

Find the possible values of the constant p. [6]

20. w/22/13

(a) Find the first three terms in ascending powers of x of the expansion of $(1 + 2x)^5$. [2]

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(b) Find the first three terms in ascending powers of x of the expansion of $(1 - 3x)^4$. [2]

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(c) Hence find the coefficient of x^2 in the expansion of $(1 + 2x)^5(1 - 3x)^4$. [2]

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TOPIC 8: Series

1. M20/12

A woman's basic salary for her first year with a particular company is \$30 000 and at the end of the year she also gets a bonus of \$600.

(a) For her first year, express her bonus as a percentage of her basic salary. [1]

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At the end of each complete year, the woman's basic salary will increase by 3% and her bonus will increase by \$100.

(b) Express the bonus she will be paid at the end of her 24th year as a percentage of the basic salary paid during that year. [5]

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2. S20/11

The sum of the first nine terms of an arithmetic progression is 117. The sum of the next four terms is 91.

Find the first term and the common difference of the progression. [4]

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3. S20/11

Each year the selling price of a diamond necklace increases by 5% of the price the year before. The selling price of the necklace in the year 2000 was $36 000.

(a) Write down an expression for the selling price of the necklace n years later and hence find the selling price in 2008. [3]

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(b) The company that makes the necklace only sells one each year. Find the total amount of money obtained in the ten-year period starting in the year 2000. [2]

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4. S20/12

The nth term of an arithmetic progression is $\frac{1}{2}(3n - 15)$.

Find the value of n for which the sum of the first n terms is 84. [5]

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5. S20/13

The first term of a progression is $\sin^2\theta$, where $0 < \theta < \frac{1}{2}\pi$. The second term of the progression is $\sin^2\theta\cos^2\theta$.

(a) Given that the progression is geometric, find the sum to infinity. [3]

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It is now given instead that the progression is arithmetic.

(b) **(i)** Find the common difference of the progression in terms of $\sin\theta$. [3]

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(ii) Find the sum of the first 16 terms when $\theta = \frac{1}{3}\pi$. [3]

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6. W20/11

A geometric progression has first term a, common ratio r and sum to infinity S. A second geometric progression has first term a, common ratio R and sum to infinity $2S$.

(a) Show that $r = 2R - 1$. [3]

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It is now given that the 3rd term of the first progression is equal to the 2nd term of the second progression.

(b) Express S in terms of a. [4]

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7. W20/12

The first, second and third terms of a geometric progression are $2p + 6$, $-2p$ and $p + 2$ respectively, where p is positive.

Find the sum to infinity of the progression. [5]

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8. W20/12

The sum, S_n, of the first n terms of an arithmetic progression is given by

$$S_n = n^2 + 4n.$$

The kth term in the progression is greater than 200.

Find the smallest possible value of k. [5]

9. W20/13

The first and second terms of an arithmetic progression are $\frac{1}{\cos^2\theta}$ and $-\frac{\tan^2\theta}{\cos^2\theta}$, respectively, where $0 < \theta < \frac{1}{2}\pi$.

(a) Show that the common difference is $-\frac{1}{\cos^4\theta}$. [4]

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(b) Find the exact value of the 13th term when $\theta = \frac{1}{6}\pi$. [3]

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10. M21/12

The first term of a progression is $\cos\theta$, where $0 < \theta < \frac{1}{2}\pi$.

(a) For the case where the progression is geometric, the sum to infinity is $\frac{1}{\cos\theta}$.

(i) Show that the second term is $\cos\theta\sin^2\theta$. [3]

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(ii) Find the sum of the first 12 terms when $\theta = \frac{1}{3}\pi$, giving your answer correct to 4 significant figures. [2]

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(b) For the case where the progression is arithmetic, the first two terms are again $\cos\theta$ and $\cos\theta\sin^2\theta$ respectively.

Find the 85th term when $\theta = \frac{1}{3}\pi$. [4]

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11. S21/11

The sum of the first 20 terms of an arithmetic progression is 405 and the sum of the first 40 terms is 1410.

Find the 60th term of the progression. [5]

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12. S21/11

The fifth, sixth and seventh terms of a geometric progression are $8k$, -12 and $2k$ respectively.

Given that k is negative, find the sum to infinity of the progression. [4]

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13. S21/12

The first, second and third terms of an arithmetic progression are a, $\frac{3}{2}a$ and b respectively, where a and b are positive constants. The first, second and third terms of a geometric progression are a, 18 and $b + 3$ respectively.

(a) Find the values of a and b. [5]

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(b) Find the sum of the first 20 terms of the arithmetic progression. [3]

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14. S21/13

(a) A geometric progression is such that the second term is equal to 24% of the sum to infinity.

Find the possible values of the common ratio. [3]

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(b) An arithmetic progression P has first term a and common difference d. An arithmetic progression Q has first term $2(a+1)$ and common difference $(d+1)$. It is given that

$$\frac{\text{5th term of } P}{\text{12th term of } Q} = \frac{1}{3} \quad \text{and} \quad \frac{\text{Sum of first 5 terms of } P}{\text{Sum of first 5 terms of } Q} = \frac{2}{3}.$$

Find the value of a and the value of d. [6]

15. W21/11

The first term of an arithmetic progression is a and the common difference is -4. The first term of a geometric progression is $5a$ and the common ratio is $-\frac{1}{4}$. The sum to infinity of the geometric progression is equal to the sum of the first eight terms of the arithmetic progression.

(a) Find the value of a. [4]

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The kth term of the arithmetic progression is zero.

(b) Find the value of k. [2]

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TOPIC 9: Differentiation

1. S20/11

The equation of a curve is $y = (3 - 2x)^3 + 24x$.

(a) Find expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

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2. M20/12

A curve has equation $y = x^2 - 2x - 3$. A point is moving along the curve in such a way that at P the y-coordinate is increasing at 4 units per second and the x-coordinate is increasing at 6 units per second.

Find the x-coordinate of P. [4]

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3. S20/12

The equation of a curve is $y = 54x - (2x - 7)^3$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

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4. W20/11

The equation of a curve is $y = 2 + \sqrt{25 - x^2}$.

Find the coordinates of the point on the curve at which the gradient is $\frac{4}{3}$. [5]

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5. W20/13

The equation of a curve is $y = 2x + 1 + \frac{1}{2x + 1}$ for $x > -\frac{1}{2}$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

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6. W20/13 (Only a)

A curve has equation $y = \frac{1}{k}x^{\frac{1}{2}} + x^{-\frac{1}{2}} + \frac{1}{k^2}$ where $x > 0$ and k is a positive constant.

(a) It is given that when $x = \frac{1}{4}$, the gradient of the curve is 3.

Find the value of k. [4]

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TOPIC 10: Further Differentiation

1. S20/12

A weather balloon in the shape of a sphere is being inflated by a pump. The volume of the balloon is increasing at a constant rate of 600 cm^3 per second. The balloon was empty at the start of pumping.

(a) Find the radius of the balloon after 30 seconds. [2]

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(b) Find the rate of increase of the radius after 30 seconds. [3]

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A point P is moving along a curve in such a way that the x-coordinate of P is increasing at a constant rate of 2 units per minute. The equation of the curve is $y = (5x - 1)^{\frac{1}{2}}$.

(a) Find the rate at which the y-coordinate is increasing when $x = 1$. [4]

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(b) Find the value of x when the y-coordinate is increasing at $\frac{5}{8}$ units per minute. [3]

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3. W20/22

Air is being pumped into a balloon in the shape of a sphere so that its volume is increasing at a constant rate of $50\,\text{cm}^3\,\text{s}^{-1}$.

Find the rate at which the radius of the balloon is increasing when the radius is 10 cm. [3]

4. W20/12

The point (4, 7) lies on the curve $y = \mathrm{f}(x)$ and it is given that $\mathrm{f}'(x) = 6x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}$.

(a) A point moves along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.12 units per second.

Find the rate of increase of the y-coordinate when $x = 4$. [3]

5. M21/12

A curve is such that $\frac{dy}{dx} = \frac{6}{(3x-2)^3}$ and $A\ (1,\ -3)$ lies on the curve. A point is moving along the curve and at A the y-coordinate of the point is increasing at 3 units per second.

(a) Find the rate of increase at A of the x-coordinate of the point. [3]

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(b) Find the equation of the curve. [4]

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6. S21/11

The equation of a curve is such that $\frac{dy}{dx} = \frac{3}{x^4} + 32x^3$. It is given that the curve passes through the point $(\frac{1}{2},\ 4)$.

Find the equation of the curve. [4]

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7. S21/11

The equation of a curve is $y = 2\sqrt{3x+4} - x$.

(a) Find the equation of the normal to the curve at the point (4, 4), giving your answer in the form $y = mx + c$. [5]

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(b) Find the coordinates of the stationary point. [3]

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(c) Determine the nature of the stationary point. [2]

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(d) Find the exact area of the region bounded by the curve, the x-axis and the lines $x = 0$ and $x = 4$. [4]

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TOPIC 11: Increasing decreasing function

1. M20/12

The function f is defined by $f(x) = \frac{1}{3x+2} + x^2$ for $x < -1$.

Determine whether f is an increasing function, a decreasing function or neither. [3]

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2. S20/11

The equation of a curve is $y = (3 - 2x)^3 + 24x$.

(a) Find expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

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Accelerating the academic skills

(b) Find the coordinates of each of the stationary points on the curve. [3]

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(c) Determine the nature of each stationary point. [2]

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3. S21/13

The function f is defined by $f(x) = \frac{1}{3}(2x - 1)^{\frac{3}{2}} - 2x$ for $\frac{1}{2} < x < a$. It is given that f is a decreasing function.

Find the maximum possible value of the constant a. [4]

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TOPIC 12: Max-Min of a function

1. M20/12

The gradient of a curve at the point (x, y) is given by $\frac{dy}{dx} = 2(x+3)^{\frac{1}{2}} - x$. The curve has a stationary point at $(a, 14)$, where a is a positive constant.

(a) Find the value of a. [3]

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(b) Determine the nature of the stationary point. [3]

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(c) Find the equation of the curve. [4]

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2. S20/12

The equation of a curve is $y = 54x - (2x - 7)^3$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

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(b) Find the coordinates of each of the stationary points on the curve. [3]

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(c) Determine the nature of each of the stationary points. [2]

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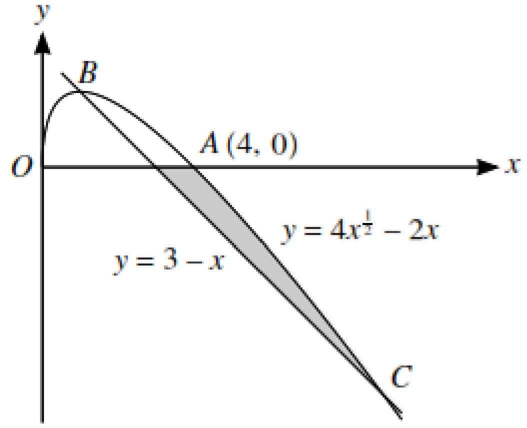
3. W20/11



The diagram shows a curve with equation $y = 4x^{\frac{1}{2}} - 2x$ for $x \geqslant 0$, and a straight line with equation $y = 3 - x$. The curve crosses the x-axis at $A\ (4, 0)$ and crosses the straight line at B and C.

(a) Find, by calculation, the x-coordinates of B and C. [4]

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(b) Show that B is a stationary point on the curve. [2]

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4. W20/13

The equation of a curve is $y = 2x + 1 + \frac{1}{2x+1}$ for $x > -\frac{1}{2}$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

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(b) Find the coordinates of the stationary point and determine the nature of the stationary point. [5]

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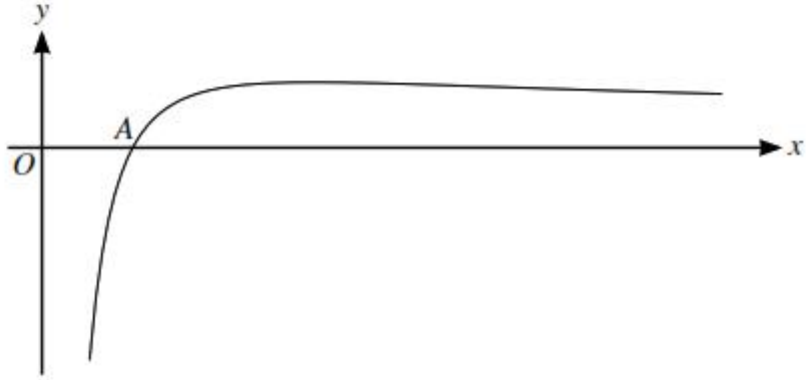
5. M21/12 (Only c)



The diagram shows the curve with equation $y = 9(x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}})$. The curve crosses the x-axis at the point A.

(c) Find the x-coordinate of the maximum point of the curve. [2]

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6. S21/12

The gradient of a curve is given by $\frac{dy}{dx} = 6(3x - 5)^3 - kx^2$, where k is a constant. The curve has a stationary point at $(2, -3.5)$.

(a) Find the value of k. [2]

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(b) Find the equation of the curve. [4]

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(c) Find $\frac{d^2y}{dx^2}$. [2]

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(d) Determine the nature of the stationary point at $(2, -3.5)$. [2]

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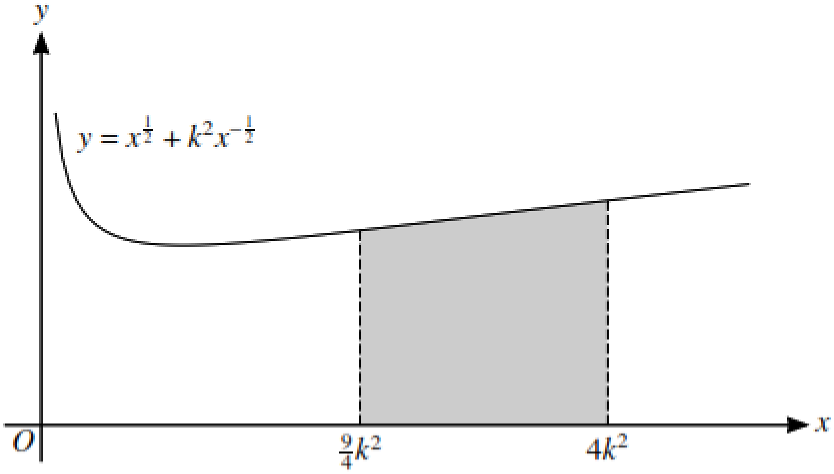
7. S21/13



The diagram shows part of the curve with equation $y = x^{\frac{1}{2}} + k^2x^{-\frac{1}{2}}$, where k is a positive constant.

(a) Find the coordinates of the minimum point of the curve, giving your answer in terms of k. [4]

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The tangent at the point on the curve where $x = 4k^2$ intersects the y-axis at P.

(b) Find the y-coordinate of P in terms of k. [4]

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The shaded region is bounded by the curve, the x-axis and the lines $x = \frac{9}{4}k^2$ and $x = 4k^2$.

(c) Find the area of the shaded region in terms of k. [3]

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8. W21/11

A curve has equation $y = f(x)$, and it is given that $f'(x) = 2x^2 - 7 - \frac{4}{x^2}$.

(a) Given that $f(1) = -\frac{1}{3}$, find $f(x)$. [4]

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(b) Find the coordinates of the stationary points on the curve. [5]

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(c) Find $f''(x)$. [1]

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(d) Hence, or otherwise, determine the nature of each of the stationary points. [2]

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TOPIC 13: Integration

1. S20/13

The equation of a curve is such that $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$. It is given that the point (4, 7) lies on the curve.

Find the equation of the curve. [4]

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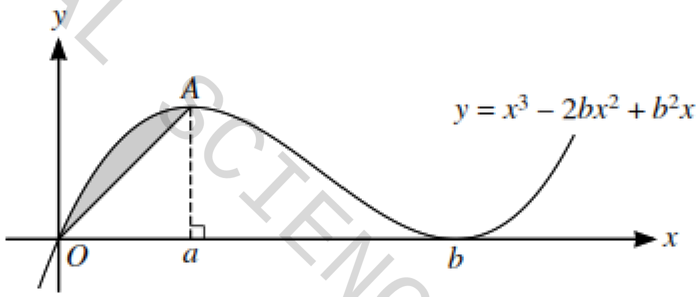
2. S20/13



The diagram shows part of the curve with equation $y = x^3 - 2bx^2 + b^2x$ and the line OA, where A is the maximum point on the curve. The x-coordinate of A is a and the curve has a minimum point at $(b, 0)$, where a and b are positive constants.

(a) Show that $b = 3a$. [4]

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(b) Show that the area of the shaded region between the line and the curve is ka^4, where k is a fraction to be found. [7]

3. W20/11

The equation of a curve is such that $\frac{dy}{dx} = \frac{1}{(x-3)^2} + x$. It is given that the curve passes through the point (2, 7).

Find the equation of the curve. [4]

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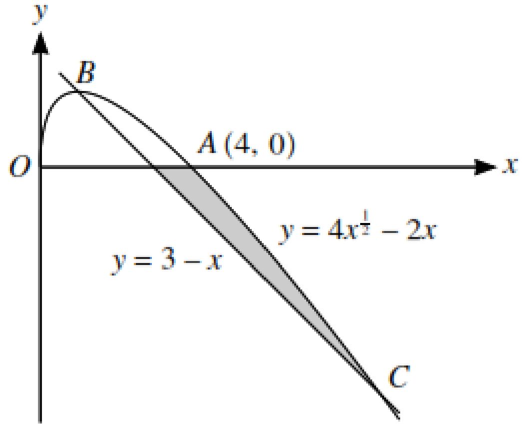
4. W20/11



The diagram shows a curve with equation $y = 4x^{\frac{1}{2}} - 2x$ for $x \geqslant 0$, and a straight line with equation $y = 3 - x$. The curve crosses the x-axis at $A\,(4, 0)$ and crosses the straight line at B and C.

(a) Find, by calculation, the x-coordinates of B and C. [4]

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(b) Show that B is a stationary point on the curve. [2]

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(c) Find the area of the shaded region. [6]

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5. W20/12

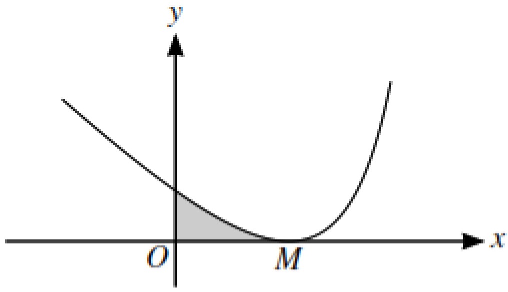
The point (4, 7) lies on the curve $y = f(x)$ and it is given that $f'(x) = 6x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}$.

(a) A point moves along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.12 units per second.

Find the rate of increase of the y-coordinate when $x = 4$. [3]

(b) Find the equation of the curve. [4]

6. W20/12



The diagram shows part of the curve $y = \frac{2}{(3-2x)^2} - x$ and its minimum point M, which lies on the x-axis.

(a) Find expressions for $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\int y\,dx$. [6]

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(b) Find, by calculation, the x-coordinate of M. [2]

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(c) Find the area of the shaded region bounded by the curve and the coordinate axes. [2]

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7. W20/13

The function f is defined by $f(x) = \dfrac{2}{(x+2)^2}$ for $x > -2$.

(a) Find $\displaystyle\int_1^{\infty} f(x)\,dx$. [3]

(b) The equation of a curve is such that $\dfrac{dy}{dx} = f(x)$. It is given that the point $(-1, -1)$ lies on the curve.

Find the equation of the curve. [2]

8. W20/13 (Only b)

(b) It is given instead that $\int_{\frac{1}{4}k^2}^{k^2} \left(\frac{1}{k}x^{\frac{1}{2}} + x^{-\frac{1}{2}} + \frac{1}{k^2}\right) dx = \frac{13}{12}$.

Find the value of k. [5]

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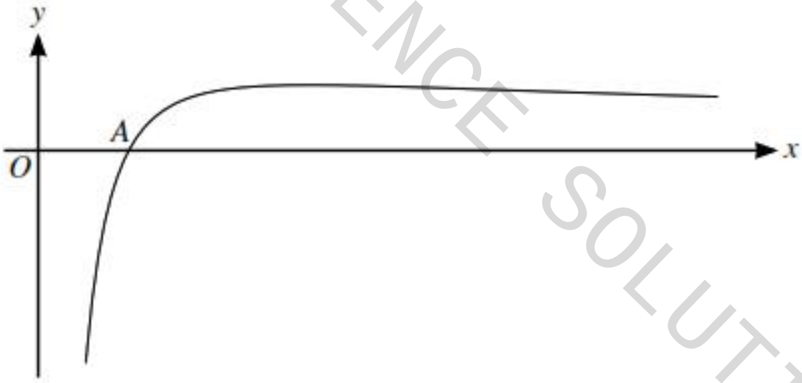
9. M21/12 (only d part)



The diagram shows the curve with equation $y = 9(x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}})$. The curve crosses the x-axis at the point A.

(d) Find the area of the region bounded by the curve, the x-axis and the line $x = 9$. [4]

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10. S21/13

A curve with equation $y = f(x)$ is such that $f'(x) = 6x^2 - \frac{8}{x^2}$. It is given that the curve passes through the point (2, 7).

Find $f(x)$. [3]

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11. W21/11 (Only part a)

A curve has equation $y = f(x)$, and it is given that $f'(x) = 2x^2 - 7 - \frac{4}{x^2}$.

(a) Given that $f(1) = -\frac{1}{3}$, find $f(x)$. [4]

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12. W21/11 (Only part a)

(a) Find $\int_1^{\infty} \frac{1}{(3x-2)^{\frac{3}{2}}}\,dx$. [4]

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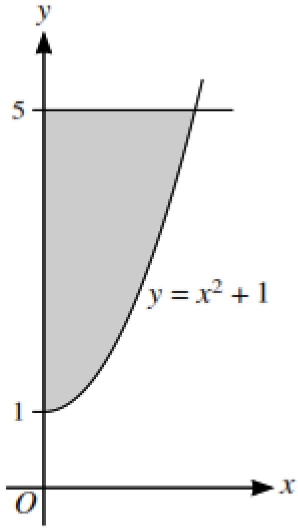
1. M20/12



The diagram shows part of the curve with equation $y = x^2 + 1$. The shaded region enclosed by the curve, the y-axis and the line $y = 5$ is rotated through 360° about the **y-axis**.

Find the volume obtained. [4]

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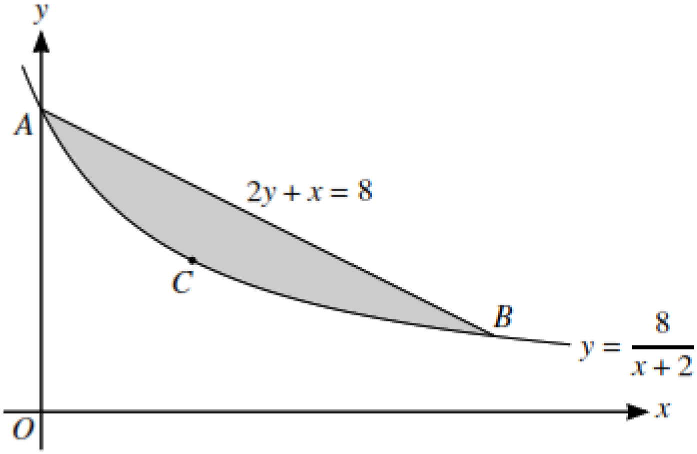
2. S20/11



The diagram shows part of the curve $y = \dfrac{8}{x+2}$ and the line $2y + x = 8$, intersecting at points A and B. The point C lies on the curve and the tangent to the curve at C is parallel to AB.

(a) Find, by calculation, the coordinates of A, B and C. [6]

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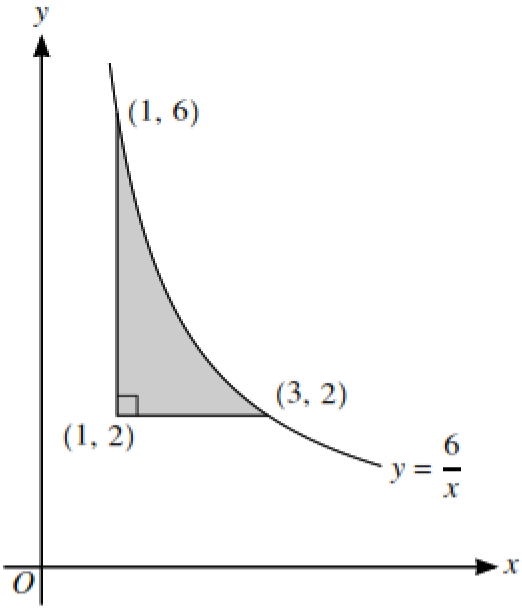
(b) Find the volume generated when the shaded region, bounded by the curve and the line, is rotated through 360° about the x-axis. [6]

3. S20/12



The diagram shows part of the curve $y = \frac{6}{x}$. The points $(1, 6)$ and $(3, 2)$ lie on the curve. The shaded region is bounded by the curve and the lines $y = 2$ and $x = 1$.

(a) Find the volume generated when the shaded region is rotated through 360° about the **y-axis**. [5]

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(b) The tangent to the curve at a point X is parallel to the line $y + 2x = 0$. Show that X lies on the line $y = 2x$. [3]

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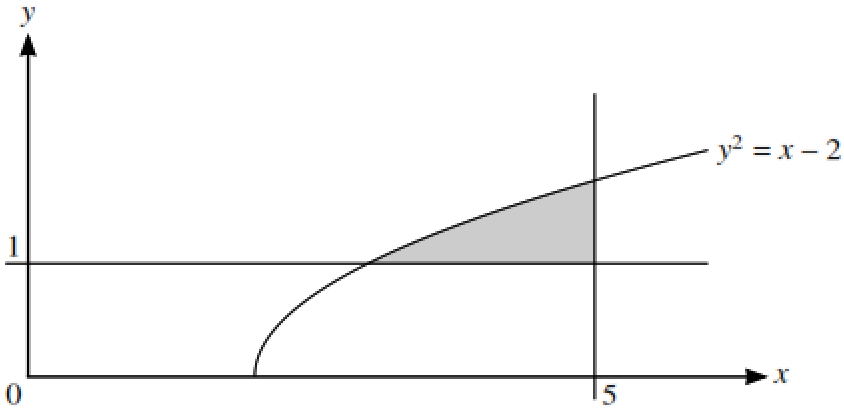
4. S21/12



The diagram shows part of the curve with equation $y^2 = x - 2$ and the lines $x = 5$ and $y = 1$. The shaded region enclosed by the curve and the lines is rotated through 360° about the x-axis.

Find the volume obtained. [6]

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5. W21/11

(a) Find $\int_1^{\infty} \frac{1}{(3x-2)^{\frac{3}{2}}}\,dx$. [4]

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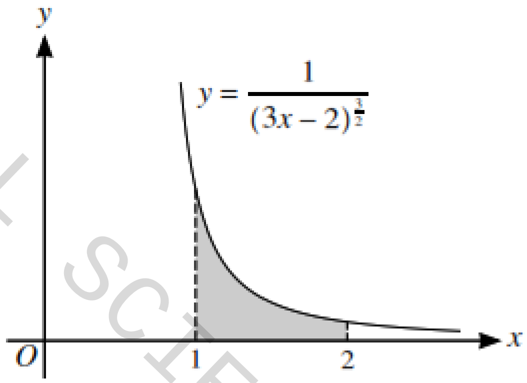

The diagram shows the curve with equation $y = \frac{1}{(3x-2)^{\frac{3}{2}}}$. The shaded region is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$. The shaded region is rotated through 360° about the x-axis.

(b) Find the volume of revolution. [4]

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The normal to the curve at the point $(1, 1)$ crosses the y-axis at the point A.

(c) Find the y-coordinate of A. [4]

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